APPENDIX: FISSION-TRACK LENGTH *C*-AXIS PROJECTION SOURCE CODE

FORTRAN source code is provided below that implements the fission-track length *c*-axis projection model presented in this paper. Appendix Table 1 lists the contents of an input data file named *testl.dat* and the corresponding output data file named *testlc.dat*.

The model presented in this paper permits any fission-track length l_i oriented at θ_i degrees to the crystallographic *c* axis in apatite to be converted to an equivalent track length parallel to the crystallographic *c* axis, $l_{c,i}$, based on the results of annealing experiments for six different apatites (five calcian fluorapatites and Durango apatite). An iterative process of calculation is required to obtain values of $l_{c,i}$ according to the following steps:

Step 1

Using Equation 1, the value of l_c is determined iteratively by varying l_c until the following expression holds within some pre-defined level of tolerance:

$$l_i = 1/\{\mathrm{sqrt}[(\sin\theta_i/l_a)^2+(\cos\theta_i/ l_c)^2]\} \quad \text{(A1)}$$

Step 2

(a) For $l_c \geq 12.96$ µm, the fission track is an elliptical-model fission track and the value of l_c obtained using Appendix Equation 1 is accepted as $l_{c,i}$. (b) However, for $l_c < 12.96$ µm, fission-track populations exhibit accelerated length reduction at relatively high angles to the crystallographic *c* axis. These populations are characterized by a progressive collapse of the fitted ellipses from relatively high angles to the *c* axis near $l_c \approx 12$ µm to increasingly low angles to the *c* axis as l_c approaches 9.31 µm, the approximate minimum l_c value to be expected before a fission-track population completely anneals and becomes unetchable. For collapsed ellipses, two equations define the mean fission-track length as a function of track angle to the crystallographic *c* axis; at relatively low angles to the *c* axis, fission-track lengths remain distributed about a partial ellipse; at relatively high angles to the *c* axis, fission-track lengths are shortened sufficiently so that they plot below the ellipse on average. A parameter defined as the accelerated-length-reduction angle, θ_{alr}, is defined corresponding to the orientation relative to the *c* axis that divides these two fission-track length behaviors. The parameter θ_{alr} is correlated with l_c according to Equation 2. For $\theta_i \leq \theta_{alr}$, the fission track is an elliptical-model fission track and the value of l_c obtained iteratively using Appendix Equation 1 is accepted as $l_{c,i}$. (c) For $\theta_i > \theta_{alr}$, most fission tracks are accelerated-length-reduction fission tracks and the mean fission-track length is modeled as a line between two specific points. The first point, ($c_1 = 0$, a_1), is the *c*-axis perpendicular intercept of the line passing through the accelerated-length-reduction fission tracks. The parameter a_1 is correlated with θ_{alr} according to Equation 3. The second point, (c_2, a_2), corresponds to the point at the angle θ_{alr} to the crystallographic *c* axis on the ellipse passing through the accompanying elliptical model fission tracks. Using Appendix Equation 1, the Cartesian coordinates of this point are given by

$$c_2 = \cos\theta_{alr}/\{\mathrm{sqrt}[(\sin\theta_{alr}/l_a)^2+(\cos\theta_{alr}/ l_c)^2]\}$$
$$a_2 = \sin\theta_{alr}/\{\mathrm{sqrt}[(\sin\theta_{alr}/l_a)^2+(\cos\theta_{alr}/ l_c)^2]\}$$

The value of l_c is determined iteratively by varying l_c until the following expression holds within some pre-defined level of tolerance.

$$l_i = \mathrm{sqrt}\{(l_i\cos\theta_i)^2 + [l_i\cos\theta_i(a_2-a_1)/(c_2-c_1) + a_1]^2\} \quad \text{(A2)}$$

For an accelerated-length-reduction fission track, the value of l_c obtained using Appendix Equation 2 is accepted as $l_{c,i}$.

Step 3

This step in program *lcproj* is used to assign the minimum possible l_c value to a fission-track length that is too short at its respective crystallographic orientation to fall within the valid model space. The value used for this parameter in *lcproj* is 7.31 µm representing 9.31 µm minus 2.00 µm; the former is the minimum l_c observed for the six selected apatites and the latter is equal to ~2 units of $\sigma_{c,mod}$. The last fission track listed in both the input and output data files in Table A1 is an example of such a fission track.

Step 4

This step in program *lcproj* is used to test for convergence of the iterative solution being implemented and to update boundary values of l_c to facilitate convergence after each iteration for which convergence has yet to be achieved.

```
      program lcproj
      dimension
      xl(2000),xthet(2000),xkin(2000),xlc(2000)
c reads 3 column length file (length,
angle to c-axis in degrees,
c kinetic parameter), projects the
lengths onto the c-axis, writes
c projected lengths to a new 3 column
length file
      call readl(n,xl,xthet,xkin)
      call projlc(n,xl,xthet,xlc)
      call writelc(n,xlc,xkin)
      stop
      end
c
      subroutine readl(n,xl,xthet,xkin)
      dimension
      xl(2000),xthet(2000),xkin(2000)
c reads 3 column length file and con-
verts angles from degrees to
c radians
      pi=3.1415927
      n=0
      open(1,file='  ')
5 n=n+1
       read(1,'(3f15.7)',end=10)xl(n),xthet(n),xkin(n)
       xthet(n)=xthet(n)*(pi/180.)
       goto 5
10    close(1)
      n=n-1
      return
      end
c
      subroutine projlc(n,xl,xthet,xlc)
      dimension
      xl(2000),xthet(2000),xlc(2000)
c projects lengths onto the c-axis with
```

```
a tolerance given by
c parameter tol
    pi=3.1415927
c parameters for Equation 1
    eq11=1.632
      eq12=-10.879
c parameters for Equation 2
      eq21=0.304
      eq22=0.439
c parameters for Equation 3
      eq31=0.1035
      eq32=-2.25
c lc value below which evidence of accel-
erated length reduction
c exists
      xlccut=12.96
c minimum lc value possible minus ap-
proximately 2 standard deviations
      xlc0=9.31-2.*1.
c tolerance level
      tol=0.001
      do 10 i=1,n
c step 1: calculate c-axis parallel
length lci value for measured
c values of li, thetai
     xlcmin=xl(i)
     xlcmax=20.
5    xlc(i)=(xlcmax+xlcmin)/2.
     xla=eq11*xlc(i)+eq12
     xltry=ell(xla,xlc(i),xthet(i))
c step 2a: test if on full ellipse or on
collapsed part of
c ellipse using cutoff given by parameter
xlccut
    if (xlc(i).lt.xlccut) then
c step 2b: if on collapsed ellipse then
calculate accelerated
c length reduction angle xthetalr for
current lci and convert to
c radians; test if xtheti is greater than
xthetalr for current lci
     xthetalr=eq21*exp(eq22*xlc(i))*(pi/
180.)
     if (xthet(i).gt.xthetalr) then
c step 2c: for xtheti greater than
xthetalr calculate parameters
c for linear collapsed part of ellipse
with lci
      c1=0.0
      a1=eq31*xthetalr*(180./pi)+eq32
      c2=
      ell(xla,xlc(i),xthetalr)*cos(xthetalr)
      a2=
      ell(xla,xlc(i),xthetalr)*sin(xthetalr)
      xseg=xl(i)*cos(xthet(i))
      yseg=((a2-a1)/(c2-c1))*xseg+a1
c equation (A.2)
      xltry=sqrt(xseg**2+yseg**2)
     endif
    endif
c step 3: if a length is too short for
the model then the model
c projects this length to the minimum lci
value possible where la
c equals 0.
    if (xlcmax-tol.le.xlc0)) then
     xlc(i)=xlc0
     goto 10
c step 4: test for convergence and adjust
minimum or maximum value
     else if (xltry.gt.(xl(i)+tol)) then
      xlcmax=xlc(i)
      goto 5
     else if (xltry.lt.(xl(i)-tol)) then
      xlcmin=xlc(i)
      goto 5
     endif
10  continue
    return
    end
c
    function ell(xla,xlc,xthet)
c equation (A.1)
    ell=(sin(xthet)/xla)**2+(cos(xthet)/
    xlc)**2
    ell=1./(sqrt(ell))
    return
    end
c
    subroutine writelc(n,xlc,xkin)
    dimension xlc(2000),xkin(2000)
c writes 3 column length file and with
the orientation angles set
c    to zero
     xthet=0.
     open(2,file='  ')
     do 10, i=1,n
      write
      (2,'(3f15.7)')xlc(i),xthet,xkin(i)
10   continue
     close(2)
     return
     end
```

Appendix Table 1. Example input and output data files for program *lcproj*

testl.dat (input)			*testlc.dat* (output)		
xl(i) (μm)	xthet (degrees)	xkin(i) (μm)	xlc(i) (μm)	xthet (degrees)	xkin(i) (μm)
16.00	87	1.50	16.47	0	1.50
16.00	3	1.51	16.00	0	1.51
13.00	87	1.52	14.63	0	1.52
13.00	3	1.53	13.01	0	1.53
12.80	87	1.54	14.51	0	1.54
12.80	45	1.55	13.95	0	1.55
12.80	10	1.56	12.92	0	1.56
8.00	87	1.57	12.82	0	1.57
8.00	45	1.58	11.30	0	1.58
8.00	10	1.59	8.69	0	1.59
3.00	87	1.60	11.61	0	1.60
3.00	45	1.61	10.63	0	1.61
9.80	10	1.62	10.12	0	1.62
1.00	10	1.63	7.31	0	1.63