

AN APPROXIMATE METHOD FOR COMPUTING THE ISOTROPIC SOUND VELOCITIES USING REFRACTIVE INDEX DATA

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ABSTRACT

A method is described for estimating the sound velocities at ambient conditions from knowledge of the chemical composition and the refractive index. These data can be determined from material existing as powder or as very small grains. The method is useful for oxide compounds, but apparently not for other compounds. By oxide compounds is meant simple oxides like MgO, silicates like Mg_2SiO_4 , and complicated minerals like tourmaline where oxygen is the dominant anion.

The method is based upon recent work which shows that for oxide compounds, the elastic moduli are functions of the specific volume only, and that the refractive indices obey Drude's law. This means that both the elastic constants and the index of refraction are functions only of the specific volume. By properly accounting for the molecular weight, it is shown that the sound velocity is a function of the index of refraction. The data on oxides shows that the sound velocity is linear with the quantity $\bar{n}^2 - 1$, where \bar{n} is the mean refractive index, for constant mean atomic weight. On the other hand, for the alkali halides the sound velocity depends upon the atomic polarizabilities and the density, as well as on $\bar{n}^2 - 1$, for constant mean atomic weight.

INTRODUCTION

Only a few physical properties can be measured on microscopically sized solids such as grains or powders; perhaps the easiest measurement is the refractive index. The measurement of sound velocity, on the other hand, is not an easy task under the best circumstances and becomes quite difficult for small samples. The difficulty is compounded if the sample is a crystalline solid of low symmetry. The relative difficulty of the two measurements is demonstrated by the fact that there is a great amount of refractive index data in the literature on oxides, while the data on sound velocity are scanty by comparison.

Any method of using refractive index data to estimate the sound velocity data is valuable to researchers concerned with the mechanical properties of minerals and inorganic compounds. In this paper we show that for oxide compounds, the longitudinal and shear sound velocities can be estimated from two properties determinable on very small samples: (1) the mean refractive index \bar{n} ; and (2) a compositional parameter called the mean atomic weight, M/p (the molecular weight divided by the number of atoms p in the formula). It turns out that this new method is apparently restricted to oxide compounds (simple oxides like MgO, silicates like Mg_2SiO_4 , and complicated minerals like tourmaline).

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A large number of oxides have the same value of M/p , so that within broad limits the index of refraction alone determines the sound velocity irrespective of phase, composition, or crystalline symmetry. This is analogous to the determination of density from the refractive index (the Gladstone-Dale law) commonly used by mineralogists.

In the subsequent discussions, by sound velocity we mean the isotropic sound velocity that a dense polycrystalline solid would have at zero porosity, and by refractive index we mean the arithmetic mean of the crystalline refractive indices.

The proposed relation between sound velocity and refractive index is based upon the fact that for oxides the elastic constants are unique functions of the specific volume (Anderson and Nafe, 1965) and the refractive index is a unique function of the density (Anderson and Schreiber, 1965). As a result, by properly accounting for the molecular weight, we can show that the sound velocity is a function of the refractive index.

The empirical correlation between sound velocity and refractive index is given in the next section. An explanation of this relation is developed later on.

DATA FOR OXIDES

It turns out that a major parameter classifying the oxides is the mean atomic weight, M/p . Most data on sound velocity exist for oxides with a mean atomic weight near 20. For such oxides, the sound velocities are given by the following empirical equations:

$$v_s = 3(\bar{n}^2 - 1) \text{ km/sec,} \quad (1)$$

$$v_p = 5(\bar{n}^2 - 1) \text{ km/sec,} \quad (2)$$

where v_s and v_p are the shear and compressional sound velocities.

These equations are plotted as solid lines in Figs. 1 and 2. The data for the oxides are listed in Table I and plotted as open circles in Figs. 1 and 2. It is seen that the correlation is valid for oxides representing various molecular weights, crystal symmetries, and compositions.

The sound velocity data are taken from tables listed by Anderson and Nafe (1965), and the refractive index data from tables listed in Ref. 3 are from Anderson and Schreiber (1965).

For oxides with the same refractive index, the mean atomic weight increases as the sound velocity decreases. This effect is shown for three garnets plotted in Figs. 1 and 2 as filled circles. The velocity data on two natural garnets were reported by Verma (1960). Garnet 1 consists of a solid solution of spessartite and almandite, and garnet 2 is predominantly almandite. The refractive index values for these garnets are given also by Verma as 1.814 and 1.817. The mean atomic weights calculated from Verma's data are 24.0 and 24.5. The third garnet is synthetic, an yt-

trium aluminum garnet grown at Bell Telephone Laboratories. The elastic constants were reported privately by Spencer. The isotropic elastic constants are computed by the Voight-Reuss-Hill averaging scheme (Anderson, 1963) yielding the following values of the sound velocities: $v_p = 8.60$ km/sec and $v_s = 4.95$ km/sec. The refractive index (Larsen and Berman, 1934) is 1.833, and the value of M/p is 23.10. The elastic con-

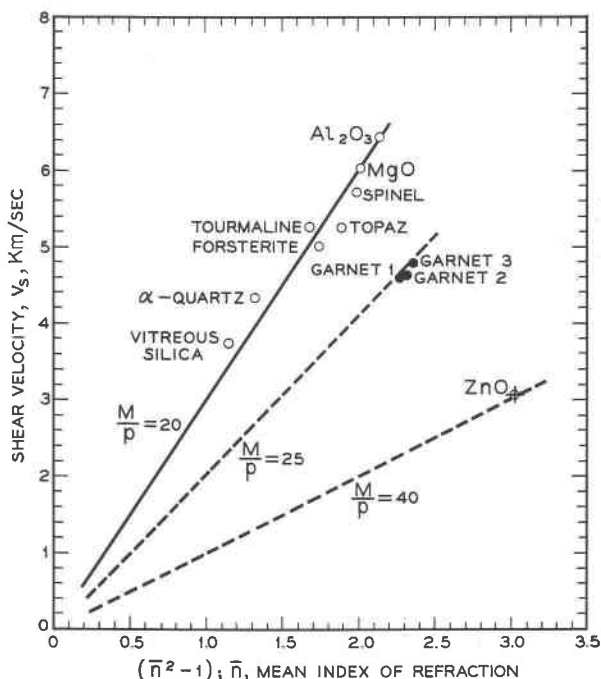


FIG. 1. Shear sound velocity versus $(\bar{n}^2 - 1)$ for oxides, at various values of M/p . The solid line is Eq. (1) for mean atomic weight of 20. The dashed lines represent expected solutions for high values of M/p .

stant data for ZnO, where the mean atomic weight is 40.6, is given by Bateman (1962). The resulting values of velocity are: $v_p = 5.96$ km/sec and $v_s = 2.84$ km/sec. These low velocities for a material with such a high value of the refractive index, 2.03, suggest that a high value of M/p materially lowers the sound velocity. These data are plotted as crossed circles in Figs. 1 and 2.

More velocity data need to be analyzed before the effect of the large values of M/p on velocity can be conclusively determined. Our best estimates are given by the dashed lines plotted in Figs. 1 and 2, which

represent guides for future work. These dashed lines must be regarded as tentative.

The solid lines in Figs. 1 and 2 representing Eq. (1) and (2) are not likely to be changed significantly by new data taken on solids with values of M/p near 20. Eq. (1) and (2) are used to compute the sound velocities of oxide compounds and minerals with $M/p = 20.5 \pm 1.5$. These minerals are

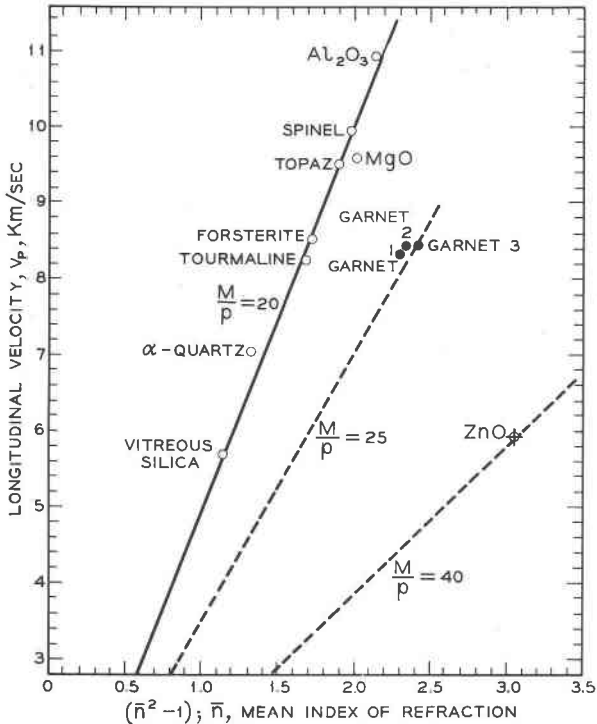


FIG. 2. Longitudinal sound velocity versus $(\bar{n}^2 - 1)$ for oxides, at various values of M/p . The solid line is Eq. (2) for mean atomic weight of 20. The dashed lines represent expected solutions for high values of M/p .

listed in Table II, along with the reported data on M/p , refractive index, density, and the computed velocities. The refractive index and density data are taken from Larsen and Berman (1934).

THE RELATIONSHIP BETWEEN SOUND VELOCITY AND REFRACTIVE INDEX FOR OXIDES

We shall now attempt to explain the empirical relationships described in the previous section. In particular, we wish to demonstrate that for oxide compounds with constant M/p , the sound velocities increase with

(\bar{n}^2-1) and are determined solely by (\bar{n}^2-1) irrespective of phase, composition, and symmetry. This is demonstrated by first discussing the velocity-density relationship and then the density-refractive index relationship.

It was shown (Anderson and Nafe, 1965) that for oxide compounds the

TABLE I. VARIATION OF MEASURED SOUND VELOCITY WITH MEASURED REFRACTIVE INDEX FOR OXIDE AND SILICATE COMPOUNDS

Phase	M/p	ρ	\bar{n}	\bar{n}^2-1	v_p	v_s
	gm/cc			km/sec		
Al ₂ O ₃	20.40	3.986	1.762	2.115	10.86	6.42
Spinel	20.32	3.63	1.727	1.986	9.94	5.66
MgO	20.20	3.57	1.736	2.01	9.57	6.03
Topaz	20.45	3.50	1.71	1.89	9.55	5.23
Forsterite	20.10	3.32	1.65	1.73	8.47	4.95
Tourmaline	19.40	3.10	1.64	1.68	8.31	5.23
α -Quartz	20.03	2.65	1.55	1.32	7.07	4.31
Vitreous Silica	20.02	2.20	1.46	1.13	5.74	3.77
Garnet 1	24.9	4.247	1.814	2.29	8.47	4.77
Garnet 2	24.3	4.183	1.817	2.30	8.52	4.77
Yttrium Aluminum Garnet	23.10	4.550	1.833	2.36	8.60	4.95
ZnO	40.6	5.676	2.03	3.04	5.96	2.84

bulk modulus at atmospheric pressure, B_0 , is related to the specific volume (per ion pair), V_0 , from compound to compound by the equation

$$B_0 = KV_0^{-x}, \quad (3)$$

where K is a constant and x is a number between 3 and 4. V_0 was defined as

$$V_0 = \frac{2M}{\rho}, \quad (4)$$

where ρ is the density. It was also shown that for a great many oxide compounds the mean atomic weight M/p is very nearly equal to the value 20.5. Consequently, for those solids with constant M/p , the bulk modulus increases from solid to solid, as the x th power of the density. The same generalization is true for the relationship between shear modulus and volume. As a consequence, the sound velocities can be defined as explicit functions of density

$$v_s = K_1(M/p) \cdot \rho^y, \quad (5)$$

$$v_p = K_2(M/p) \cdot \rho^z. \quad (6)$$

TABLE II. CALCULATED SOUND VELOCITIES FROM REFRACTIVE INDEX DATA FOR SUBSTANCES OBEYING BIRCH'S LAW ($M/p \cong 20$)

(ranked in order of increasing refractive index)

Substance	Ideal Chemical Formula	Molecular Weight M	Mean Atomic Weight M/p	Measured Refractive Index \bar{n}	Measured Density ρ	$\bar{n}^2 - 1$	Calc. Shear Velocity	Calc. Long. Velocity
							$3(\bar{n}^2 - 1)$	$5(\bar{n}^2 - 1)$
km/sec								
Fused Silica	SiO ₂	60.09	20.03	1.459	2.20	1.129	3.4	5.6
Tridymite	SiO ₂	60.09	20.03	1.471	2.27	1.164	3.5	5.8
Cristobalite	SiO ₂	60.09	20.03	1.486	2.34	1.208	3.6	6.0
Leucite	KAlSi ₃ O ₈	218.26	21.82	1.501	2.47	1.253	3.8	6.3
Keatite	SiO ₂	60.09	20.03	1.519	2.50	1.307	3.9	6.5
Carnegieite	NaAlSiO ₄	142.07	20.29	1.521	2.51	1.313	3.9	6.6
Orthoclase	KAlSi ₃ O ₈	278.35	21.41	1.522	2.55	1.316	3.9	6.6
Anorthoclase	KNaAl ₂ Si ₆ O ₁₆	540.6	20.79	1.524	2.58	1.323	4.0	6.6
Microcline	KAlSi ₃ O ₈	278.35	21.41	1.526	2.56	1.328	4.0	6.6
Albite	NaAlSi ₃ O ₈	262.25	20.17	1.530	2.61	1.340	4.0	6.7
Chalcedony	SiO ₂	60.09	20.03	1.535	2.63	1.356	4.1	6.8
Carnegieite	NaAlSiO ₄	142.07	20.29	1.536	2.62	1.359	4.1	6.8
Quartz	SiO ₂	60.09	20.03	1.547	2.65	1.393	4.2	7.0
Anorthite	CaAl ₂ Si ₂ O ₈	278.22	21.40	1.583	2.77	1.506	4.5	7.5
Enstatite	MgSiO ₃	100.41	20.08	1.590	2.87	1.528	4.6	7.6
Coesite	SiO ₂	60.06	20.03	1.595	2.92	1.544	4.5	7.7
Sarcosite	Ca ₃ Al ₂ Si ₃ O ₁₂	498.47	21.67	1.608	2.93	1.586	4.8	7.9
Akermanite	(Mg, Ca) ₂ Si ₂ O ₇	272.66	~22.00	1.635	3.12	1.673	5.0	8.4
Andalusite	Al ₂ SiO ₆	162.05	20.25	1.639	3.15	1.686	5.1	8.4
Mullite	Al ₄ Si ₂ O ₁₃	425.94	20.28	1.644	3.23	1.702	5.1	8.5
Mullite	Al ₆ Si ₂ O ₁₃	425.94	20.28	1.647	3.03	1.713	5.1	8.6
Forsterite	Mg ₂ SiO ₄	140.73	20.10	1.652	3.22	1.729	5.2	8.6
Enstatite	MgSiO ₃	100.41	20.08	1.654	3.18	1.736	5.2	8.7
Clinoenstatite	MgSiO ₃	100.41	20.08	1.655	3.28	1.739	5.2	8.7
Jadeite	NaAlSi ₃ O ₆	202.16	20.21	1.659	3.43	1.752	5.3	8.8
Sillimanite	Al ₂ SiO ₅	162.05	20.25	1.677	3.23	1.779	5.3	8.9
Olivine	(Mg, Fe) ₂ SiO ₄	?	(20.10?)	1.671	3.34	1.792	5.4	9.0
Diopside	CaMgSi ₂ O ₆	216.58	21.65	1.676	3.28	1.809	5.4	9.0
Hypersthene	MgSiO ₃	100.41	20.08	1.688	3.37	1.849	5.5	9.2
Schefferite	MgCaSi ₂ O ₄	184.58	23.07	1.688	3.39	1.849	5.5	9.2
Jeffersonite	MgCaSi ₂ O ₆	216.58	21.65	1.694	3.39	1.870	5.6	9.4
Pigeonite	MgSiO ₃	100.41	20.08	1.697	3.42	1.880	5.6	9.4
Pyrope	Mg ₃ Al ₂ Si ₃ O ₁₂	405.85	20.29	1.705	3.51	1.907	5.7	9.5
Kyanite	Al ₂ SiO ₆	162.05	20.25	1.720	3.60	1.960	5.9	9.8
Spinel	MgAl ₂ O ₄	142.28	20.32	1.723	3.60	1.990	6.0	10.0
Periclase	MgO	40.32	20.16	1.736	3.58	2.010	6.0	10.0
Corundum	Al ₂ O ₃	101.96	20.39	1.762	4.00	2.100	6.3	10.5
Stishovite	SiO ₂	60.09	20.03	1.806	4.28	2.262	6.8	11.3

In Eq. (5) and (6) the constants K_1 and K_2 are written as functions of the mean atomic weight M/p , to account for substitution of density for volume in Eq. (1). The values of the exponents in Eq. (5) and (6), y and z , will depend upon the value of x in Eq. (3) and the corresponding exponent in a similar expression for the shear modulus. If x is 3, then y and z should be 1. In any case y and z will be close to unity, though perhaps slightly larger.

Equation (3) also holds for alkali halides, sulfides, selenides, tellurides, or group IVA solids; but in these classes of solids the value of x is between 1 and $4/3$, not between 3 and 4. As a consequence, Eq. (3) does not lead to velocity-refractive index relationships similar to Eq. (5) and (6).

We now proceed to the density-refractive index correlation. It has been shown, that for the same class of compounds for which Eq. (3) is valid, Drude's law is valid, that is (Anderson and Schreiber, 1965)

$$\frac{\bar{n}^2 - 1}{\rho} = K_3(M/p) \quad (7)$$

from material to material. In other words, for oxides with a constant value of M/p , the ambient density is determined solely by the mean refractive index. The value of the constant diminishes as the value of M/p increases.

Substituting Eq. (7) into Eq. (5) and (6), we find

$$v_s = k_1(M/p) \cdot (\bar{n}^2 - 1)^y, \quad (8)$$

$$v_p = k_2(M/p) \cdot (\bar{n}^2 - 1)^z, \quad (9)$$

where y and z are close to unity, and k_1 and k_2 are new functions of the mean atomic weight.

We see that Eq. (8) and (9) are the same as Eq. (1) and (2) if the exponents y and z are unity. We further see that the slope of the curve between velocity and $(\bar{n}^2 - 1)$ is dependent upon the value of M/p .

THE RELATIONSHIP BETWEEN SOUND VELOCITY AND REFRACTIVE INDEX FOR ALKALI HALIDES

Figures 1 and 2 and Eq. (1) and (2) show that in oxides the ratio of sound velocity to $\bar{n}^2 - 1$ is constant for constant M/p . That is, density, structure, and molecular polarizability do not affect the ratio $v_p / (\bar{n}^2 - 1)$. We now demonstrate that molecular polarizability and density, as well as mean atomic weight, affect the ratio $v_p / (\bar{n}^2 - 1)$ in alkali halides.

It has been shown (Anderson and Nafe, 1965) that to a good approximation the relationship between bulk modulus, B_0 , and specific volume (per ion pair), V_0 , from compound to compound in the alkali halide group, is given by:

$$B_0 V_0 = \text{constant.} \quad (10)$$

We now replace V_0 by using the appropriate formula for the refractive index. We are faced with the fact that there are two theoretical formulas to choose between: the Lorentz-Lorenz relation and the Drude relation. Mott and Gurney (1940) have discussed the application of both these formulas to the alkali halides and have come to the conclusion that it is difficult to choose between them on theoretical grounds, but that on ex-

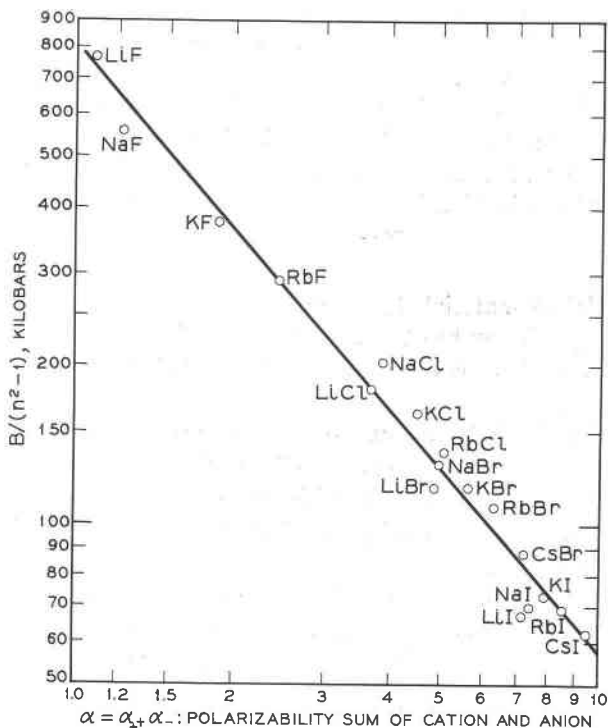


FIG. 3. The ratio $B_0/(\bar{n}^2-1)$, where B_0 is bulk modulus, versus the polarizability sum for alkali halides. The solid line is Eq. (13).

perimental grounds the Drude relationship is slightly more consistent with the data. Taking the Drude formula

$$n^2 - 1 = 4\pi \frac{N\alpha}{V_0}, \quad (11)$$

where α is the polarizability of each ion pair and N is Avogadro's number, and substituting (11) into (10), we have

$$\frac{B_0}{n^2 - 1} = \frac{\text{constant}}{\alpha}, \quad (12)$$

which should hold for the alkali halides. Replacing α by $\alpha_+ + \alpha_-$, where these refer to the polarizability of the positive and negative ion, we have:

$$\frac{B_0}{n^2 - 1} = \frac{\text{constant}}{\alpha_+ + \alpha_-}. \quad (13)$$

Eq. (13) is substantiated by plotting $B_0/(n^2 - 1)$ versus $(\alpha_+ + \alpha_-)$ in Fig. 3. The values of B_0 are taken from Anderson (1963) and the values of the

TABLE III. VARIATION OF BULK MODULUS WITH MOLECULAR POLARIZABILITY FOR ALKALI HALIDES

Solid	Bulk Modulus	Index of Refraction Function	Cation Polarization	Anion Polarization	Molecular Polarization	Ratio: Bulk Modulus / $\bar{n}^2 - 1$
	B_0	$n^2 - 1$	α_+	α_-	$\alpha = \alpha_+ + \alpha_-$	$B_0/(\bar{n}^2 - 1)$
	kilobars					kilobars
LiF	698	0.92	0.03	1.04	1.07	756
NaF	456	0.74	0.18	1.04	1.22	657
KF	319	0.85	0.83	1.04	1.87	375
RbF	273	0.93	1.40	1.04	2.44	294
LiCl	315	1.75	0.03	3.66	3.69	180
NaCl	252	1.25	0.18	3.66	3.84	201
KCl	182	1.13	0.83	3.66	4.49	161
RbCl	162	1.19	1.40	3.66	5.06	136
LiBr	257	2.16	0.03	4.77	4.80	119
NaBr	211	1.62	0.18	4.77	4.95	130
KBr	155	1.33	0.83	4.77	5.60	117
RbBr	138	1.33	1.40	4.77	6.17	104
CsBr	156	1.78	2.42	4.77	7.19	88
LiI	188	2.80	0.03	7.10	7.13	67
NaI	161	1.91	0.18	7.10	7.28	84
KI	124	1.69	0.83	7.10	7.93	73
RbI	112	1.63	1.40	7.10	8.50	69
CsI	125	2.03	2.42	7.10	9.52	62

ionic polarizabilities and the index of refraction are taken from Waxler and Weir (1965). The pertinent data are listed in Table III. Eq. (13) shows that molecular weight and density do not affect the ratio of modulus to $\bar{n}^2 - 1$. The dimensions of velocity, since it includes density as well as modulus, must therefore lead to a different dependence from that seen in Eq. (13). Manipulation of Eq. (10) and (11) leads to

$$\frac{\sqrt{B/\rho}}{\bar{n}^2 - 1} = \text{constant} \frac{\sqrt{M/\rho}}{\rho(\alpha_+ + \alpha_-)} \quad (14)$$

By analogy, the ratio of the longitudinal modulus to $\bar{n}^2 - 1$ should be

$$\frac{v_p}{\bar{n}^2 - 1} = \text{constant} \frac{\sqrt{M/\rho}}{\rho(\alpha_+ + \alpha_-)} \quad (15)$$

Comparing Eq. (15) with Eq. (7), we see that, at constant mean atomic

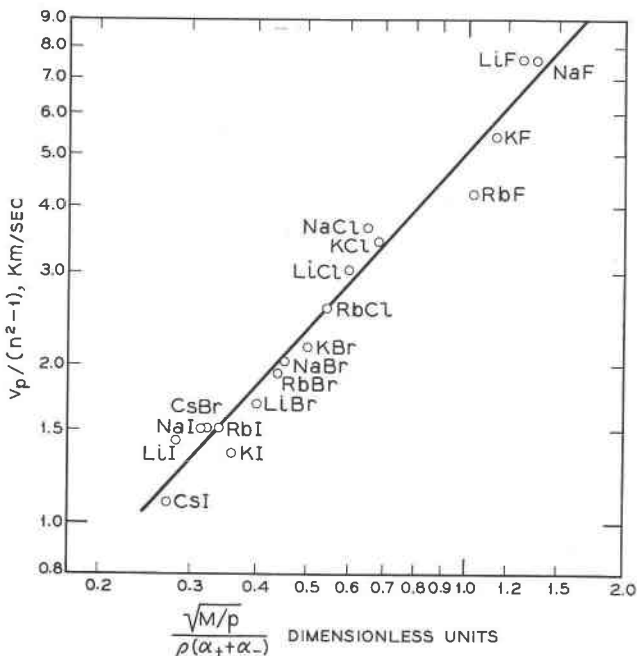


FIG. 4. The ratio $v_p / (\bar{n}^2 - 1)$, where v_p is the longitudinal sound velocity, versus $\sqrt{M/\rho} / \rho(\alpha_+ + \alpha_-)$ for alkali halides. The solid line is Eq. (15).

weight, for alkali halides v_p is determined by density and molecular polarization as well as index of refraction, while for oxides v_p is determined only by index of refraction.

A plot of the data for alkali halides is given in Fig. 4 which confirms the validity of Eq. (15). The pertinent data are listed in Table IV.

DISCUSSION

The primary reason that Eq. (1) and (2) work for oxides is that for a great number of oxide compounds the mean atomic weight changes very little from compound to compound, even though the density may change

considerably. This constancy of M/ρ , demonstrated in Table II, is not characteristic of other classes of solids. In particular, the atomic weight of the alkali metals, alkali halides, and the covalent elements and compounds changes greatly from solid to solid in a group. Consequently one cannot replace the modulus-volume relationship, such as Eq. (3), with a simple velocity-density relationship.

TABLE IV. VARIATION OF SOUND VELOCITY WITH PARAMETER $\sqrt{M/\rho}/\rho\alpha$ FOR ALKALI HALIDES

Solid	Mean Atomic Weight	Long Velocity	Shear Velocity	Density	Molecular Polarization	Index of Refraction Function	$\sqrt{M/\rho}$	Ratio: Long. Velocity $/n^2-1$
	M/ρ	v_p	v_s	ρ	$\alpha = \alpha_+ + \alpha_-$	$n^2 - 1$	$\frac{\sqrt{M/\rho}}{\rho\alpha}$	$v_p/(n^2-1)$
		km/sec		gm/cc				km/sec
LiF	12.97	7.14	4.29	2.64	1.07	0.92	1.27	7.76
NaF	21.00	5.67	3.33	2.81	1.21	0.74	1.35	7.64
KF	29.05	4.64	2.58	2.53	1.87	0.85	1.14	5.46
RbF	52.25	3.95	2.10	2.88	2.44	0.93	1.03	4.25
LiCl	21.20	5.25	3.06	2.08	3.69	1.75	0.60	3.01
NaCl	29.22	4.55	2.59	2.16	3.84	1.25	0.65	3.64
KCl	37.28	3.94	2.11	1.99	4.49	1.13	0.68	3.45
RbCl	60.47	3.08	1.61	2.80	5.06	1.19	0.54	2.59
LiBr	43.43	3.62	2.01	3.47	4.80	2.16	0.40	1.68
NaBr	51.45	3.28	1.81	3.20	4.95	1.62	0.45	2.03
KBr	59.50	2.88	1.51	2.75	5.60	1.33	0.50	2.17
RbBr	82.70	2.58	1.30	3.35	6.17	1.33	0.44	1.94
CsBr	106.20	2.70	1.55	4.43	7.19	1.78	0.32	1.52
LiI	66.93	2.85	1.51	4.06	7.13	2.80	0.28	1.43
NaI	74.68	2.73	1.50	3.67	7.28	1.91	0.32	1.51
KI	83.01	2.54	1.31	3.13	7.93	1.69	0.36	1.35
RbI	106.40	2.20	1.11	3.55	8.50	1.63	0.34	1.52
CsI	129.91	2.21	1.30	4.51	9.52	2.03	0.27	1.10

The second reason that Eq. (1) and (2) apply to oxides is that the bulk modulus and shear modulus are strongly dependent upon the volume, but not on the valence. Alkali metals, alkali halides, fluorides, and divalent compounds depend on the value of V_0 only to the inverse first power. Consequently, even if the mean atomic weight were constant, the velocity would not increase with density. It would be virtually independent of density. A further complication is that in most nonoxides the sound velocity depends upon the valence.

Another important difference between the oxides and the other classes of solids mentioned is the fact that Drude's law, Eq. (7), is uniformly applied to oxide compounds. That is, for oxide compounds with constant

M/p , the constant K_3 in Eq. (7) does not vary significantly from compound to compound. But for alkali halides, as shown by Eq. (11), the index of refraction changes from compound to compound because the polarizability changes (Table III).

The behavior of the oxide compounds from compound to compound is very much like the behavior of a classical solid under pressure. That is to say, if an alkali metal or an alkali halide is compressed, a new velocity and a new refractive index will be associated with the new density; and the subsequent relationship between sound velocity and refractive index produced by compression will be similar to that special relationship for oxides, Eq. (1) and (2), which is produced by compositional variation. This will now be demonstrated in the following equations.

The variation of bulk modulus with pressure is independent of pressure and is near the number 4 for a great many solids. Taking

$$\frac{dB}{dP} = \beta \cong 4 \quad (16)$$

and recalling that

$$B = - \frac{dP}{dV/V},$$

we have

$$\frac{dB}{B} = - \beta \frac{dV}{V}. \quad (17)$$

The above integrates into

$$\frac{B}{B_0} = \left(\frac{V}{V_0} \right)^{-\beta}. \quad (18)$$

However, since the value of dB/dP is always near 4 (that is, it has nearly the same value as x in Eq. (3)), the variation of B with V due to pressure using Eq. (18) is nearly the same as the variation of B_0 with V_0 due to compositional changes expressed by Eq. (3). The sound velocity defined from the bulk modulus alone is

$$\phi = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{B_0}{\rho_0}} \left(\frac{\rho}{\rho_0} \right)^{(\beta-1)/2}, \quad (19)$$

where β is the constant dB/dP . Eq. (19) is the same form shown by Eq. (5) and (6). If dB/dP is 3, the value of the exponent of ρ in Eq. (19) is unity. Probably the exponent of ρ is slightly larger than unity for most solids. By analogy, similar relations to Eq. (19) hold for the shear and longitudinal sound velocities.

The density-refractive index relationship for many solids under pressure obeys either the Lorentz-Lorenz law or Drude's law (Waxler and Weir, 1965). (Exceptions include Diamond and perhaps MgO.) The sub-

ject has been reviewed by Hamann (1957). Consequently solids under pressure obey some functional relationship equivalent to Eq. (7), although Eq. (7) has been presented to represent data for different oxides at ambient conditions. Thus the formalism which leads from Eq. (3) to Eq. (8) and (9) holds for most solids under pressure.

The distinction of oxide compounds is that not only do they obey Eq. (8) and (9) as individual solids under pressure, but that as a class the sound velocity and refractive index vary from compound to compound at ambient conditions according to Eq. (8) and (9).

It seems clear that as far as the isotropic values of the optical constants and the velocity constants are concerned, oxide compounds with constant M/p near 20 or 21 can be treated as one solid with numerous polymorphic states. This is an important simplification since there are a large number of such oxide compounds, as demonstrated by Table II. Furthermore, these solids include many which are important to current research in geophysics, electronic materials, and ceramics.

Finally, we comment on those few oxides which fail to conform to Eq. (5) and (6). Such oxides will be those which either do not obey Eq. (3) or Eq. (7). One example is TiO_2 , which has an abnormally high value of the refractive index for its density and molecular weight. This is probably due to the same cause which leads to a high dielectric constant. Von Hippel (1954) ascribes the high dielectric constant to the TiO_6 constellation which has binding in transition between polar and nonpolar types. Another example is ZrO_2 .

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