# DIFFERENTIAL DISPERSION MEASUREMENT OF REFRACTION INDEX

# JOHN MCANDREW, Division of Mineralogy, CSIRO, University of Melbourne, Parkville, Australia, 3052

#### ABSTRACT

The wavelengths at which the refractive index of a solid matches those of two or more immersion liquids are plotted on Hartmann dispersion paper against the refractive indices at the D wavelength,  $n_p$ , of the liquids. These points establish a line which has a slope representing the difference in dispersion of the solid and liquids, and intersects the D wavelength ordinate at  $n_p$  of the solid. There is no need to determine the dispersion lines of the solid or liquids to ascertain  $n_p$ of the solid from wavelength variation measurements. The error in  $n_p$  introduced by using the differential dispersion line in place of the dispersion line can be kept to 0.0001, which is insignificant relative to other errors common to both methods.

#### INTRODUCTION

Microscopic immersion measurement of refractive index at ambient temperature using wavelength dispersion is notably rapid, convenient, and accurate. The method deserves much wider utilization, particularly as recent developments in high intensity electrical globes, and of continuously variable interference filters, simplify wavelength variation of microscope illumination.

It does however require measurement of the refractive index of liquids at the particular wavelengths of matching, or determination of their individual dispersion curves. The present note shows that when using liquids of similar dispersion, such as in a series mixed from two components, it is only necessary to know their refractive index  $n_D$  at the standard sodium D wavelength of 589.3 nm to determine that of the solid at this wavelength. This further simplifies the dispersion method of refractive index measurement, while not requiring liquids at exactly 0.005 spacing of refractive index as does the chart of Watkin (1958). The procedure can also be applied to immersion measurements made at different temperatures with the same liquid.

# Method

The normal procedure used for dispersion measurements is to plot the refractive index, on a linear scale, against the wavelength  $\lambda$  nm of matching, on a scale inversely proportional to ( $\lambda - 200$ ) (Fig. 1). Hartmann dispersion graph paper constructed to these scales is convenient to use. JOHN McANDREW



Fig. 1. Dispersion line and differential dispersion line of xenotime  $\omega$ .

A different procedure is to plot on Hartmann paper  $n_D$  of the liquids against the actual wavelength at which the solid matches them. The line through these points represents the differential dispersion of the solid and liquids and intersects the standard D wavelength at  $n_D$  of the solid, as of course does the dispersion curve (Fig. 1). The differential dispersion curve is sufficiently close to a straight line to determine  $n_D$  to within 0.001 by linear interpolation or extrapolation from two suitably located points on it. Only  $n_D$  of the liquids need be measured to determine this differential dispersion curve, avoiding the refractometry of the liquids at varying wavelengths which is normally used in locating the dispersion curve of the solid.

A straight differential dispersion line is obtained when the dispersion of the solid and each liquid corresponds to a straight line on Hartmann paper, and the liquids have the same dispersion  $(n_F - n_C)$ . This follows from Figure 2, which complies with these conditions. A' and B' are points on the dispersion line of the solid, A and B the equivalent points on the differential dispersion line plotted by the new procedure. For liquids with the same dispersion, and hence parallel dispersion lines, the dotted triangles are similar. The shaded right angle triangles, geometrically dependent on these, are also similar, with equal angles at their common apex C. ACB is therefore a straight line, and linear interpolation between A and B locates C and  $n_p$ .

The physical significance of ACB, as the dispersion of the solid minus the dispersion of the liquid, relative to  $n_D$  of the solid, is apparent from Figure 2. Dispersion of the liquids normally exceeds that of the solid, so the slope of the differential dispersion line is reverse to that of the dispersion lines of the solid and liquids.

### DISCUSSION

This method requires liquids of neighbouring refractive index which do not differ greatly in dispersion  $(n_F - n_C)$ . It is thus applicable to double variation measurements using the one liquid (Emmons, 1928; 1943), plotting the wavelength of matching against  $n_D$  of the liquid at the temperature of matching. Suitable liquids for single, dispersion variation are mixtures of end components, with adjacent liquids differing in refractive index by  $\frac{1}{4}$  to  $\frac{1}{3}$  of the dispersion  $(n_F - n_C)$ .

The differential dispersion line obtained from measurements at



WAVELENGTH, NM.



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ambient slide temperature (determined to  $0.1^{\circ}$ C with a thermistor probe) has been found to determine  $n_D$  of minerals to within 0.001 of measurements by the dispersion line. This error accords with the precision with which matching of the refractive index of the solid and liquid can be normally observed by the Becke line. The two methods are however theoretically equivalent only if the liquids of different  $n_D$  used in a determination have the same dispersion, a condition which can be only approximately met in practice.

# ERROR INTRODUCED BY LINEAR PLOTTING

As the error introduced by using the differential dispersion line in place of the dispersion line to determine refractive index is less than 0.001, it has been indirectly ascertained, using refractive indices known to 0.0001 or better at monochromatic wavelengths emitted by mercury, sodium and hydrogen lamps for axinite (Möller, 1958), spodumene (Bottcher, 1956), calcite (Gifford, 1902; 1910), and chrysolite (Ernst, 1925). The dispersion curves of these minerals were plotted on Hartmann paper, together with those for selected liquids in the monochlornaphthalene-diiodomethane series differing in refractive index by approximately half their dispersion  $(n_F - n_c)$ . Liquids of adjacent refractive index differed by 3 percent to 4 percent in dispersion. The scale of plotting was sufficient to clearly read refractive index to 0.0001, and wavelength to less than the variation producing a 0.0001 change in refractive index. The Hartmann dispersion paper used, Selector No. 397<sup>1</sup>/<sub>2</sub> of Schleicher and Schull, varied from a scale of  $1/(\lambda - 1)$ 200.0) at 450 nm to  $1/(\lambda - 196.4)$  at 650 nm, relative to the location of the 400 nm and 700 nm wavelengths.

The intersection of the dispersion curve of each mineral with those of the liquids determines points on the differential dispersion line of the minerals to within 0.0001 in refractive index.  $n_D$  can then be determined by linear interpolation or extrapolation to the D wavelength from pairs of these points. The error in  $n_D$  introduced by using this differential dispersion plotting procedure is listed in Table 1. It does not exceed 0.0001 when two fairly widely spaced points on the differential dispersion line are used, such points being between 470 nm and 665 nm, either straddling 589.3 nm or with one point near 589.3. Interpolation from more widely spaced points gives greater error. Extreme extrapolation from wavelengths well away from D, and outside that used in dispersion measurements, gives errors of up to 0.0007 when the differential dispersion curve is taken to be linear.

The differential dispersion plotting procedure when appropriately used within the wavelength range of ca. 480 nm to 650 nm, on minerals

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# TABLE 1. ERROR IN $n_D$ ARISING FROM LINEAR INTERPOLATION OR EXTRAPOLATION OF DIFFERENTIAL DISPERSION.

Mineral	Wavelengths	of matching	Error in $n_{j}$	D
Normal wavelength working range				
axinite, β	479.8	580.9	0.0001	
spodumene, β	521.7	653	0.0001	
calcite, ω	474.0	600	0.0001	
chrysolite, a	517.0	665	0.0001	
chrysolite, a	448.3	665	0.0002	
Extreme extrapolation				
axinite,β	418.4	479.8	0.0007	
spodumene, a	445.0	521.7	0.00045	
chrysolite,a	448.3	517.0	0.0004	

without a strong absorption band in this region and hence with dispersion close to linear on the Hartmann scale, can thus be expected to, of itself, introduce an error of not greater than about 0.0001. This is the error when using liquids differing in dispersion  $(n_F - n_C)$  by not more than 3 percent per 0.010 difference in refractive index.

Such an error is less than the standard deviation of *ca.* 0.0002 found in visual dispersion refractive index measurements under exceptionally favorable circumstances (Morse, 1968). It is appreciably smaller than the experimental error normally obtained with this method, and also less than the error in measurement of refractive indices of the comparison liquids on an Abbe type refractometer, unless particular care is taken (Fisher, 1958; Fujii and Bloss, 1962).

Differential dispersion measurement of  $n_D$  is thus normally limited in accuracy by the error in matching the refractive index of the solid and liquids, and in determining the refractive index of the liquids.

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The procedure is more than adequate in accuracy for immersion refractive index determination as well as being well suited for such measurements, particularly using fixed grain mounts (Olcott, 1960) or a spindle stage (Wilcox, 1959; Jones, 1968).

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