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Frequency distribution of plagioclase extinction angles: precision of the Michel-Lévy technique

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Abstract

The precision and accuracy of the Michel-Lévy method of plagioclase determination in thin section depend on the probability that an angle close to the maximum extinction angle will be observed in a given set of measurements. By assuming that the grains have a uniform random orientation, the probability density function of the extinction angle can be calculated. Knowledge of the density function allows one to calculate the probability of observing an angle close to the maximum angle, as a function of the number of measurements. These probabilities are surprisingly high; for most compositions, a set of ten measurements gives a 95 percent probability of predicting a composition within five mole percent An of the true value.

Introduction

The Michel-Lévy "statistical" method for thin-section determination of plagioclase composition was introduced in the last century (Michel-Lévy, 1894), and is probably the most useful optical method available. However, the method is subject to systematic errors because it depends on finding the maximum value of a randomly-distributed extinction angle. If the true maximum angle is not measured, the resulting anorthite content, as predicted from the angle, will either be too low (for compositions more calcic than An_{20}) or too high (for compositions more sodic than An_{20}). The precision and accuracy of the technique depend on the probability of observing an angle sufficiently close to the maximum angle, and thus on the frequency distribution of extinction angles. This paper discusses the statistics of plagioclase extinction angles.

Method

In this paper, the plagioclase grains are assumed to have a uniform random orientation, and only those with (010) vertical are considered. Many rocks (*e.g.* those with trachytic fabrics) do not meet this first condition, but it is a convenient starting point for discussion of the statistics of extinction angles.

The extinction angle θ, defined as the angle between the fast ray x′ and the trace of (010), can be determined on a stereonet by using the construction given in Bloss (1961, p. 228–229). Analysis of the stereonet solution shows that, for grains with (010) vertical, θ is given by

$$\theta(\rho) = \tfrac{1}{2}|\arctan[\tan\alpha_1 \sin(\beta_1 + \rho)] + \arctan[\tan\alpha_2 \sin(\beta_2 + \rho)]| \quad (1)$$

where ρ is the angle between the crystallographic *c* axis and the microscope axis, and α_1, α_2, β_1, and β_2 are composition-dependent constants which depend on the orientations of the optic axes of the plagioclase grain. These constants are easily derived from the optical data of Burri *et al.* (1967; low-temperature plagioclase data abstracted in Mizutani, 1975).

Figure 1 is a plot of θ as a function of ρ for two compositions. For all compositions, θ ranges between zero and a maximum angle. The Michel-Lévy determinative curve plots this maximum extinction angle against composition. Figure 2 is such a determinative plot, derived using equation 1 and the low-temperature data cited above (19 points from An_0 to An_{90}). It agrees well with the Michel-Lévy curve, especially for intermediate compositions.

The extinction angle θ is a function of the random variable ρ, so θ is randomly distributed. ρ is uniformly distributed between 0 and 2π (under the assumption made at the start of this section), so the density function of θ can be calculated by change-of-

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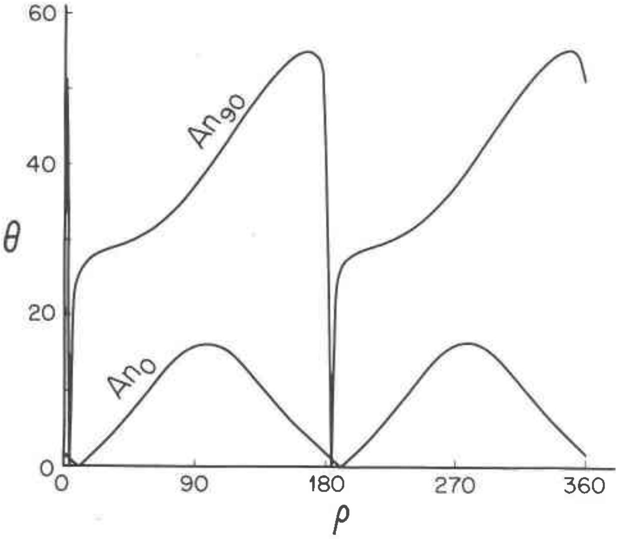

Fig. 1. Plot of the extinction angle θ as a function of the rotation angle ρ, for compositions An_0 and An_{90}.

variable techniques (Mood and Graybill, 1963, p. 220–225). Call this calculated density function $p(\theta)$. Figure 3 shows plots of $p(\theta)$ for compositions An_0 and An_{90}. For most compositions, $p(\theta)$ resembles the curve for An_0; for very calcic compositions a secondary peak is present at about half the maximum angle. Now for any composition, the probability of observing an angle between any two limits is equal to the area under $p(\theta)$ between those limits. For all compositions, $p(\theta)$ goes to infinity as θ approaches the maximum angle. Therefore, in any random observation, the probability of observing an angle near the maximum is much greater than the probability of observing an angle near any particular smaller value.

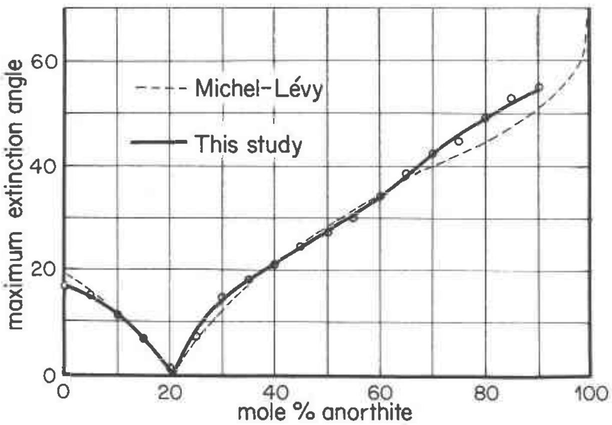

Fig. 2. Comparison of the curve plotting maximum extinction angle *vs.* composition, determined in this study, with the Michel-Lévy curve taken from Heinrich (1965, p. 364).

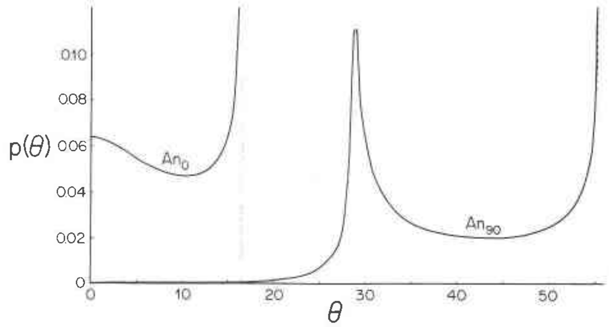

Fig. 3. Plot of the probability density function of the extinction angle θ, for compositions An_0 and An_{90}. Dashed lines are maximum angles to which the curves are asymptotic.

Now suppose that θ_{max} is the maximum extinction angle for a given composition, and θ_e is some angle less than θ_{max} which yields an apparent composition in error by e mole percent An when applied to the determinative curve of Figure 2. The probability that a randomly chosen grain will show an extinction angle less than θ_e is equal to the area under the curve $p(\theta)$ to the left of θ_e. If n randomly chosen grains are measured, the probability that *at least one* observed angle will be greater than θ_e is equal to one minus the probability that *none* of the observed angles are greater than θ_e, or

$$P = 1 - \left[\int_0^{\theta_e} p(\theta)d\theta\right]^n$$

In Figure 4 these probabilities are plotted as a function of n, for two different tolerance limits: e = 5 (Fig. 4a) and e = 10 (Fig. 4b). The probabilities are quite high; for most compositions, even as few as three measurements gives better than a 70 percent chance of being within 10 mole percent of the true composition. Ten measurements, the number recommended in most textbooks, give a very high probability of achieving the correct value. The probability of an accurate measurement reaches unity for composition near An_{20}, since for those compositions the extinction angle is so small that significant deviation from the true maximum angle is impossible.

Conclusion

If judiciously applied, the Michel-Lévy method can give highly reproducible results, in spite of its "statistical" nature. Ten measurements give a high probability of being within 5 mole percent An of the true composition, and the bias in the calculated composition is small. However, the probabilities in Figure 4 should not be used to put precise confidence

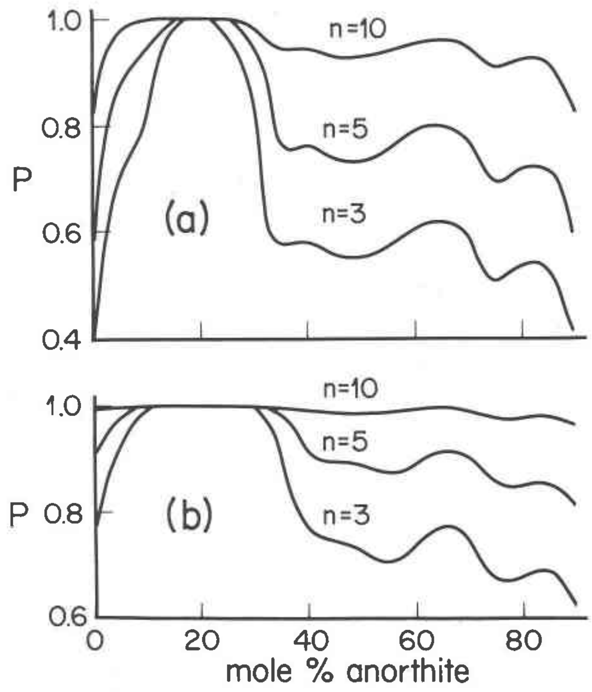

Fig. 4. Probability P of predicting a composition within (a) 5 mole % An and (b) 10 mole % An of the true composition, as a function of n (the number of measurements) and composition.

limits on optical plagioclase determinations, because several other factors enter into the true error. Some of these are: (1) non-uniform grain orientation; (2) chemical zoning; (3) change in extinction angle with change in structural state, for a given composition; (4) errors in the data used to calculate Figure 2; (5) deviations of (010) from vertical.

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