A Simple Method for the Measurement of Biabsorption Coefficients in Pleochroic Crystals Using a Normal Petrological Microscope

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Abstract

When the polarizer of a normal petrological microscope is rotated at between 2.25 and 6.25 revolutions per second with the analyzer in position and a biabsorbant section on the stage, positions of minimum flicker are observed through the eyepiece as the stage is rotated. If at these positions, θ is the angle subtended between the principal vibration axis with maximum transmission in the section and the privileged direction of the analyzer, the biabsorption coefficient of the section is given either by the function $\lambda_0 \ln \tan \theta/2\pi t$ (if positive) or $\lambda_0 \ln \cot \theta/2\pi t$ (if negative), where λ_0 is the wavelength in a vacuum of the light source used and t the thickness of the section.

Introduction

Recording the nature of differential absorption of light by birefringent crystals is common practice in mineralogical microscopic investigations. However, a quantitative approach to the study of this phenomenon for general application is lacking. Except for the methods outlined by Mandarino (1959a), which involve either the use of a double image plate and the visual matching of the brightness of the two resultant images, or else a specially constructed polarizing microscope and compensator plates, the problem has received little attention. Nevertheless, an accurate technique for determining the biabsorption coefficient is available using any normal petrological microscope without the need for sophisticated ancillary equipment.

Theory

The light intensity (I) leaving the upper polar (*analyzer*) of a polarizing microscope is related to a, the amplitude of the light emergent from the lower polar (*polarizer*), and to α, the angle between the privileged directions of the two polars by the function:

$$I = ka^2 \cos^2 \alpha$$

where k is a constant. The value of I can thus vary between 0 (for $\alpha = 90°$) and ka^2 (for $\alpha = 0°$).

If a section of a uniaxial crystal cut parallel to the optic axis is placed between the polarizer and analyzer, the light of amplitude a emerging from the polarizer is resolved into two orthogonal components on entering the crystal. These components on leaving the crystal will be out of phase with one another by an amount depending upon the birefringence of the crystal and the thickness of the section. On resolution by the analyzer into the same plane, interference between the two components will occur, causing a change in the resultant intensity. The magnitude of this change will, of course, alter with the value of α (Hartshorne and Stuart, 1970). However, if this variation in the intensity of the light transmitted by the analyzer with α can be eliminated, the resultant intensity will be the sum of the intensities of the two components or a fixed proportion thereof. The theory of the technique is based upon this assumption, and a method whereby it may be achieved in practice is given later.

1. Conditions of Transmission for Non-Pleochroic Crystals

Let us determine the intensity of the light transmitted by the analyzer for different values of α when one principal vibration axis of the crystal section described above subtends an angle θ with the privileged direction of the analyzer.

(A) If $\alpha = 0°$ (Fig. 1), the amplitude of the component leaving the section along the reference principal vibration axis will be $a \cos \theta$, and $a \sin \theta$ will be the amplitude of the orthogonal component. On entering the analyzer these two amplitudes will be resolved into the plane of the privileged direction
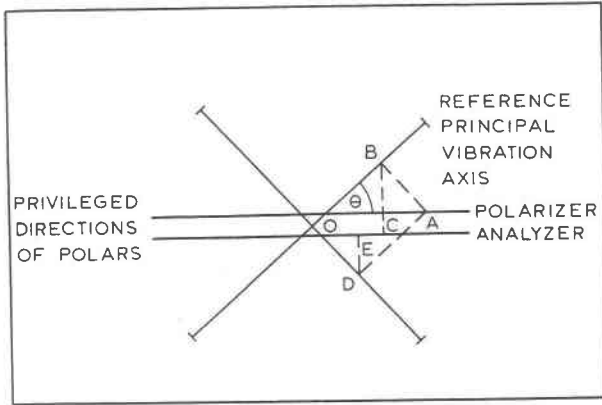
FIG. 1. Vector diagram of amplitudes for $\alpha = 0°$. For the sake of clarity, the positions of the polarizer and analyzer privileged directions are shown separated.

$\overline{OA}$ (the amplitude of the light leaving the polarizer) $= a$.

$\overline{OB}$ (the amplitude of the component along the principal vibration axis at angle θ to the privileged direction of the analyzer) $= a \cos \theta$.

$\overline{OC}$ (the amplitude of the component of $\overline{OB}$ in the privileged direction of the analyzer) $= a \cos^2 \theta$.

$\overline{OD}$ (the amplitude of the component normal to $\overline{OB}$) $= a \sin \theta$.

$\overline{OE}$ (the amplitude of the component of $\overline{OD}$ in the privileged direction of the analyzer) $= a \sin^2 \theta$.

of the analyzer and thus become $a \cos^2 \theta$ and $a \sin^2 \theta$ respectively. The intensity of the light leaving the analyzer is proportional to the sums of the squares of these two amplitudes, and thus to:

$$a^2 \sin^4 \theta + a^2 \cos^4 \theta \tag{1}$$

(B) If $\alpha = 90°$ (Fig. 2), the amplitudes of the components leaving the crystal are, as above, $a \cos \theta$ and $a \sin \theta$, but the amplitude of the component along the reference principal vibration axis is now $a \sin \theta$. On resolution by the analyzer these amplitudes become $a \cos \theta \sin \theta$ and $a \sin \theta \cos \theta$. The intensity of the light leaving the analyzer is therefore proportional to the sums of the squares of these amplitudes and thus to:

$$2a^2 \sin^2 \theta \cos^2 \theta \tag{2}$$

(C) For a more general solution let us consider the case when $90° > \alpha > 0°$ (Fig. 3), where α is measured in the same direction as θ; then, using the same reasoning as above, the intensity of the light leaving the analyzer is proportional to:

$$a^2 \cos^2 \theta \cos^2 (\alpha - \theta) + a^2 \sin^2 \theta \sin^2 (\alpha - \theta) \tag{3}$$

If $\alpha = \theta$, this simplifies to:

$$a^2 \cos^2 \theta \tag{4}$$

2. Conditions of Equality for Non-Pleochroic Crystals

Let us now find the values of θ necessary to obtain equality of intensity of light transmitted by the analyzer for various values (i and ii) of α.

(A) (i) $\alpha = 0°$;
     (ii) $\alpha = 90°$.
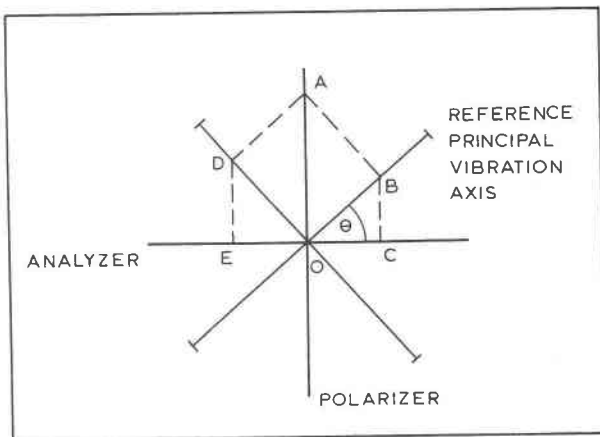
If such equality exists between (i) $\alpha = 0°$ and (ii)

FIG. 2. Vector diagram of amplitudes for $\alpha = 90°$. Notation as in Figure 1.

$\overline{OA} = a$.

$\overline{OB} = a \sin \theta$.

$\overline{OC} = a \sin \theta \cos \theta$.

$\overline{OD} = a \cos \theta$.
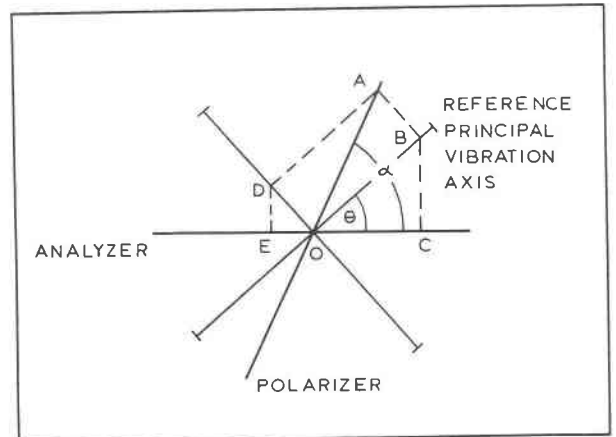
$\overline{OE} = a \cos \theta \sin \theta$.

FIG. 3. Vector diagram of amplitudes for $90° > \alpha > 0°$. Notation as in Figure 1.

$\overline{OA} = a$.

$\overline{OB} = a \cos (\alpha - \theta)$.

$\overline{OC} = a \cos (\alpha - \theta) \cos \theta$.

$\overline{OD} = a \cos (\alpha - \theta)$.

$\overline{OE} = a \sin (\alpha - \theta) \sin \theta$.

$\alpha = 90°$, then Eq. (1) equals Eq. (2) so that:

$$a^2 \sin^4 \theta + a^2 \cos^4 \theta = 2a^2 \sin^2 \theta \cos^2 \theta,$$

whereupon $\theta = 45°$.

(B) (i) $90° > \alpha > 0°$;
  (ii) $\alpha = 90°$.

From equations (2) and (3), using the same reasoning:

$$a^2 \cos^2 \theta \cos^2 (\alpha - \theta) + a^2 \sin^2 \theta \sin^2 (\alpha - \theta)$$
$$= 2a^2 \sin^2 \theta \cos^2 \theta$$

so that

$$2 \cos^4 \theta - \cos^2 \theta - 2 \cos^2 \theta \sin^2 (\alpha - \theta)$$
$$+ \sin^2 (\alpha - \theta) = 0;$$

whereupon $\cos \theta = \sin (\alpha - \theta)$ (which is true only when $\alpha = 90°$, the equality used)

or $2 \cos^2 \theta = 1$

and $\theta = 45°$.

(C) (i) $\alpha = \theta$;
  (ii) $\alpha = n\theta$, where n is any integer.

From equations (3) and (4):

$$a^2 \cos^2 \theta \cos^2 (\alpha - \theta) + a^2 \sin^2 \theta \sin^2 (\alpha - \theta)$$
$$= a^2 \cos^2 \theta$$

so that $(\tan^2 \theta - 1) \sin^2 (n\theta - \theta) = 0$
then $\sin^2 (n\theta - \theta) = 0$;
hence $n\theta = \theta$
and $n = 1$ (which is the equality used)
or $\theta = 0$ (which is invalid)
or $\theta = 45°$.

We may therefore conclude that when there is equal transmission in both principal vibration directions of the section, the intensity of the light transmitted by the analyzer remains constant for any value of α if $\theta = 45°$.

3. Conditions of Transmission for Pleochroic Crystals

Let us now consider the case of a section in which biabsorption occurs, by assuming complete transmission of the component along the reference principal vibration axis, while transmission of the orthogonal component is reduced to $1/x$ of its non-

absorbed value. Conditions otherwise remain as shown in Figures 1 to 3.

(A) If $\alpha = 0°$, the amplitudes of the components leaving the section are $a \cos \theta$ in the direction of the reference principal vibration axis, and $a \sin \theta/x$ in the orthogonal direction. On resolution by the analyzer these become $a \cos^2 \theta$ and $a \sin^2 \theta/x$ respectively, so that the intensity of the light leaving the analyzer is proportional to:

$$\frac{a^2 \sin^4 \theta}{x^2} + a^2 \cos^4 \theta \qquad (5)$$

(B) If $\alpha = 90°$, the amplitudes of the components leaving the section are now $a \sin \theta$ in the direction of the reference principal vibration axis, and $a \cos \theta/x$ in the orthogonal direction. On resolution by the analyzer these become $a \sin \theta \cos \theta$ and $a \cos \theta \sin \theta/x$ respectively, so that the intensity of the light leaving the analyzer is now proportional to:

$$a^2 \sin^2 \theta \cos^2 \theta + \frac{a^2 \cos^2 \theta \sin^2 \theta}{x^2} \qquad (6)$$

(C) With $90° > \alpha > 0°$, where α is measured in the same direction as θ and the same conditions apply as above, then the amplitudes of the components leaving the section are $a \cos (\alpha - \theta)$ in the direction of the reference principal vibration axis, and $a \sin (\alpha - \theta)/x$ in the orthogonal direction. On resolution by the analyzer these become $a \cos (\alpha - \theta) \cos \theta$ and $a \sin (\alpha - \theta) \sin \theta/x$ respectively, so that the intensity of the light leaving the analyzer is now proportional to:

$$a^2 \cos^2 (\alpha - \theta) \cos^2 \theta + \frac{a^2 \sin^2 (\alpha - \theta) \sin^2 \theta}{x^2} \qquad (7)$$

If $\alpha = \theta$ this simplifies to:

$$a^2 \cos^2 \theta \qquad (8)$$

4. Conditions of Equality for Pleochroic Crystals

Again let us find values of θ necessary to obtain equality of intensity of light transmitted by the analyzer for various values (i and ii) of α.

(A) (i) $\alpha = 0°$;
  (ii) $\alpha = 90°$.

From equations (5) and (6):

$$\frac{a^2 \sin^4 \theta}{x^2} + a^2 \cos^4 \theta$$
$$= a^2 \sin^2 \theta \cos^2 \theta + \frac{a^2 \cos^2 \theta \sin^2 \theta}{x^2}$$

so that $\tan^4 \theta - (x^2 + 1) \tan^2 \theta + x^2 = 0$

$$\text{and } \theta = 45°$$
$$\text{or } \tan \theta = x.$$

(B) (i) $\alpha = 0°$;
   (ii) $\alpha = \theta$.

From equations (5) and (8):

$$\frac{a^2 \sin^4 \theta}{x^2} + a^2 \cos^4 \theta = a^2 \cos^2 \theta$$

so that $x^2 + 1 = \tan^2 \theta + (x^2 + 1) \cos^2 \theta$
   and $\tan \theta = x$.

(C) (i) $\alpha = \theta$;
   (ii) $\alpha = n\theta$, where n is any integer.

From equations (7) and (8):

$a^2 \cos^2 \theta \cos^2 (\alpha - \theta)$

$$+ \frac{a^2 \sin^2 \theta \sin^2 (\alpha - \theta)}{x^2} = a^2 \cos^2 \cdot \theta$$

so that $\left(\frac{\tan^2 \theta}{x^2} - 1 \right) \sin^2 (n\theta - \theta) = 0$;

hence either $\sin^2 (n\theta - \theta) = 0$
   and $n\theta = \theta$
   so that $n = 1$ (which is the equality used)
   or $\theta = 0$ (which is invalid)
   or $\tan \theta = x$.

We may therefore conclude that a position exists for a biabsorbant section at $\theta = \tan^{-1} x$ when the intensity of the light transmitted by the analyzer remains independent of the value of α. This is consistent with the equalities derived for non-pleochroic sections; for when $x = 1$, $\tan^{-1} x = 45°$.

Effect of Interference on Light Transmitted by the Analyzer

In the above discussion we have ignored the effect of interference produced by a phase difference resulting between the two components of coherent light on leaving the birefringent section and resolution by the analyzer. Nevertheless, such interference must result in a change in the intensity of the light transmitted by the analyzer both with variation of the retardation involved and with variation of α. The final intensity therefore is not the sum of the square of the two component amplitudes but becomes proportional to the square of their resultant amplitude. Variations in retardation in any section of uniform thickness are, however, absent and so need not be considered, but variations in α cause

a marked change in the value of the resultant amplitude, reaching a maximum between the limits of $\alpha = 0°$ and $90°$ when $\theta = 45°$. In order that the equalities derived above may remain true it is therefore necessary to eliminate this effect. Fortunately this may be achieved by simply reducing the thickness of the section, or inserting a compensator, until the total retardation produced is $(2n - 1)\lambda_0/4$; where n is any integer and λ_0 the wavelength in a vacuum of the light to be used. The two components leaving the section will then possess the characteristics of sine and cosine functions. On resolution by the analyzer the sum of the square of the amplitude of these functions gives the square of the amplitude of their resultant. However, as noted above, the square of the resultant amplitude determines the intensity of the light transmitted by the analyzer. This relationship is thus identical in form to equations (1) to (8). All equalities derived assuming no interference therefore also hold true for this retardation.

Comparison of Light Intensities

In order to determine the value of x we must therefore compare the intensities of light transmitted by the analyzer for different values of α while the value of θ for the section on the microscope stage is adjusted. Fortunately, the petrological microscope provides within its normal construction the basic requirements of a flicker photometer, a simple device which enables the intensity of different light sources to be readily compared. Briefly, its principle is that two light sources are made available to an observer in moderately rapid alternation, that is at between about 10 to 25 alternations per second. If the sources are of equal intensity no flickering is seen. By using a small electric motor to rotate the polarizer of a petrological microscope (Durrance, 1967), a speed of 2.25 to 6.25 revolutions per second will produce alternations between $\alpha = 0°$ and $\alpha = 90°$ within this range. However, by rotating the polarizer in this fashion we are producing light intensities transmitted by the analyzer for all values of α. Thus only if the intensity of light transmitted by the analyzer is independent of α will no flickering be observed through the eyepiece of the microscope. As noted above, this condition is only fulfilled when a birefringent section is placed on the microscope stage with the principal vibration axis of maximum transmission at an angle θ $(= \tan^{-1} x)$ to the privileged direction of the analyzer.

x ($= \tan \theta$) thus gives a measure both of the amplitude ratio of the components along the two principal vibration axes in the section, and the orientation of the principal vibration axis showing maximum transmission. By normal methods these results may then be related to the ordinary (ω) and extraordinary (ϵ) vibration paths.

Biabsorption Coefficient

The amplitude a_t of light transmitted by a medium possessing an absorption index K is given by the function $a_t = a_i e^{-2\pi t K/\lambda}$ where a_i is the amplitude of the incident light, t the path length in the medium and λ the wave length of the light in the medium (Condon, 1967).

In the case of a birefringent medium the values of K and λ for the ordinary wave path (K_ω, λ_ω) will differ from those for the extraordinary wave path (K_ϵ, λ_ϵ). Transmitted amplitudes are thus:

$$a_{t_\omega} = a_i e^{-2\pi t K_\omega/\lambda_\omega}$$

$$\text{and } a_{t_\epsilon} = a_i e^{-2\pi t K_\epsilon/\lambda_\epsilon};$$

hence

$$\frac{a_{t_\omega}}{a_{t_\epsilon}} = e^{2\pi t (K_\epsilon/\lambda_\epsilon - K_\omega/\lambda_\omega)}.$$

In terms of intensities of transmitted light (I_t):

$$\frac{I_{t_\omega}}{I_{t_\epsilon}} = \frac{a_{t_\omega}^2}{a_{t_\epsilon}^2} = e^{4\pi t (K_\epsilon/\lambda_\epsilon - K_\omega/\lambda_\omega)};$$

therefore

$$\frac{K_\epsilon}{\lambda_\epsilon} - \frac{K_\omega}{\lambda_\omega} = \frac{\ln\left(\dfrac{I_{t_\omega}}{I_{t_\epsilon}}\right)}{4\pi t}.$$

Mandarino (1959a) gives the expression:

$$K_\epsilon - K_\omega = \frac{\lambda_0 \ln\left(\dfrac{I_{t_\omega}}{I_{t_\epsilon}}\right)}{4\pi t} = Q$$

where $K_\epsilon - K_\omega$ is defined as the biabsorption coefficient (Q), and λ_0 is the wavelength of light in a vacuum. In this equation he assumed (incorrectly) that λ_0 is unaltered in value in passing through the medium. In practice, Q is therefore better expressed as:

$$\lambda_0\left(\frac{K_\epsilon}{\lambda_\epsilon} - \frac{K_\omega}{\lambda_\omega}\right)$$

This modification does not, however, question the usefulness of the biabsorption coefficient.

In the rotating polarizer method described above, the value of x equals or exceeds unity, hence $\tan^{-1} x$ must equal or exceed 45°. Therefore if $a_{t_\omega} > a_{t_\epsilon}$ (positive biabsorption, because then $K_\epsilon > K_\omega$):

$$\frac{a_{t_\omega}}{a_{t_\epsilon}} = x = \tan \theta$$

and

$$Q = \frac{\lambda_0 \ln \tan \theta}{2\pi t}.$$

Alternatively if $a_{t_\omega} < a_{t_\epsilon}$ (negative biabsorption, because then $K_\epsilon < K_\omega$):

$$\frac{a_{t_\omega}}{a_{t_\epsilon}} = \frac{1}{x} = \cot \theta$$

and

$$Q = \frac{\lambda_0 \ln \cot \theta}{2\pi t}.$$

Accuracy

To achieve maximum accuracy in the measurement of Q using this method, it is advisable to determine the average of the value of θ, for minimum flicker, for each of the four 90° quadrants of rotation of the section. With an ordinary, well-adjusted microscope and visual location of the position of minimum flicker an angular limit of $\theta \pm 0.5°$ is then easily obtained. After the acquisition of a little skill on the part of the observer this can, without difficulty, be reduced to $\theta \pm 0.25°$.

The limit to the accuracy with which θ may be determined is the only source of error in the method, and is independent of both the wavelength of the light used and the thickness of the section. However, the percentage error in Q, resultant upon this angular limit, is dependent upon the magnitude of θ. That is, for small positive or negative values of Q (where $\theta \approx 45°$) the error is a maximum. This relationship is identical to that experienced by Mandarino (1959b) who used a double image method for the measurement of Q. A detailed comparison of the accuracy of the rotating polarizer and double image methods is given in Table 1.

Apart from the broad inverse correlation between percentage error and the magnitude of Q, Table 1 also shows that even with an angular limit of $\theta \pm 0.5°$ the rotating polarizer method achieves an accuracy of the same order as that found using the double image method. As would be expected, therefore, when the angular limit of the rotating polarizer method is $\theta \pm 0.25°$ a greater accuracy is obtained

TABLE 1.   Percentage Errors for Double Image and Rotating
Polarizer Methods

$\varrho \times 10^6$	ρ^0	Double image	Rotating polarizer $\pm 0.5^0$	$\pm 0.25^0$
-1	46.2	40	40	20
-20	50.3	10	9.5	4.8
-40	55.3	4.5	5	2.5
-60	59.7	3	3.7	1.9
-80	63.7	2.5	2.7	1.4
-100	67.2	2	2.8	1.4
-120	70.3	2	2.7	1.4
-140	73.1	2	2.7	1.4
-160	75.3	2	2.7	1.4
-180	77.3	2	2.7	1.4
-200	79.1	2	3	1.5

than with the double image method. Furthermore, the rotating polarizer method lends itself to refinement, without the need for sophisticated ancillary equipment, should even greater accuracy be required. This is achieved by monitoring, on an oscilloscope, the output from a photovoltaic cell mounted on the eyepiece of the microscope. As the section is rotated on the stage, the amplitude of the alternating signal varies. The position of minimum amplitude yields θ. Angular limits in the order of $\theta \pm 0.05°$ can be obtained with this arrangement.

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