APPENDIX

to

Geobarometry from host-inclusion systems: the role of elastic relaxation

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Summary

The purpose of this Appendix is to show that the solution for elastic relaxation in host-inclusion systems found by Zhang (1988) is a special case of the more general solution provided in the current paper.

Introduction

In the main body of the paper we discussed the calculation of the residual pressure on an inclusion surrounded by a host phase, which arises from changes in pressure and temperature. The final inclusion pressure $P_{I,end}$ is comprised of two parts:

$$P_{I,end} = P_I^* + \Delta P_{I,relax}$$

The part P_I^* arises solely from constraining the volume change of the inclusion phase to that of the host phase, without any mutual relaxation. It can be calculated from the EoS of the two phases. By use of the concept of the isomeke, we showed that the mutual elastic relaxation of the host and inclusion that gives rise to $\Delta P_{I,relax}$ comes from a fractional volume change of the inclusion of $\varepsilon_H K_{21}$. The ε_H is the fractional volume change (i.e. the volume strain) of the host due to the isothermal pressure reduction from the pressure P_{foot} on the isomeke to $P_{H,end}$, and the parameter K_{21} is an elastic interaction parameter whose value is dependent on the elastic properties of both the host and inclusion:

$$K_{21} = \frac{K_I - K_H}{K_I + \frac{4}{3} G_H}$$

By using linear elasticity and assuming that the elastic parameters are invariant with pressure, Zhang (1998) calculated that the fractional volume change of the inclusion due to relaxation is:

$$\frac{\Delta \mathrm{V_{I,relax}}}{\mathrm{V_0}} = \frac{3\left(P_{I,end} - P_{H,end}\right)}{4G_H} \qquad \text{(A1)}$$

This, from the definition of bulk modulus as $K = -V\frac{\partial P}{\partial V}$ corresponds to a pressure change of:

$$\Delta P_{I,relax} = \frac{-3K_I\left(P_{I,end} - P_{H,end}\right)}{4G_H} \quad \text{(A2)}$$

In the main body of the paper we noted that this expression for relaxation provides a good estimate of the relaxation even when the elasticity of the host and inclusion is not linear, provided that:

1. The host is relatively stiff with a high shear modulus G_H.
2. That the difference between P_{foot} and $P_{I,thermo}$ is small.

Here we show algebraically that the Zhang (1998) result is a special case of the more general solution that we have derived.

Derivation of the Zhang solution: recovery to P=0

To derive the Zhang (1998) solution, we now impose the conditions of linear elasticity upon our general result. For linear elasticity, the rate of change of volume with pressure is constant, so we can define the elastic response of a material as $\frac{\Delta \mathrm{V}}{\mathrm{V}_0} = -\frac{\Delta P}{K}$, with the quantity K also a constant. Note that this requires the use of a reference volume V_0. Therefore with 'linear elasticity' the value of K is only the true bulk modulus in the immediate vicinity of V_0. We choose the foot of the isomeke, P_{foot} as our reference point, where $V_{H,foot} = V_{I,foot} = V_0$. In order to make the derivation clear, we first derive an expression for the volume relaxation strain for the case when the final external pressure $P_{H,end}$ =0.

The final volume strain of the host alone is then, for linear elasticity:

$$\varepsilon_H = \frac{V_{H,end} - V_0}{V_0} = \frac{P_{foot}}{K_H} \qquad \text{(A3)}$$

The volume relaxation strain is then, from our result:

$$\frac{\Delta V_{I,relax}}{V_0} = -\varepsilon_H K_{21} = -\frac{K_{21}}{K_H} P_{foot} \qquad \text{(A4)}$$

This defines the volume relaxation in terms of the original pressure of the system on the isomeke. We now find an expression for P_{foot} so that we can express the volume relaxation in terms of the final inclusion pressure $P_{I,end}$. The final pressure of the inclusion after relaxation is defined from the volume strain of our solution as:

$$\varepsilon_H (1 - K_{21}) = -\frac{\left(P_{I,end} - P_{foot}\right)}{K_I} \qquad \text{(A5)}$$

Thus:

$$\frac{P_{foot}}{K_H}(1 - K_{21}) = \frac{\left(P_{foot} - P_{I,end}\right)}{K_I} \qquad \text{(A6)}$$

Re-arrangement yields: $P_{foot} = P_{I,end}\left[1 - \frac{K_I}{K_H}(1 - K_{21})\right]^{-1}$ (A7)

Substitution of this expression for P_{foot} in equation (A4) yields:

$$\frac{\Delta V_{I,relax}}{V_0} = -\frac{K_{21}}{K_H} P_{I,end} \left[1 - \frac{K_I}{K_H}(1 - K_{21}) \right]^{-1} \qquad \text{(A8)}$$

By taking $\frac{K_{21}}{K_H}$ inside:

$$\frac{\Delta V_{I,relax}}{V_0} = -P_{I,end} \left[\frac{K_H}{K_{21}} - \frac{K_I}{K_{21}} + K_I \right]^{-1}$$

$$\frac{\Delta V_{I,relax}}{V_0} = -P_{I,end} \left[\frac{(K_H - K_I)}{K_{21}} + K_I \right]^{-1}$$

$$\frac{\Delta V_{I,relax}}{V_0} = -P_{I,end} \left[-(K_I + \frac{4}{3} G_H) + K_I \right]^{-1}$$

$$\frac{\Delta V_{I,relax}}{V_0} = -P_{I,end} \left[-\frac{4}{3} G_H \right]^{-1}$$

$$\frac{\Delta V_{I,relax}}{V_0} = \frac{3P_{I,end}}{4G_H} \qquad \text{(A9)}$$

This is the result of Zhang (1998) and others when the final external pressure is zero. We therefore see that the result of Zhang (1998) does not arise from the assumption of linear elasticity over the entire decompression from entrapment conditions, but can be derived instead from the assumption of linear elasticity over the decompression of the host from P_{foot} to $P_{H,end}$. This explains why it can be a good approximation to the correct approach.

Derivation of the Zhang solution: recovery to any P

The derivation for the case when the final external pressure $P_{H,end} \neq 0$ follows exactly the same steps, but the algebra to reduce the expressions is a little more painful.

The final volume strain of the host alone is now:

$$\varepsilon_H = \frac{V_{H,end} - V_0}{V_0} = \frac{\left(P_{foot} - P_{H,end}\right)}{K_H} \qquad \text{(A10)}$$

And the volume relaxation strain is therefore:

$$\frac{\Delta V_{I,relax}}{V_0} = -\varepsilon_H K_{21} = -\frac{K_{21}}{K_H}\left(P_{foot} - P_{H,end}\right) \qquad \text{(A11)}$$

Equation (A5) remains the same, but substitution for the host strain produces instead of (A6):

$$\frac{\left(P_{foot} - P_{H,end}\right)}{K_H}(1 - K_{21}) = -\frac{\left(P_{I,end} - P_{foot}\right)}{K_I}$$

Solving this for P_{foot} leads to:

$$P_{foot} = \left[P_{I,end} + P_{H,end}(1 - K_{21})\frac{K_I}{K_H}\right]\left[1 - \frac{K_I}{K_H}(1 - K_{21})\right]^{-1}$$

Substitution of this expression for P_{foot} in equation (A11) yields:

$$\frac{\Delta V_{I,relax}}{V_0} = \frac{K_{21}}{K_H} P_{H,end} - \frac{K_{21}}{K_H}\left[P_{I,end} + P_{H,end}(1-K_{21})\frac{K_I}{K_H}\right]\left[1-\frac{K_I}{K_H}(1-K_{21})\right]^{-1}$$

With the manipulation from the first derivation, this immediately reduces to:

$$\frac{\Delta V_{I,relax}}{V_0} = \frac{K_{21}}{K_H} P_{H,end} + \frac{3}{4G_H}\left[P_{I,end} + P_{H,end}(1-K_{21})\frac{K_I}{K_H}\right]$$

Re-arranging yields:

$$\frac{\Delta V_{I,relax}}{V_0} = \frac{3P_{I,end}}{4G_H} + P_{H,end}\left[\frac{K_{21}}{K_H} + \frac{3}{4G_H}(1-K_{21})\frac{K_I}{K_H}\right]$$

$$\frac{\Delta V_{I,relax}}{V_0} = \frac{3P_{I,end}}{4G_H} + \frac{3P_{H,end}}{4G_H}\left[\frac{4G_H K_{21}}{3K_H} + (1-K_{21})\frac{K_I}{K_H}\right]$$

The term in the square bracket reduces to simply the value -1, so the final expression becomes:

$$\frac{\Delta V_{I,relax}}{V_0} = \frac{3\left(P_{I,end} - P_{H,end}\right)}{4G_H}$$

Thus the result derived from linear elasticity is a special case of the more general solution presented in the main body of this paper.

Zhang, Y. (1998) Mechanical and phase equilibria in inclusion–host systems. Earth and Planetary Science Letters, 157, 209-222.