

SUPPLEMENTARY MATERIAL

A. DERIVATION OF EQUATION (3)

In equation (1), C_i^{std} and $I_{i,net,qnt}^{std}$ are constants, and G and $I_{i,net,qnt}^{unk}$ are variables. As G is a correction factor for the matrices, G can be approximated as a constant for certain mineral phases as follows:

$$C_i^{unk,phase} = \alpha_i^{phase} I_{i,net,qnt}^{unk,phase}, (A. 1)$$

where,

$$\alpha_i^{phase} = \frac{C_i^{std} G}{I_{i,net,qnt}^{std}} \approx Const,$$

and α is a correction factor that depends on the mineral phase and element in question.

The use of equation (A. 1) to quantify X-ray maps requires conversion of mapped peak X-ray intensities to net X-ray intensities. This conversion is not straightforward because background X-ray intensities are generally not mapped. On the other hand, referential spot analysis measures background X-ray intensities. Thus, we first convert the mapped peak X-ray intensities to peak X-ray intensities under the conditions of referential spot analysis, and then correct the background.

Conversion of mapped peak X-ray intensities to peak X-ray intensities under quantitative analytical conditions is achieved by the following equation:

$$I_{i,pk,qnt}^{unk,phase} = \beta_i I_{i,pk,map}^{unk,phase}, (S. 2)$$

where, β is a correction factor that depends on the element of interest, and β is independent of matrices because the ratio of peak X-ray intensity among different analytical conditions is determined from the probe current and probe diameter.

Background corrections for the calibrated peak X-ray intensities are achieved by subtracting the average of the quantified background X-ray intensities from the corresponding element within the corresponding phase, γ . The value of γ is approximately constant regardless of the compositions of the phases, as discussed in the main text, and it is calculated as follows:

$$\gamma_i^{phase} = \frac{1}{N^{phase}} \sum_{j=1}^{N^{phase}} I_{i,j,bg,qnt}^{unk,phase} \approx Const., (S. 3)$$

where, N is the number of quantitative analyses of a particular phase.

Equations (A. 1), (A. 2), and (A. 3) provide the following:

$$C_i^{unk,phase} = \alpha_i^{phase} (\beta_i I_{i,pk,map}^{unk,phase} - \gamma). (S. 4)$$

In practice, we quantify the composition of pixels, some of which sample multiple phases. The composition of a pixel is equal to the mean of the compositions of the phases within the pixel, weighted by the abundance of the phases (Fig. 1). Thus, we sum the product of equation (A. 4) and the abundances of the corresponding phases in the pixels in equation (3) as follows:

$$C_i^{unk} = \sum_{phase} X^{phase} \alpha_i^{phase} (\beta_i I_{i,pk,map}^{unk,phase} - \gamma_i^{phase}), (3)$$

where, X is the abundance of the phases in a pixel, equivalent to the membership grade from the clustering analysis, whose sum is 1.

B. LOCAL BULK ROCK COMPOSITIONS IN TABLE 1

A simple approach to estimate local bulk compositions and their errors from the gray-shaded areas shown in Figure 10 is to average the compositions of all pixels in each area weighted by the density of the pixels and propagate the analytical errors resulting from referential spot analysis and X-ray mapping. Using this approach, standard errors become negligibly small because of the large number of data points (i.e.,

pixels). Yet, the error must arise due to inhomogeneity in selected areas. To consider the effect of inhomogeneity, we employed the bootstrap method (Davidson & Hinkley, 1997).

First, local bulk compositions are estimated for each 1-pixel \times 500-pixel column parallel to the layering structure of the sample in a target area. During this step, local bulk compositions are estimated by weighted means that weight pixels by their densities. Second, the local bulk compositions of the columns are randomly re-sampled with a replacement of 535 and 110 times for the garnet-rich matrix and pyroxene-rich layer, respectively. The number of re-sampling is equal to the number of columns (i.e. width of the pixels) in the gray shaded areas (Fig. 10). The re-sampled local bulk composition is estimated by the average weighted with the average density of the columns. Third, this random re-sampling and local bulk estimation was repeated 2000 times. The iterative re-sampling considers inhomogeneity by creating datasets with various chemical compositions such as those that re-sampled each column of a layer nearly equally, those that re-sampled garnet-rich columns selectively from a layer, etc. Finally, an average and a standard deviation are estimated from 2000 re-sampled local bulk compositions. Table 1 shows the former as the local bulk compositions and a double of the standard deviation as the error of the local bulk compositions.