Abstract:

Olivine, the most abundant mineral in the upper mantle, plays a key role in controlling the thermodynamic properties in the Earth’s and planetary interiors, like the temperature distribution along the adiabatic geotherm. In this study, we conducted simultaneously high-pressure ($P$) and high-temperature ($T$) Raman measurements on a San Carlos olivine sample, in an externally-heated diamond anvil cell (DAC). The intrinsic anharmonic parameters, $a_i$, are calculated as
functions of both pressure and temperature, and the isochoric ($C_V$) and isobaric ($C_P$) heat capacities are computed at various $P-T$ conditions with the anharmonic correction, which are important for the heat transport properties. The harmonic heat capacities are $C_V = 807.7$ J/kg/K and $C_P = 815.4$ J/kg/K at the ambient condition, with anharmonic contribution of $\Delta C = 7.9$ J/kg/K. Relative to the previous vibrational measurements conducted at high-$P$ or high-$T$ conditions, this simultaneous high-$P,T$ experiment indicates that the anharmonic contribution to heat capacities is overestimated if the anharmonic parameters ($a_i$) are treated as constants like before. The pressure effect is marginal on the intrinsic anharmonic contribution to thermodynamic properties, whereas it is much more significant on the external anharmonicity (thermal expansivity). The pressure dependence of $C_P$ ($dC_P/dP$, in J/kg/K/GPa) increases from -3.14 at 300 K to -1.94 at 700 K, and then decreases smoothly to -5.03 at 1800 K. Combining the derived high-$P,T$ capacity with reliable $P-V-T$ equation of state (EOS) for olivine, we further modeled the thermodynamic Grüneisen parameter, $\gamma(P,T)$. The Grüneisen parameter is important for the connection between isothermal and adiabatic compressions of minerals, which can be decreased by approximately 5% with the anharmonic correction at high temperatures. The modeled adiabatic bulk modulus and bulk sound velocity can be expressed as: $K_S(T,P)$ (GPa) = 127.5(1) + 4.32(5)$P - 0.018(1)(T-300)$ and $V_\Phi(T,P)$ (km/s) = 6.22(2) + 0.069(3)$P - [3.74(15)-0.075(13)\cdot P] \cdot 10^{-4}\cdot (T-300)$. The adiabatic temperature gradient, $dT_\Phi/dP$, almost independent of pressure, is equal to 13.40(16) and 12.35(16) K/GPa in the harmonic and anharmonic models, respectively. This study provides a useful example for modeling the radial temperature distribution in adiabatic planetary mantles.
Keywords: Olivine; simultaneously high-\(P,T\) Raman spectra; intrinsic anharmonic parameter; heat capacity; thermodynamic Grüneisen parameter; adiabatic temperature profile

1. Introduction

Olivine, with a typical composition approximately \((\text{Mg}_{0.9}\text{Fe}_{0.1})_2\text{SiO}_4\) (e.g. Agee 1998), is the most abundant mineral in the upper mantle (60 ~ 70 vol.%) above the 410-km seismic discontinuity. It is also an important component in other terrestrial planets, like Mercury and Mars, as well as some asteroids (Sanchez et al. 2014; Namur et al. 2016; Khan et al. 2018). Its thermodynamic properties play a dominant role in the physics and chemistry of the Earth and planetary interiors. For example, temperature is fundamental for modeling the dynamics of the mantle, and the adiabatic \(T\)-gradient in olivine is the major factor of controlling the geotherm in the upper mantle considering its abundance (Katsura et al. 2010; Katsura 2022). Among the physical parameters related to adiabatic procedures, the thermodynamic Grüneisen parameter (\(\gamma\)) is key for calculating thermal pressures at high temperatures (Brown and McQueen 1986; Huang et al. 2010, 2018), and heat capacities are also important for connecting the thermal transport properties of thermal conductivity and diffusivity (Zhang et al. 2019, 2023).

In order to derive these thermodynamic properties, we need both precise \(P-V-T\) EOS and heat capacities at high-\(P,T\) conditions (Liebfried and Ludwig 1961; Brown and Shankland 1981; Anderson et al. 1991). Quite a few studies have been reported on \(P-V-T\) Equations of state (EOSs) for mantle-composition olivines (e.g., Liu and Li 2006; Dorogokupets et al. 2015; Angel et al. 2018; Katsura 2022), among which Angel et al. (2018) gave a thorough review of the existing datasets. Furthermore, vibrational spectra are widely adopted to model high-\(T\) heat capacities for minerals (e.g. Kieffer 1979, 1980; Hofmeister 1987; Ross and Navrotsky 1987,

On the other hand, harmonic (or quasi-harmonic) approximation for thermodynamic properties would deviate from reality at high temperatures, since the atomic vibrations inside the lattice structure cannot be simplified as harmonic oscillators any more under such conditions. Then, anharmonicities must be taken into consideration (Stacey and Isaak 2003; Anderson 2007; Stacey and Hodgkinson 2019). There are generally two types of anharmonicities: (1) ‘external anharmonicity’ arising from asymmetrical bonding (potential) (e.g. Richet et al. 1977), which is associated with the thermal expansion behaviors of minerals; (2) ‘intrinsic anharmonicity’ describing deviation of the heat capacity $C_V$ from the Dulong-Petit limit at high temperatures (Anderson et al. 1991). For the case of intrinsic anharmonicity, an anharmonic mode parameter, $a_i$, is introduced to evaluate the magnitude of the anharmonic correction, which is defined as the variation of a vibrational frequency ($v_i$) with temperature at constant volume. The intrinsic anharmonicity has profound impact on modelling the thermodynamic properties of minerals at high-$P,T$ conditions in the Earth’s interior, such as the heat capacities (Kieffer 1979; Cynn et al. 1996a, 1996b; Holland and Powell 1998; Wu 2015), equations of state (Oganov and Dorogokupets 2003, 2004; Dorogokupets and Oganov, 2004), as well as equilibrium isotopic fractionations (Polyakov and Kharlashina 1994; Polyakov 1998; Zhu et al. 2019). Previous studies consistently indicated that the reported $a_i$ parameters are negative for many silicates, such as olivine (Gillet et al. 1991, 1997; Liu et al. 2021), clinohumite (Liu et al. 2019b) and garnets in a cubic structure (Gillet et al. 1992), supporting that the anharmonic contribution is positive to the heat capacities.

In this study, we conducted in situ Raman measurements on a San Carlos olivine sample at simultaneously high-$P$ and high-$T$ conditions, taking advantage of an externally-heated diamond
anvil cell. It is noted that previous vibrational spectroscopic measurements on olivine samples were obtained at ambient conditions (Burns and Huggins 1972; Guyot et al. 1986; Kolesov and Tanskaya 1996; Gaisler and Kolesov 2007; Mouri and Enami 2008; Ishibashi et al. 2012; Breitenfeld et al. 2018), or at only high-temperature \((P = 0 \text{ GPa})\) or high-pressure \((T = 300 \text{ K})\) conditions (Chopelas 1990; Gillet et al. 1991, 1997; Liu and Mernagh 1993; Wang et al. 1993; Hushur et al. 2009; Manghnani et al. 2013; Yang et al. 2015; Santamaria-Perez et al. 2016; Liu et al. 2019a, 2021). Based on this experiment, we can have a systematical check on the pressure effect on thermodynamic properties at high-temperature conditions (including the heat capacities, Grüneisen parameter, and adiabatic \(P\)-\(T\) profile). Moreover, the anharmonic parameters \((a_i)\) are traditionally simplified as constants, independent of temperature or pressure (e.g., Gillet et al. 1991, 1997; Liu et al. 2021). Taking advantage of this simultaneously high-\(P\),\(T\) Raman measurement, we will also examine the pressure and temperature effects on the intrinsic anharmonicity.

2 Experimental Methods

A natural San Carlos olivine sample, from a peridotite rock collected in Arizona, USA, was adopted for this experiment, and the compositional analysis by electron probe microanalyzer (EPMA) gave the formula of \(\text{Mg}^{1.753(8)}\text{Fe}^{0.223(2)}\text{Ni}^{0.0064(1)}\text{Ca}^{0.0019(2)}\text{Mn}^{0.0030(4)}\text{Si}^{1.012(6)}\text{O}_4\) (approximately Fo50Fa11) (Liu et al. 2021). \textit{In situ} Raman spectra at simultaneous high-temperature and high-pressure conditions were carried out on a Horiba LabRAM HR Evolution system, which is equipped with a micro confocal Raman spectrometer and an 1800 line/mm grating. A green beam \((\lambda = 532 \text{ nm})\) was excited from a Nd YAG laser source operated at 20 mW, and the spectrometer was calibrated with a silicon single crystal as reference, whose fundamental
Si-Si stretching mode is detected at 520.4(1) cm\(^{-1}\). A SLM Plan 50× objective lens with long focal distance was utilized to focus the incident laser beam onto the samples.

A pair of 400-μm-culet diamond anvils, with very low fluorescence, was aligned inside an externally-heated BX90-type diamond anvil cell (outer diameter: 5 cm and height: 4.8 cm) ([Fig. S1](#)). In total, we conducted six runs of high-pressure Raman measurements at temperatures of 300, 400, 500, 600, 700 and 800 K. Taking the run at \(T = 400\) K for example, a rhenium gasket (5×5×0.2 mm\(^3\)) was initially pre-indent to a thickness of 35 μm, and a 260-μm-diameter hole was drilled at the center of the indentation as the sample chamber. A selected sample chip (approximately 50-60 μm in diameter), two or three annealed ruby chips (3 μm in diameter) and commercial cubic boron nitride (cBN) crystals (20-30 μm, pressure standard) were loaded in the sample chamber between two NaCl layers, which had been cold compressed to a thickness about 10 μm.

We had utilized Ar and a methanol-ethanol-water mixture as the pressure transmission media before, and found that the pressure was automatically increased by more than 7 GPa when the temperature was increased from 300 K to 600 K. The thermal pressure was as small as 2 GPa in the solid NaCl media even when the temperature was raised to 800 K. Hence, we chose NaCl as the pressure medium so as to have a good coverage in the pressure-temperature field below 14 GPa (as in the upper mantle). The DAC was typically sealed below 1 GPa, which was measured at ambient temperature.

A pyrophyllite ceramic cylinder (outer diameter: 22 mm, inner diameter: 6.5 mm, height: 4 mm) was placed outside the DAC, and coiled by a Pt wire (in 200 μm diameter) which serves as the external heating source. A K-type thermocouple was placed onto the diamond anvil in the cylinder side, approximately 0.4 mm away from the culet, and temperature was controlled by an
automated controller with an uncertainty within 5 K. The pyrophyllite ceramic was filled with protection gas (98 vol.% Ar + 2 vol.% H₂) throughout the high-temperature measurements to prevent potential oxidation of the diamond anvils at high temperatures. Initially, we increased the temperature up to 800 K and maintained it for 15 minutes, so as to anneal differential stress in the NaCl medium. Weidner et al. (1994) examined the differential stress in NaCl at various pressure-temperature conditions. They found that the differential stress increases steadily to approximately 0.3 GPa around 2 GPa, and remains almost constant at higher pressure at \( T = 300 \) K. At \( P = 6 \) GPa, the differential stress was observed to decrease rapidly at elevated temperature, and approached 0 GPa above 673 K, under which hydrostatic compression should be achieved inside the DAC chamber. This phenomenon is also observed in the laser-heated DAC experiments, and the annealed differential stress in NaCl was kept within 1 GPa even up to 1 megabar (Dorfman et al. 2010, 2012; Uts et al. 2013). Hence, the differential stress in the NaCl medium is expected to be no more than 0.3 GPa throughout this high-\( P,T \) experiment, which is smaller than the measurement uncertainties from the pressure standard.

Next, temperature was decreased to 400 K and the pressure was measured to be 2.4 GPa. Pressure inside the chamber was gradually increased up to 13.1 GPa at intervals of 0.3 – 1.3 GPa, and the temperature was maintained at 400 K throughout this experimental run. In total, 11 Raman spectra were obtained, and each one was collected in the range of 100 – 1200 cm\(^{-1} \) with an accumulation of 5 times and a duration of 3 minutes for each time. Noted that the full width at half maxima (FWHM) of ruby \( R_1 \) line gets much wider especially above 600 K, causing severe uncertainty in pressure determination. Hence, pressures at high-\( T \) conditions were calibrated by the cBN pressure scale (Datchi et al. 2004), which were measured before and after collection of each spectrum with a difference typically within 0.3 GPa. In the following runs at higher...
temperatures, we pre-indentied new Re gaskets, and repeated the experimental procedure mentioned above. The $P$-$T$ conditions for our experiment are summarized in Fig. 1. On the other hand, Liu et al. (2021) measured high-$T$ Raman spectra on this San Carlos olivine sample up to 1193 K in a Linkam THMS 1500 heating stage at the ambient pressure, and high-$P$ Raman spectra up to 19 GPa in DAC in an Argon pressure medium at room temperature. Noted that these high-$P$ Raman spectra collected in NaCl (at $T = 300$ K) are consistent with the previous ones obtained in Ar throughout this experimental $P$-range below 14 GPa.

The software package of Peakfit v4.12 was utilized for the analyses of the Raman spectra including fitting background and distinguishing peaks. In total, 75 Raman spectra were recorded at various $P$-$T$ conditions, and up to 10 modes are fitted by Gaussian peak shapes in each spectrum. The phonon-phonon interaction is not so significant to obviously change the peak shapes or symmetry even at high temperatures. The background can be simply treated as a slightly inclined line from 100 to 1100 cm$^{-1}$, since the spectra were collected through the diamond windows with very low fluorescence.

3 Results and discussion

3.1 Vibrational modes at various $P$-$T$ conditions

The (Mg,Fe)$_2$SiO$_4$ olivine crystallizes in an orthorhombic structure (space group: $Pbnm$) with 4 formula units in a unit cell ($Z = 4$). Consequently, there are 84 vibrational modes, including 3 acoustic modes and 81 optical modes (Fateley et al. 1971; Farmer and Lazarev 1974; Iishi 1978; Hofmeister 1987; Rao et al. 1988; Chopelas 1991; Kolesov and Geiger 2004). The optical vibrations are further divided into three groups: 36 Raman-active ones ($11 A_g + 11 B_{1g} + 7$...
B$_2$g + 7 B$_3$g), 35 IR-active ones (9 B$_{1u}$ + 13 B$_{2u}$ + 13 B$_{3u}$), and 10 inactive modes (10A$_u$). On the other hand, the optical modes can also be divided according to their attributions: (1) 45 lattice vibrations (5 A$_g$ + 5 B$_{1g}$ + 4 B$_{2g}$ + 4 B$_{3g}$ + 7 A$_u$ + 7 B$_{1u}$ + 6 B$_{2u}$ + 7B$_{3u}$) associated with the translations of Mg$^{2+}$/Fe$^{2+}$ cations (denoted as T(M)) as well as translations and rotations of SiO$_4$ units (T(Si) and R(Si), respectively), which are distributed in the frequency range below 482 cm$^{-1}$; (2) 8 symmetric ($v_2$, 1 A$_g$ + 1 B$_{1g}$ + 1 B$_{2g}$ + 1 B$_{3g}$ + 1 A$_u$ + 1 B$_{1u}$ + 1 B$_{2u}$ + 1 B$_{3u}$) and 12 asymmetric ($v_4$, 2 A$_g$ + 2 B$_{1g}$ + 1 B$_{2g}$ + 1 B$_{3g}$ + 1 A$_u$ + 1 B$_{1u}$ + 2 B$_{2u}$ + 2 B$_{3u}$) O-Si-O bending modes inside SiO$_4$ tetrahedra in the range from 349 to 644 cm$^{-1}$; (3) 4 symmetric ($v_1$, 1 A$_g$ + 1 B$_{1g}$ + 1 B$_{2u}$ + 1 B$_{3u}$) and 12 asymmetric ($v_3$, 2 A$_g$ + 2 B$_{1g}$ + 1 B$_{2g}$ + 1 B$_{3g}$ + 1 A$_u$ + 1 B$_{1u}$ + 2 B$_{2u}$ + 2 B$_{3u}$) internal Si-O stretching modes above 820 cm$^{-1}$.

The sets of vibrational peaks detected in the simultaneous high-$P$ and high-$T$ spectra (Fig. 2(a-c)) were consistent with those obtained at ambient conditions, and the vibrational signals become weaker and broader at elevated temperatures and pressures inside the DAC chambers. The fitted vibrational frequencies are listed in Table S1 in the supporting materials. The intensities of the vibrational bands vary at elevated temperature due to the Bose-Einstein phonon population function, while we focus on the frequencies (positions of the vibrational bands) in this Raman measurement. No phase transition has been observed in these measurements up to 15 GPa and 800 K, and these modes gradually shift to higher frequencies with increasing pressure, whereas to lower frequencies at elevated temperatures. The most intensive Si-O stretching modes in the frequency ranges of 812 – 828 cm$^{-1}$ and 835 – 860 cm$^{-1}$ were always detected as the characteristic peaks for olivine.

The vibrational frequencies ($v_i$) are generally believed to be in negative linear dependence with the volume ($V$) in isotropic approximation (e.g., Hofmeister and Mao 2002;...
Based on a recently proposed Mie-Grüneisen-Debye (MGD) thermal EOS in the 3rd-order Birch-Murnaghan form for olivine with the mantle composition (Angel et al. 2018), we can approximate the volume as a function of \( P \) (GPa) and \( T \) (K) as:

\[
V/V_0 = 1 + 3.8(2) \times 10^{-5} \cdot (T-300) - 6.5(3) \times 10^{-3} \cdot P - 1.3(1) \times 10^{-6} \cdot (T-300) \cdot P
\]

in the \( P-T \) range of \( 0 < P < 14 \) GPa and \( 300 < T < 1800 \) K (\( V_0 \) is the ambient volume), which will be illustrated more in the following discussion and Fig. 4(d). The fitting residual for \( V/V_0 \) is within \( \pm 1.8 \times 10^{-3} \) with \( R^2 = 0.996 \). Hence, we fit \( v(T,P) \) to Eqn (1) for the combination of the datasets from Liu et al. (2021) and this study, which is in a similar functional form as \( V/V_0 \) above:

\[
v_i(T,P) = v_0 + a \cdot (T - 300) + b \cdot P + k \cdot (T - 300) \cdot P
\]

where \( v_0 \) is the vibrational frequency at ambient conditions, and the variables of \( T \) and \( P \) are in the units of K and GPa, respectively. The fitted coefficients of \( a \), \( b \) and \( k \) are listed in Table 1 for each of the Raman-active modes. The \( a \) (cm\(^{-1}\)/K) and \( b \) (cm\(^{-1}\)/GPa) coefficients from this fitting are generally consistent with the \( T- \) and \( P- \) derivatives of \( v_i \)’s from Liu et al. (2021), whereas the coefficient \( k \) for the third term on the right side of Eqn. (1) (i.e., the cross term for both \( P \) and \( T \)) is typically in a magnitude no more than \( 1.1 \times 10^{-3} \) cm\(^{-1}\)/(GPa·K).

It is noted that a systematic deviation up to 5 cm\(^{-1}\) would be caused from the current fitting results for \( v_i \)’s at 800 K and high pressures, if this cross item was ignored due to a lack of simultaneous high-\( P,T \) dataset. In order to further illustrate the fitting results, we selected the strong Si-O stretching band around 823 cm\(^{-1}\) as an example. Variation of this mode with pressure is plotted in Fig. 3(a) at various temperatures, and linear regression is fitted between \( v_i \) and \( P \) at each temperature. At \( T = 300 \) K, the data points measured in NaCl (this study, solid circles) are in agreement with those measured in Ar (Liu et al. 2021, open circles). The spectra connected at simultaneous high-\( P,T \) inside DAC are also consistent with the previous high-\( T \) measurement in...
the heating stage at $P = 0$ GPa. The fitting residuals ($\Delta v_i$) are generally within $\pm 1.5$ cm$^{-1}$ (Fig. 3b), which is typical for these vibrational modes. In addition, the $P$- and $T$-dependences for the other measured vibrational modes are also illustrated in supplementary Fig. S2(a-c).

3.2 The mode Grüneisen and intrinsic anharmonic parameters

The shift of vibrational frequency at high-$T$ and high-$P$ conditions is generally viewed as a combination of two contributions (e.g. Gillet et al. 1991, 1997): (1) a pure volumetric contribution due to thermal expansion and compression, which could be simply interpreted as quasi-harmonicity and (2) an intrinsic anharmonic contribution which is in positive correlation with temperature and independent with volume variation. Firstly, the isobaric ($\gamma_{iP}$) and isothermal ($\gamma_{iT}$) mode Grüneisen parameters are introduced to describe variation of the frequency ($v_i$) with temperature and pressure, respectively:

$$\gamma_{iP}(T, P) = \left( \frac{\partial \ln v_i}{\partial P} \right)_T = -\frac{1}{\alpha(T,P)} \cdot \left( \frac{\partial \ln v_i(T,P)}{\partial T} \right)_P$$  (2)

$$\gamma_{iT}(T, P) = \left( \frac{\partial \ln v_i}{\partial P} \right)_T = K_T(T, P) \cdot \left( \frac{\partial \ln v_i(T,P)}{\partial P} \right)_T$$  (3)

where $\alpha$ and $K_T$ are the thermal expansion coefficient and isothermal bulk modulus at high-$P,T$ conditions, respectively. Next, the intrinsic anharmonic parameter ($a_i$), describing variation of the frequency with $T$ at constant volume, can be derived from the difference between $\gamma_{iT}$ and $\gamma_{iP}$:

$$a_i(T, P) = \left( \frac{\partial \ln v_i}{\partial T} \right)_V = \alpha(T, P) \cdot \left[ \gamma_{iT}(T, P) - \gamma_{iP}(T, P) \right] = \left( \frac{\partial \ln v_i}{\partial T} \right)_P + \alpha \cdot K_T \cdot \left( \frac{\partial \ln v_i}{\partial P} \right)_T$$  (4)

In this calculation, we adopt the MGD $P$-$V$-$T$ EOS by Angel et al. (2018), which is constructed in the 3rd-order Birch-Murnaghan form: $K_{T0} = 126.3(2)$ GPa at the ambient condition...
with a pressure derivative of $K_0' = 4.54(6)$, the Debye temperature $q_{D0} = 644(9)$ K, and the 
Grüneisen parameter $\gamma_0 = 1.044(4)$ with a volume dependence of $q = 1.9(2)$ as in $\gamma = \gamma_0 \cdot \left(\frac{V}{V_0}\right)^q$.
The isothermal bulk modulus and thermal expansion coefficient are calculated as a function of 
temperature ($300 < T < 1800$ K), at the selected pressures of 0, 3, 6, 9, 12 and 14 GPa (Fig. 4a,b and Table S2 in the supporting materials). The $K_T$ modulus decreases at elevated temperature, 
and increases with increasing pressure, whereas positive $T$-dependence and negative $P$-
dependence are observed for $\alpha$. The calculated $\alpha(T)$ profile at $P = 0$ GPa is consistent with that 
for Fo$_{87}$Fa$_{13}$ olivine (Kroll et al. 2012), with discrepancy typically within $\pm 5$ %. The product of 
$\alpha K_T$ (Fig. 4c) increases rapidly below 700 K while smoothly at higher temperature, and its 
magnitude decreases by 9.7 % from 0 to 14 GPa, at $T = 1800$ K. Finally, the volume at high-$P,T$ 
conditions (Fig. 4d) is derived according to the definition of the thermal expansion coefficient:

$$V(T, P) = V(300K, P) \cdot \exp\left[\int_{300K}^{T} \alpha(T, P) \cdot dT\right]$$

(5)

which is also listed in Table S2 and will be utilized in the following calculations. Besides, the 
Anderson-Grüneisen parameter ($\delta_T$) is also extensively adopted to describe the isobaric variation 
of $K_T$ with volume, as defined in Eqn (6) (e.g., Anderson 1995; Angel et al. 2018):

$$\delta_T = -\frac{1}{\alpha K_T} \left(\frac{\partial K_T}{\partial T}\right)_P = -\left(\frac{\partial \ln K_T}{\partial \ln V}\right)_P$$

(6)
The computed $\delta_T(P,T)$ for olivine is also plotted in Fig. 4(e), which exhibits negative $T$-
dependence, since the product of $\alpha K_T$ increases at elevated temperature for a constant pressure.

Based on the calculated $K_T$ and $\alpha$ at high-$P,T$ conditions, we computed the $\gamma(T,P)$ (Eqn 
2) and $\gamma_P(T,P)$ (Eqn 3) parameters for the vibrations (Fig. 5a,b for a representative example of
the mode at 823 cm$^{-1}$). The isothermal and isobaric Grüneisen parameters are distributed in the
value ranges of 0.54 – 1.5 and 0.35 – 0.66, respectively, both of which exhibit negative $T$- and positive $P$-dependences. The anharmonic parameter $a_i (10^{-5} \text{ K}^{-1})$ is systematically in negative values (Fig. 5c), and its magnitude decreases from 1.26 - 1.33 at $T = 300 \text{ K}$ to 0.92 – 1.02 at 1800 K. It should be noted that all the $1\sigma$ uncertainties from measurements, including temperature ($\sim 1 \text{ K}$), pressure ($< 0.3 \text{ GPa}$) and frequency fitting ($< 1.5 \text{ cm}^{-1}$), would be propagated in the following calculation for thermodynamic properties (Grüneisen and anharmonic mode parameters, heat capacities, thermodynamic Grüneisen parameters $\gamma$, bulk modulus $K_S$ and velocity $V_\Phi$, as well as adiabatic $T-P$ profile), which are also included in supplementary Table S2.

On the other hand, Hofmeister and Mao (2002) pointed out that for some particular vibrations, such as internal Si-O stretching modes and transition of MO$_6$, it is appropriate to utilize the polyhedral bulk moduli ($K_X$) to calculate the isothermal mode Grüneisen parameters (Eqn. 3) in the polyatomic structures. They adopted the moduli of 147 GPa for MgO$_6$ and 423 GPa for SiO$_4$ in Mg$_2$SiO$_4$-spinel (Hazen and Yang 1999), to calculate the $\gamma_T$ parameters for structure-related forsterite. A following high-$T$ structure refinement on Mg-pure ringwoodite gave the averaged polyhedral thermal expansion coefficients ($\alpha_X$) of 2.6(4)$\times 10^{-5} \text{ K}^{-1}$ for MgO$_6$ while 0.9(3)$\times 10^{-5} \text{ K}^{-1}$ for SiO$_4$ (Ye et al. 2012). Consequently, the product of $\alpha_X K_X$ is about 3.8(6) MPa/K for both MgO$_6$ and SiO$_4$, which is also very close to that for the bulk (Fig. 4c) from $P-V-T$ EOS fitting (Angel et al. 2018). It may cause some discrepancies when we routinely compute the mode Grüneisen parameters of $\gamma_T$ and $\gamma_P$ with only bulk $\alpha$ and $K_T$, but the impact on the anharmonic mode parameter ($a_i$) should be insignificant. It is therefore that the $a_i$ parameters, rather than $\gamma_T$ or $\gamma_P$, will be utilized in the following calculation for the anharmonic
contribution to the thermodynamic properties.

3.3 Anharmonic contribution to heat capacities

According to Kieffer’s model for the vibrational contribution to the isochoric heat capacities $C_V$ in poly-atomic minerals (Kieffer 1979, 1980), the optical modes, including both Raman-active and IR-active ones, can be divided into several continua based on their attributions. For the case of forsterite, a model of density of states (DOSs) has been adopted to calculate the heat capacity (e.g. Gillet et al. 1991) as: (1) 52 modes in continuum I from 105 to 482 cm$^{-1}$, including 45 lattice vibrations and 7 symmetric O-Si-O bending ones ($v_2$); (2) 13 modes in continuum II in the range of 505 - 644 cm$^{-1}$, including 12 asymmetric O-Si-O bending vibrations ($v_4$) and 1 $v_2$; (3) 16 Si-O stretching ($v_1$ and $v_3$) modes in continuum III from 825 to 975 cm$^{-1}$. It is widely known that the vibrational frequencies systematically shift to lower frequencies with substitution of Mg$^{2+}$ by Fe$^{2+}$ in olivine (e.g. Breitenfeld et al. 2018), and the upper ($v_{ui}$) and lower ($v_{li}$) limits of each continuum in San Carlos olivine have been modified as: 103 – 478, 499 – 639 and 822 – 973 cm$^{-1}$ for continua I, II and III, respectively (Liu et al. 2021).

Then, the capacity of $C_V$ (J/(mol·K)) in harmonic approximation can be derived as below:

$$C_V(T) = \left(\frac{\partial U}{\partial T}\right)_V = 3 \cdot n \cdot R \sum_{i=1}^{3} \frac{n_i}{N} \int_{x_{li}}^{x_{ui}} \frac{x_i^2 \exp(x_i) dx_i}{(x_{li} - x_{ui})(\exp(x_i) - 1)^2} = 3 \cdot \frac{n}{N} \cdot R \sum_{i=1}^{3} C_{vi}^h (T) \quad (7)$$

where $n$ (7) and $N$ (84) are the numbers of atoms per formula and vibrational modes per unit cell, respectively. $R$ is the gas constant and $C_{vi}^h$ is the microscopic heat capacity from each continuum in harmonic approximation. The dimensionless frequency is defined as $x_i = h \cdot c \cdot \nu / (k_B T)$, where $h$, $c$ and $k_B$ are the Planck, light speed in vacuum, and Boltzmann constants, individually.
On the other hand, the intrinsic anharmonic contribution becomes essential and necessary for accurate determination of thermodynamic properties, especially at high temperatures (Gillet et al. 1991; Polyakov 1998; Dorogokupets and Oganov 2004; Wu 2015). Since the $a_i$ parameters have already been determined as a function of both temperature and pressure in the above section, we propose a new approach to compute the anharmonic contribution to the heat capacity $C_V$ at high temperatures, which is similar to that from Gillet et al. (1991):

$$C_V(T, P) = 3 \cdot \frac{n}{N} \cdot R \sum_{i=1}^{3} C_V^i(T, P) \cdot (1 - 2 \int_0^T a_i(T, P) \cdot dT)$$  \hspace{1cm} (8)

Following the above computation procedure for the example of 823 cm$^{-1}$, we calculated the averaged anharmonic parameters for the modes in Continua I, II and III (Fig. 6(a-c)). The magnitudes of the $a_i$ parameters systematically increase with increasing pressure, indicating that the intrinsic contribution to thermodynamics would become higher at elevated pressure. The $a_i$ parameters for the internal vibrations inside the rigid units of SiO$_4$ are typically in smaller magnitudes, as compared those for the rotations or translations of these units (Liu et al. 2019b). When $T \to 0$ K, the thermal expansion coefficient decreases to zero, and consequently, the $a_i$ parameters also approach 0 according to Eqn (4). In order to have a precise evaluation of the anharmonic contribution to thermodynamics, the $a_i(T)$ curves are fitted in 6$^{th}$-order polynomials for temperature ($R^2 \geq 0.99$), and the corresponding coefficients ($c_i$, $i = 1, 2...6$, with $c_0$ fixed at 0) are listed in Table S3 in the supporting materials.

The vibrational modes generally shift to higher frequencies at elevated pressure, as supported by both high-pressure Raman (e.g. Gillet et al. 1991; Liu et al. 2021; this study) and infrared (e.g. Hofmeister et al. 1989; Wang et al. 1993; Hofmeister 1997) spectra on olivine-group minerals. The $P$-dependences of $v_i$’s ($dv_i/dP$, cm$^{-1}$·GPa$^{-1}$) from these studies are compared in Fig. 7, and there are no clearly systematical differences for the slopes between the Raman and
infrared measurements. To simplify calculation, we adopt the averaged rates of 3.2(4), 2.3(3) and 3.2(7) cm\(^{-1}\)/GPa for the measured modes in continua I, II and III, respectively. Correspondingly, high-pressure \(C_1(T)\) profiles in Kieffer’s model are computed according to the pressure effect on the vibrational frequencies. The harmonic \(C_1(T)\) at selected pressures is illustrated in Fig. 8a and listed in Table S2. The value of \(C_1(0 \text{ GPa})\) is 5 % higher than that for \(C_1(14 \text{ GPa})\) at 300 K, while this difference decreases to less than 0.2 % at 1800 K (Fig. 8b).

The correlation between the isochoric and isobaric (\(C_P\)) heat capacities is expressed below:

\[
C_P(T, P) = C_V(T, P) + T \cdot V \cdot \alpha(T, P)^2 \cdot K_T(T, P) \tag{9}
\]

where \(V\) is the molar volume (in the unit of m\(^3\)/mol). According to the calculated variables of \(\alpha\), \(K_T\) and \(V\) at high-\(T, P\) conditions above, we also compute the \(C_P(T,P)\) profiles (Fig. 8c). Similarly, the isobaric heat capacity also decreases at elevated pressure, and the \(C_P\) difference between 0 and 14 GPa is as large as about 5 % at \(T = 1800\) K (Fig. 8d), since both the volume and thermal expansion coefficient decrease at elevated pressure.

The anharmonic contribution to the heat capacities (\(\Delta C = C_{\text{anh}} - C_{\text{har}}\)) is plotted at various \(P-T\) conditions in Fig. 8(e), which is consistently positive since the \(a_i\) parameters are in negative values. At \(T = 1800\) K and \(P = 0\) GPa, the anharmonic contribution is equal to 7.9 J\(\cdot\)mol\(^{-1}\)\(\cdot\)K\(^{-1}\), which is 22 % smaller than the previous calculation (Liu et al. 2021). In the previous model, the \(a_i\) parameters were treated as constants, independent of temperature. However, this study reveals that the \(a_i\) parameters decrease at elevated temperature. Meanwhile, even at the high temperature of 1800 K, \(\Delta C(14 \text{ GPa})\) is only 1.6 J\(\cdot\)mol\(^{-1}\)\(\cdot\)K\(^{-1}\) higher than \(\Delta C(0 \text{ GPa})\), which accounts for less than 1 % of the magnitude of \(C_1(1800\) K) (Fig. 8f). Hence, the variation of \(C_P\) at high pressure
mostly comes from the pure volumetric variation with pressure (i.e. \(P-V-T\) EOS), instead of the variation of vibrational frequencies. In order to check the validation of anharmonic contribution, the modeled \(C_P(T)\) is compared with the high-\(T\) measurements on the end-members of forsterite and fayalite at \(P = 0\) GPa (Fig. 9) (Gillet et al. 1991; Saxena et al. 1993; Benisek et al. 2012). Fe\(^{2+}\) incorporation seems to increase the isobaric heat capacity as implied by the comparison between Mg\(_2\)SiO\(_4\) and Fe\(_2\)SiO\(_4\). The harmonic \(C_P\) profile for Fo\(_{89}\)Fa\(_{11}\) located even below those for Fo\(_{100}\), while the anharmonic correction provides a more reasonable model between forsterite and fayalite.

This simultaneous high-\(P,T\) Raman measurement indicates that the pressure effect on the intrinsic anharmonicity is much smaller, as compared with that on the external anharmonicity (thermal expansivity). When the pressure was increased from 0 to 14 GPa, the thermal expansion coefficient, \(\alpha(T)\), was decreased by 40 (at 300 K) – 48 (at 1800 K) % (Fig. 4b). In conclusion, the intrinsic anharmonic contribution is still important for the thermodynamic properties at high temperatures and ambient pressure. When extrapolated to simultaneously high-\(P,T\) conditions, it is reasonable to just consider the pure volumetric contribution (from \(P-V-T\) EOS) for the pressure effect, and the consequent uncertainty should be within 1 % for the cases of the heat capacities as discussed above.

3.4. Thermodynamic Grüneisen parameters and bulk seismic velocities

Based on the derived thermal expansion coefficient, isothermal bulk modulus and isochoric heat capacity at high \(P-T\) conditions in the above sections, we can also derive the thermodynamic Grüneisen parameter (\(\gamma\)) (e.g. Liebfried and Ludwig 1961; Anderson et al. 1991)
as below:

$$\gamma(T, P) = \left(\frac{\partial P}{\partial T}\right)_V \cdot V = \frac{\alpha(T,P) \cdot K_T(T,P) \cdot V(T,P)}{C_V(T,P)}$$  \hspace{1cm} (10)$$

It is noted that this $\gamma$ parameter is different from the isothermal and isobaric mode Grüneisen parameters above. The former ones refer to variation of local vibrational modes with pressure and temperature, while this thermodynamic $\gamma$ parameter can be used to relate the bulk thermal energy to the thermal pressure based on MGD $P-V-T$ equation. In this study, the connection of mode and thermodynamic Grüneisen parameters is fulfilled by the intrinsic anharmonic contribution (the $a_i$ parameters in Eqns 4 and 8) to the heat capacity $C_V$. The $\gamma(T,P)$ parameters are computed in both harmonic and anharmonic models (Fig. 10(a,b)), which are fitted as 3rd-polynomial functions of $T$, and the coefficient for each item is fitted as a linear function of $P$ with $R^2 > 0.98$:

$$\gamma_{har}(T, P) = (0.001 \cdot P - 0.128) \cdot x^3 + (-0.003 \cdot P + 0.531) \cdot x^2 + (0.002 \cdot P - 0.646) \cdot x - 0.013 \cdot P + 1.351$$  \hspace{1cm} (11)$$

$$\gamma_{anh}(T, P) = (0.001 \cdot P - 0.130) \cdot x^3 + (-0.003 \cdot P + 0.523) \cdot x^2 + (0.003 \cdot P - 0.677) \cdot x - 0.013 \cdot P + 1.349$$  \hspace{1cm} (12)$$

where the fractional temperature $x = T/1000$.

At a given temperature $T$, the thermodynamic Grüneisen parameter systematically decreases at elevated pressure, due to compression of the volume. In addition, the $\gamma$ parameters decrease significantly from 300 to 700 K due to rapidly increasing $C_V$. Above 900 K, $\gamma_{har}$ increases smoothly by nearly 5 % up to $T = 1800$ K, whereas $\gamma_{anh}$ remains almost constant. The anharmonic $\gamma$ parameter is smaller than the harmonic one, since the intrinsic anharmonicity has
positive contribution to $C_V$, and the difference is about 1% at $T = 300$ K, which increases to 8–9% at 1800 K. Besides, both the $\gamma_{\text{har}}$ and $\gamma_{\text{anh}}$ parameters at the selected pressures are plotted as a function of normalized volume ($V/V_0$) (Fig. 10c), where $V_0$ is the volume at the ambient condition.

These high-$T$ Grüneisen parameters (above 800 K) in both harmonic and anharmonic models agree with those fitted from the recent MGD $P$-$V$-$T$ EOSs for olivines (e.g., Dorogokupets et al. 2015; Angel et al. 2018; Katsura 2022), whereas Liu and Li (2006) predicted a much higher $\chi(V)$ parameter. In addition, our modeled $\gamma_0$ at the ambient condition is 4% smaller than that from Isaak (1992). It is noted that in a typical MGD $P$-$V$-$T$ EOS fitting approach, the heat capacity and thermal free energy are approximated by the harmonic Debye model, which basically considers the contributions from the three acoustic modes (one longitudinal and two transverse), and ignores the optical vibrations at higher frequencies. In the case of olivine with 28 atoms (4 formula units) in its unit cell, there are totally 81 optical modes together with 3 acoustic ones, and the contribution from the optical vibrations should also be considered for accurate calculation of the thermodynamic properties. Strictly speaking, the thermodynamic Grüneisen parameter cannot be simply described as a function of volume for silicate minerals with multiple atoms, and two variables out of $P$, $V$, $T$ are needed, like $\chi(P,T)$, $\chi(V,T)$ or $\chi(V,P)$.

The relationship between adiabatic ($K_S$) and isothermal ($K_T$) bulk moduli is expressed below:

$$K_S(T, P) = K_T(T, P) \cdot (1 + \alpha(T, P) \cdot \gamma(T, P) \cdot T) \quad (13)$$

Based on the calculated $K_T$, the thermal expansion coefficient $\alpha$, and the thermodynamic
Grüneisen parameters at simultaneously high-\(P,T\) conditions, we can derive \(K_S(T,P)\) in both the harmonic and anharmonic models (Fig. 11a). The anharmonic contribution is marginal to \(K_S\), which only causes a reduction of 0.6 – 0.7 GPa even at the high temperature of 1800 K. Our model yields \(K_{S0} = 127.5(1)\) GPa with a \(T\)-derivative of \((\partial K_S/\partial T)_P = -0.018(1)\) GPa/K and a \(P\)-derivative of \(K'_S = 4.32(5)\). Next, the bulk sound velocity \(V_\Phi, \text{km/s}\) can be further calculated based on the derived \(K_S\) and density \(\rho\) (Fig. 11b):

\[
V_\Phi(T, P) = \sqrt{\frac{K_S(T,P)}{\rho(T,P)}} \tag{14}
\]

The difference between harmonic and anharmonic velocities is also tiny, which is still within 0.02 Km/s even at \(T = 1800\) K. The anharmonic velocity is fitted as a function of \(T\) (K) and \(P\) (GPa) as:

\[
V_\Phi(T,P) = 6.22(2) + 0.069(3)\times P - [3.74(15) - 0.075(13)\times P]\times 10^{-4}\times (T-300). \tag{14}
\]

On the other hand, Liu et al. (2005) directly measured the seismic velocities of \(V_P\) and \(V_S\) on San Carlos olivine up to 8 GPa and 1073 K, and the bulk sound velocity can also be computed according to the relationship of \(V_\Phi^2 = V_P^2 - 4/3\cdot V_S^2\). An agreement is met between both these studies with a discrepancy approximately 1 %, which is comparable to the uncertainty of measurement.

### 3.5. Adiabatic temperature profile of olivine

The thermodynamic Grüneisen parameter can be adopted to calculate the adiabatic temperature profile in the mantle as following (Ramakrishnan et al. 1978; Boehler et al. 1979; Brown and Shankland 1981; Katsura 2022):

\[
\gamma(P,T) = - \left(\frac{\partial \ln T_S}{\partial \ln V}\right)_S = \frac{K_S(P,T)}{T_S} \cdot \frac{dT_S}{dP} \tag{15}
\]

where the subscript \(S\) stands for an adiabatic process (with constant entropy). We further
compute the adiabatic temperature, $T_S$, profiles in both the harmonic and anharmonic models (Fig. 12), which can be approximated as a linear function of pressure ($0 \leq P \leq 14$ GPa) with $R^2 > 0.997$:

$$T_S^{har}(P) = [0.0089(1) \cdot T_0 - 1.028(6)] \cdot P + T_0$$

(16)

$$T_S^{anh}(P) = [0.0078(1) \cdot T_0 - 0.487(5)] \cdot P + T_0$$

(17)

where $T_0$ is the ‘starting’ temperature anchored at $P = 0$ GPa (i.e. $T_3(0 \text{ GPa})$). The anharmonic thermal gradient of $dT_S/dP$ (K/GPa) is slightly smaller than that in the harmonic approximation (Table S4 in the supplementary materials), since the anharmonic correction has negative contribution to the $\gamma$ parameter. The $T_S$ profile with $T_0 = 1600$ K matches a recently proposed adiabatic geotherm in the upper mantle by Katsura (2022), which is a revision for the previous one (Katsura et al. 2010) based on the updated $P$-$V$-$T$ EOSs and phase diagram, as well as $P$-correction for the thermocouple in multi-anvil press. The corresponding slope of $dT_S/dP$ is 13.40(16) and 12.35(16) K/GPa for the harmonic and anharmonic models, respectively, and the reported value is 12.04(4) K/GPa (Katsura 2022). As suggested by Eqns (16) and (17), the pressure gradient of the adiabatic $T_S$ profile is independent of pressure, and in positive dependence with the starting temperature $T_0$, which increases from 2.21 (at $T_0 = 300$ K) to 14.05 K/GPa (at 2000 K) with the anharmonic correction.

4 Implications for thermodynamic properties in the Earth and planetary mantles

Previous vibrational spectra on olivine were obtained at high-$T$ or high-$P$ conditions, and the derived anharmonic parameters ($a_i$) were constants, independent with temperature or pressure. We conducted simultaneous high-$P$,$T$ Raman measurements, and modeled the $a_i$
parameters as a function of both temperature and pressure. One of the major findings from this study is that the anharmonic contributions to the thermodynamic properties, like heat capacities, could be overestimated if the $a_i$ parameters were treated as constants like before. Pressure has significant impact on the external anharmonicity (thermal expansivity), while a marginal effect on the intrinsic anharmonicity. Hence, when modeling the thermodynamic properties of minerals at high-$P,T$ conditions in the Earth and planetary interiors, it is still important to conduct the anharmonic correction at high temperatures, and the pressure effect could be simply attributed to variation of volume based on $P-V-T$ EOSs. The anharmonic contribution can be ignored for the high-$T$ bulk moduli of $K_T$ and $K_S$.

The thermodynamic Grüneisen parameter of olivine, $\gamma(P,T)$, is precisely determined based on combination of isothermal compression (for $P-V-T$ EOS) and vibrational spectroscopy (for heat capacity). Noted that temperatures could not be directly measured in early shockwave experiments, but were inferred according to the Grüneisen parameters from $P-V-T$ EOSs (e.g., Ahrens et al. 1971; Jackson and Ahrens 1979; Syono et al. 1981; Mosenfelder et al. 2007). At present, new technology has been introduced for measuring shock temperatures based on the gray-body Planck spectra (e.g., Ozaki et al. 2009; Sekine et al. 2016), but only very high temperatures (well above 3000 K) could be accurately determined. Hence, reliable $\gamma$ parameter is still key for constraining shock temperatures especially in the $T$-ranges relevant to the Earth and many planetary mantles.

This calculation for the Grüneisen parameter further provides an example for constraining the radial (adiabatic) temperature distributions, which is useful in various geodynamic settings inside Earth’s and planetary mantles where convection is active. In the mantle of a terrestrial planet, the pressure increases more quickly with increasing depth (i.e.
larger \( dP/dz \), where \( z \) is the depth in the mantle), if the radius (gravity) is greater. Then, the gradient of the geotherm (\( dT_S/dz \) in K/km) would be consequently larger in the adiabatic compression. In this case, we can expect that \( dT_S/dz \) in the Earth’s mantle is larger than those in the early lunar and Martian mantles (e.g. Ziethe et al. 2009; Šrámek and Zhong 2010), when their mantle convections were active and volcanism was widespread on their planetary surfaces. In addition, temperature is also the main factor of controlling the depth of the 410-km discontinuity, which is caused by olivine-wadsleyite phase transition (e.g. Katsura et al. 2004). Precise adiabatic \( T \)-gradient is also important for constraining variations of the 410-km discontinuity under various geodynamic settings.

The heat capacities of olivine at high-\( P,T \) conditions, \( C_P \) and \( C_V \), are also fundamental for constraining the thermal transport properties in the Earth’s and planetary interiors (e.g. Dubuffet et al. 1999; Tosi et al. 2013). The correlation between thermal conductivity (\( \lambda \)) and thermal diffusivity (\( \kappa \)) is expressed as: \( \lambda(P,T) = C(P,T) \times \rho(P,T) \times \kappa(P,T) \) (Zhang et al. 2019; 2023), where the density \( \rho(P,T) \) can be computed from \( P-V-T \) EOS, while the heat capacity \( C \) is taken as average of \( C_V \) and \( C_P \). Our modeled heat capacities for San Carlos olivine are \( C_{V,har} = 807.7 \) J/kg/K and \( C_{P,har} = 815.4 \) J/kg/K with anharmonic contribution of \( \Delta C = 7.9 \) J/kg/K at the ambient condition, which is consistent with that for Fo\(_{90}\) olivine (815 J/kg/K) extrapolated from the measurements on \( \lambda(P,T) \) and \( \kappa(P,T) \) (Zhang et al. 2019). Meanwhile, the high-\( P \) vibrational spectroscopic measurements give the \( P \)-derivates of isobaric heat capacity (\( dC_P/dP \), in the unit of J/kg/K/GPa) for olivine increases from -3.14 at 300 K to -1.94 at 700 K, and then decrease gradually to -5.03 at 1800 K, which is comparable to those from the simultaneous high-\( P \) measurements on thermal conductivities and diffusivities (Osako et al. 2004; Zhang et al. 2019). Hence, the datasets from different experiments could be combined together to have more precise
constraint on the heat transport properties of minerals at high-$P,T$ conditions, which are important for mantle convection, subduction dynamics, as well as cooling histories of rocky planets.

Acknowledgments

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Figure captions:

**Figure 1.** Pressure-temperature conditions of the Raman measurements from this study (solid circles) and Liu et al. (2021) (open circles). Part of the data points from Liu et al. (2021) is shown in the figure for \( T \leq 800 \text{ K} \) and \( P \leq 14 \text{ GPa} \). The vertical dashed line stands for the pressure at the 410-km seismic discontinuity.

**Figure 2.** Selected high-pressure Raman spectra with the backgrounds (approximated as inclined lines) subtracted, which were obtained at the temperatures of (a) 400, (b) 600 and (c) 800 K. The peak positions are labelled for the spectrum obtained at \( T = 400 \text{ K} \) and \( P = 2.7 \text{ GPa} \) as a representative, and its signal below 700 cm\(^{-1}\) is also magnified for clarity.

**Figure 3.** (a) Pressure dependence of the vibrational mode around 823 cm\(^{-1}\) at various temperatures, and a linear regression (solid line) is fitted for the dataset at each temperature (solid circles: this study; open circles: Liu et al., 2021). (b) Fitting residual for the vibrational frequency and the dashed horizontal lines stand for \( \Delta v_i = \pm 1.5 \text{ cm}^{-1} \).

**Figure 4.** The calculated (a) isothermal bulk modulus \( K_T \), (b) thermal expansion coefficient \( \alpha \), (c) product of \( K_T \alpha \), (d) volume variation \( V/V_0 \) and (e) Anderson-Grüneisen parameter as a function of temperature at \( P = 0, 3, 6, 9, 12 \) and 14 GPa, based on the \( P-V-T \) EOS for mantle composition olivine (Angel et al. 2018). \( V_0 \) is the unit-cell volume at the ambient condition. The black dashed
curve stands for the thermal expansion coefficient of $\alpha(T, 0 \text{ GPa})$ Fo$_{87}$Fa$_{13}$ olivine from Kroll et al. (2012).

**Figure 5.** The (a) isothermal and (b) baric mode Grüneisen parameters as well as (c) the magnitude of intrinsic anharmonic parameters ($\alpha_i$) for the mode around 823 cm$^{-1}$ at various $P-T$ conditions. The $1\sigma$ uncertainties for the mode Grüneisen parameters are smaller than sizes of the curves, and the statistical uncertainty for the $\alpha_i$ parameter is exhibited as gray area for the curve at $P = 0 \text{ GPa}$ as a representative in (c).

**Figure 6.** The averaged magnitudes for the $\alpha_i$ parameters in the continua of (a) 103 – 478 cm$^{-1}$ for translations of $\text{M}^{2+}$ (T(M)) and SiO$_4$ units (T(Si)), as well as symmetric O-Si-O bending modes ($v_2$), (b) 499 – 639 cm$^{-1}$ for asymmetric O-Si-O bending modes ($v_4$) and one symmetric O-Si-O bending mode ($v_2$), (c) 822 – 973 cm$^{-1}$ for symmetric ($v_1$) and asymmetric ($v_3$) Si-O stretching modes. The gray areas surrounding the curves at $P = 0 \text{ GPa}$ stand for the $1\sigma$ uncertainty of the anharmonic parameters.

**Figure 7.** Pressure dependencies of the vibrational frequencies ($dv_i/dP$) for olivine-group minerals from high-$P$ Raman (solid circles for Chopelas et al. 1990; Liu et al. 2021; This study) and infrared (open circles for Hofmeister et al. 1989, 1997; Wang et al. 1993) at $T = 300 \text{ K}$.

**Figure 8.** The temperature profiles of the harmonic (a,b) isochoric and (c,d) isobaric heat capacities, and (e,f) the anharmonic contribution ($\Delta C = C_{\text{anh}} - C_{\text{har}}$) at the selected pressures. The
pressure effect on the isochoric and isobaric heat capacities, and the anharmonic contribution are illustrated in (b), (d) and (f), respectively, in which the 1σ uncertainties are shown as gray areas for the case of \( P = 0 \) GPa. The Dulong-Petit limit for \( C_V \) at extremely high temperature is plotted as horizontal dashed line in (a).

**Figure 9.** The modeled \( C_P(T, 0 \) GPa) for San Carlos olivine in both harmonic (dashed curve) and anharmonic (solid curve), and comparison is made with the high-\( T \) measurements on the heat capacities of forsterite and fayalite (Gillet et al. 1991; Saxena et al. 1993; Benisek et al. 2012). The uncertainties for the heat capacities (< 0.25 J/K/mol) are smaller than the sizes of the curves.

**Figure 10.** Temperature dependence of the thermodynamic Grüneisen parameters at various pressures, in both (a) harmonic (dashed curves) and (b) anharmonic (solid curves) models. The \( \gamma \) parameter is also plotted as a function of normalized volume \((V/V_0)\) in (c), and compared with Those fitted in \( P-V-T \) EOSs for olivines from Isaak (1992) \((\gamma_0 = 1.26 \) at the ambient condition), Angel et al. (2018) \((\gamma = 1.044(4) \times (V/V_0)^{1.9(2)})\), Liu and Li (2006) \((\gamma = 1.27(5) \times (V/V_0)^{5.64(142)})\), Katsura (2022) \((\gamma = 1.00(2) \times (V/V_0)^{2.4(5)})\), and Dorogokupet et al. (2015) \((\gamma = 1.066 \times (V/V_0)^{2.225})\). The statistical uncertainties for the Grüneisen parameters are smaller than the sizes of the curves.

**Figure 11.** The (a) adiabatic bulk modulus and (b) bulk sound velocity at high-\( P,T \) conditions. The propagated 1σ uncertainties for \( K_S \) and \( V_\phi \) are smaller than the sizes of the plotted curves.
Figure 12. The modeled adiabatic temperature profiles as a function of pressure in both harmonic (dashed) and anharmonic (solid) models. The starting temperature ($T_0$) at $P = 0$ GPa is marked for each curve, and the averaged mantle geotherm (Katsura 2022) is plotted as a bold brown curve. The uncertainty for the temperature is exhibited as shaded area if larger than the size of the curve.
Table 1. The fitted parameters for the measured high-$P, T$ vibrational frequency ($v_i$, in the unit of cm$^{-1}$) according to Eqn (1).

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<th>symmetry</th>
<th>$v_0$ (cm$^{-1}$)</th>
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<th>$b$ (cm$^{-1}$GPa$^{-1}$)</th>
<th>$k \times 10^3$ (cm$^{-1}$GPa$^{-1}$K$^{-1}$)</th>
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Supplementary figure captions

Figure S1. Configuration of external resistive heating diamond anvil cell.

Figure S2(a-c). Variations of the vibrational frequencies with pressure at high temperatures.

Table S1. The fitted vibrational frequencies (cm\(^{-1}\)) for San Carlos olivine at various P-T condition.

Table S2. The thermodynamic properties for olivine at high temperatures and high pressures.

Table S3. The averaged anharmonic parameters for different frequency continua in olivine, which are fitted as a 6\(^{th}\)-order polynomial function of T at various pressures: \(a_i (10^{-6} \text{ K}^{-1}) = c_1(T/1000)^6 + c_2(T/1000)^5 + c_3(T/1000)^4 + c_4(T/1000)^3 + c_5(T/1000)^2 + c_6(T/1000)\) with \(c_0 = 0\) to satisfy that \(a_i\) approaches zero at \(T = 0 \text{ K}\).

<table>
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Table S4. The fitted slope of $dT_s/dP$ for the adiabatic temperature profile in both harmonic and anharmonic models, and $T_0$ is the starting temperature at $P = 0$ GPa.
Figure 3

(a) Li et al. 2021 and this study for $823 \text{ cm}^{-1}$.

(b) Pressure vs. $\Delta \nu$ diagram.
Figure 4

(a) $K_t$ (GPa) vs. Temperature (K)

(b) $\alpha$ (10$^{-5}$/K) vs. Temperature (K)

(c) $K_t'/(\alpha T)$ vs. Temperature (K)

(d) $V/V_0$ vs. Temperature (K)

(e) $\delta_t$ vs. Temperature (K)

Kroll et al., 2012
Figure 7

Raman Active Modes
- △ Fo\textsubscript{100} (Choeals 1990)
- ■ Fo\textsubscript{89}Fa\textsubscript{11} (this study)
- ♦ Fo\textsubscript{58}Fa\textsubscript{42} (Liu et al. 2021)

IR Active Modes
- △ Fo\textsubscript{100} (Wang et al. 1993; Hofmeister et al. 1989)
- ♦ Fa\textsubscript{100} (Hofmeister et al. 1989)
- ○ CaMgSi\textsubscript{2}O\textsubscript{4} (Hofmeister et al. 1997)
Figure 9
Figure 11

(a) Graph showing the relationship between $K_s$ (GPa) and temperature (K) for different pressures: 14 GPa, 12 GPa, 9 GPa, 6 GPa, 3 GPa, and 0 GPa. The graph compares the harmonic model (dashed line) and the anharmonic model (solid line).

(b) Graph showing the relationship between $V_0$ (km/s) and temperature (K) for different pressures: 14 GPa, 12 GPa, 9 GPa, 6 GPa, 3 GPa, and 0 GPa. The graph compares the harmonic model (dashed line) and the anharmonic model (solid line).
Figure 12

- - - - harmonic

anharmonic

mantle geotherm (Katsura, 2022)

$T_0 = 1800$ K

$T_0 = 1600$ K

$T_0 = 1400$ K

$T_0 = 1200$ K

$T_0 = 1000$ K

$T_0 = 800$ K

$T_0 = 600$ K

$T_0 = 400$ K

Pressure (GPa)

$T_s$ (K)