The analyses of ludwigite from Hungary by Ludwig and Sipöcz and from Montana by Schaller show it to be essentially a magnesium borate plus the magnetite molecule, with part of the magnesium replaced by a subordinate amount of ferrous iron. The Riverside mineral, however, is essentially a ferrous borate plus the magnetite molecule with part of its iron replaced by a subordinate amount of magnesia. Its distinctive difference in composition, and its manifest difference in structural and optical characters from ludwigite, justifies the writer in proposing the new name *vonsenite*, after its discoverer, for the mineral.

THE GOLDSCHMIDT TWO-CIRCLE METHOD. CALCULATIONS IN THE HEXAGONAL SYSTEM

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Forms and Symbols

The gnomonic projection of a hexagonal crystal presents a grouping of face-poles of hexagonal or trigonal pattern. The axes of reference used to determine the symbols may be either of two sets of lines; each set intersects mutually at 60 degrees; the two sets are turned to each other 30 (or 90) degrees. Symbols and axial ratio may be determined from either set, and may later be transformed to accord with a choice of the other. The determination as to which set is to be used in a given case is somewhat arbitrary, and the dual choice introduces some confusion in the study of this system.

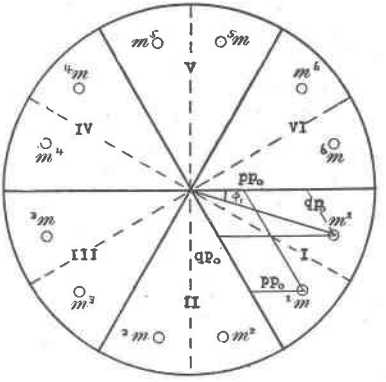

Fig. 27

In figure 27 the two sets of axes are shown by heavy and dotted lines; there is also shown the numbering of the sectants and the method of indicating by exponents the position of a face (m) on the crystal. In the holohedral crystals all sectants are identical. For two faces the coördinates are shown in the figure, and the Goldschmidt symbol is derived as in the systems with rectangular axes, by measuring along any two adjacent axes the coördinate lengths which fix the position of the face-pole.

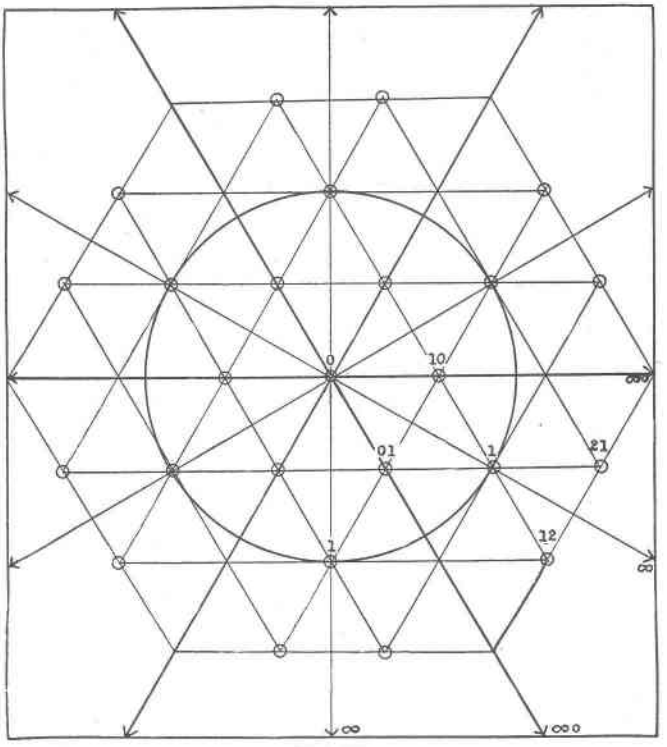

FIG. 28

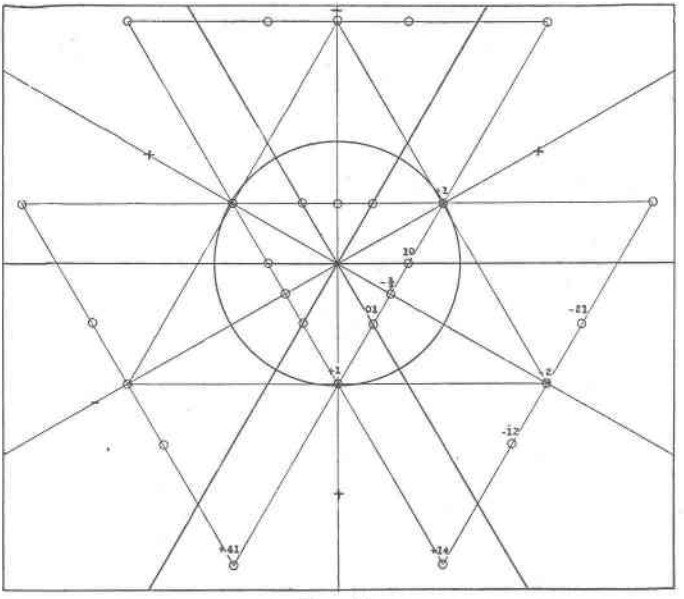

FIG. 29

For m^1 the coördinates are pp_0 and qp_0; for ^1m, qp_0 and pp_0.

Since p_0, the element of a given species, is common to all symbols, the values p and q, whole numbers or fractions, indicate the position of the face-pole and constitute the symbol. This symbol, pq, gives directly the Bravais-Miller symbol ordinarily used for this system, thru the following equation:

$$pq = (p.q.\ p + q.1)$$

For example: $21 = (2.1.\bar{3}.1.)$ $\quad 10 = (1.0.\bar{1}.1)$

$\frac{1}{2}1 = (1.2.\bar{3}.2)$ $\quad 11 = (1.1.\bar{2}.1)$

In figure 28 is shown a projection of beryl with one of each of the seven types of forms of the hexagonal system, which receive the following symbols and occupy the given positions in the Goldschmidt system:

	Gdt. (G_1)	Bravais	Position of Face-pole
Basal Pinacoid	0	0001	Center of coördinates.
Prism, 1st Order	$\infty 0$	$10\bar{1}0$	In the direction of face 10 or of the coördinate axes.
Prism, 2nd Order	∞	$11\bar{2}0$	In the direction of face 1 or of the intermediate axes.
Pyramid, 1st Order	p0	p0$\bar{p}$1	On coördinate axes.
	e.g. 10	$10\bar{1}1$	
Pyramid, 2nd Order	p	p.p.$\bar{2}\bar{p}$.1	On intermediate axes.
	e.g. 1	$11\bar{2}1$	
	$\frac{1}{2}$	$11\bar{2}2$	
Dihexagonal Pyramid	pq	p.q.$\overline{p+q}$.1	Any position between two sets of axes. 2 faces in each sectant.
	e.g. 21	$21\bar{3}1$	
	$\frac{2}{3}\ \frac{1}{3}$	$21\bar{3}3$	
Dihexagonal Prism	$\frac{p}{q}\infty$	p.q.$\overline{p+q}$.0	In the direction of face pq; 2 faces in each sectant.
	e.g. 2∞	$21\bar{3}0$	

This choice of axes is indicated in the *Winkeltabellen* by a symbol (G_1) placed after the axial ratio table at the head of each hexagonal mineral. It corresponds to a position of the crystal in which the second order pyramid ($11\bar{2}1$) is taken as unit form and is used chiefly for minerals of pronounced hexagonal habit such as apatite, beryl and quartz.

If c be the length of the vertical axis of a crystal as given, say, in Dana, then c_1, the length of the vertical axis in position G_1 is $\sqrt{3}c$.

$$p_0 = \frac{2}{3}c_1 = \frac{2}{\sqrt{3}}c \qquad c_1 = \frac{3}{2}p_0 \qquad c = \frac{\sqrt{3}}{2}p_0$$

Thus for quartz, $c = 1.09997$; $c_1 = \sqrt{3}\, c = 1.9051$; $p_0 = \frac{2}{3} c_1 = 1.2701$.

If the alternative set of axes be chosen (Position "G_2"), the Goldschmidt symbols are derived as before from the adjacent axes and have a similar form, but with a different value of p_0. In this case the pyramid of first order (or rhombohedron), $(10\bar{1}1)$, is taken as unit form and c_2 is equal to c.

$$p_0 = \tfrac{2}{3} c_2,\ c_2 = \tfrac{3}{2} p_0,\ p_0\ (G_1) = \sqrt{3} p_0 (G_2).$$

Thus for calcite:—$c = c_2 = 0.8543$; $p_0 = \frac{2}{3} c_2 = 0.5695$.

The Goldschmidt symbol, pq, in position G_2, does not yield the correct Bravais symbol; it must first be transformed to the G_1 equivalent by means of the following transformation equation.

$$pq\ (G_2) = \frac{p + 2q}{3} \cdot \frac{p - q}{3}\ (G_1).$$

For example:

G_2	G_1	Bravais
21	$\frac{4}{3}\ \frac{1}{3}$	$41\bar{5}3$
$2\frac{1}{2} = \frac{4}{2}\ \frac{1}{2}$	$\frac{2}{2}\ \frac{1}{2}$	$21\bar{3}1$
$1 = 11$	$1\ 0$	$10\bar{1}1$
10	$\frac{1}{3}\ \frac{1}{3}$	$11\bar{2}3$

The position G_2 is generally adopted in the *Winkeltabellen* for crystals of rhombohedral symmetry. It should be noted that altho Goldschmidt regarded the requirement just stated in deriving Bravais symbols from G_2 symbols thruout the *Index der Krystallformen*, he abandoned it in the *Winkeltabellen*, where he derived the Bravais symbols directly from the G_2 symbols. Consequently the Bravais symbols given there for all minerals with the G_2 heading are not conformable to usage. In the later *Atlas der Krystallformen* both types of Bravais symbols are given, which helps to clear up the confusion.

The transformation equation for the reverse change of G_1 to G_2 symbols is as follows: $pq\ (G_1) = (p + 2q) \cdot (p - q)\ (G_2)$.

For further discussion of the relation of the two positions, for proof of the various equations given above, and for transformation equations from Goldschmidt symbols to Miller, Naumann and other symbols, reference should be made to the Introduction of the *Index der Krystallformen*, pp. 29–36 and 44–53.

Figure 29 shows a few forms of a rhombohedral mineral (calcite) in gnomonic projection. The sectants are alternately positive and negative, II, IV and VI being +; I, III and V −.

This brings the unit positive rhombohedron of the crystal towards the front, the customary position. The positive or negative character of a form is indicated in Goldschmidt symbols by the appropriate sign placed before the symbol. In this figure as in figure 28 the value of p_0 is given graphically by the distance measured along the axis from the coördinate center to the face 10.

The further subdivision of the sectants into right and left halves for a mineral of trirhombohedral symmetry is well shown in the gnomonic projection reproduced in a paper on willemite (G_2) by Palache and Graham.[1] The distribution of face-poles in the rhombohedral-trapezohedral symmetry type is shown in the splendid folio plate of quartz in Goldschmidt's *Atlas der Krystallographische Projectionsbilder*, Berlin, 1887. In the same atlas are numerous plates of rhombohedral minerals of varying habit.

Calculation of the Element, p_0, and of Symbols.

A hexagonal crystal having been measured, it is projected by means of the v and h angles (Gnom. Proj., p. 70) and thus a graphic solution for the unknown values is obtained. The calculation follows only where the measurements are sufficiently good and numerous to insure increased accuracy in the results. From the plot or directly from the goniometer readings it will be evident that a number of the v angles differ from one another by almost exactly 30 degrees or some multiple (60°, 90°, 120°). The first meridian is to be placed at or near one of these readings, the exact value being obtained by averaging them. The G_1 or G_2 position is determined by the choice of either one of two of them, 30 degrees apart; the decision must be made by studying the habit of the crystal or by comparing the angles made by terminal faces with those given in the *Winkeltabellen* if the species of the crystal is known. For example, in a crystal of calcite the zero meridian will be so chosen that one face of the cleavage rhombohedron will have a φ of $+ 90°$.

The position of v_0 having been established, it is subtracted from each v to obtain φ. In addition it is necessary to find for each face φ_1, which is the angle to the nearest axis or the difference between φ and the nearest multiple of 60 degrees; and also the value $(60° - \varphi_1)$. ρ for each face is also obtained by subtracting

[1] *Am. J. Sci.*, **36**, 641, 1913.

h_0 from h. For an illustration of the form in which these calculations are made by Goldschmidt, reference is made to an article by him on vanadinite.[2]

In figure 30 is shown a gnomonic projection of a sectant of a hexagonal crystal with the angular relations of a face-pole, pq, whose angles φ_1 and ρ are known. From this figure, using the sine-law of plane trigonometry, we obtain the following relations:

$$\frac{pp_0}{\tan \rho} = \frac{\sin (60° - \varphi_1)}{\sin 60°}; \qquad \frac{qp_0}{\tan \rho} = \frac{\sin \varphi_1}{\sin 60°},$$

whence

$$pp_0 = \frac{\sin (60° - \varphi_1) \tan \rho}{\sin 60°}; \qquad qp_0 = \frac{\sin \varphi_1 \tan \rho}{\sin 60°}.$$

The formulas are simpler for the following special cases:

$$p0,\ \varphi_1 = 0\ ; \qquad pp_0 = \tan \rho; \qquad qp_0 = 0;$$

$$pp,\ \varphi_1 = 30°; \qquad pp_0 = qp_0 = \frac{\tan \rho}{\sqrt{3}}.$$

$$\text{Prisms: } \frac{p}{q} \infty; \qquad \frac{p}{q} = \frac{\sin (60° - \varphi_1)}{\sin \varphi_1}.$$

From the values for the coördinates of each face derived by the application of the above formulas the element p_0 can be obtained as an average value, using for p and q the values shown by the graphic solution. Or if p_0 is known, the exact values of p and q are to be found from the same coördinates.

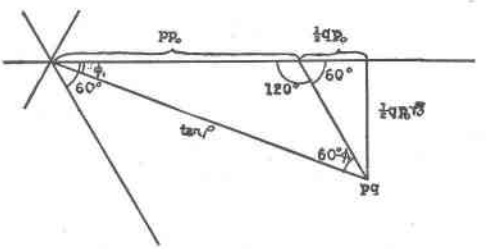

FIG. 30

CALCULATION OF ANGLES φ AND ρ FROM THE ELEMENT p_0 AND SYMBOLS.

In figure 30, using the right-angled triangle, the following relations may be established:

[2] *Z. Kryst. Min.*, **32**, 561, 1900.

$$\tan \varphi_1 = \frac{\frac{1}{2}qp_0\sqrt{3}}{pp_0 + \frac{1}{2}qp_0} = \frac{q\sqrt{3}}{2p + q} = \frac{\sqrt{3}}{2n + 1} \quad \text{where } n = \frac{p}{q},$$

$$\tan^2 \rho = (pp_0 + \tfrac{1}{2}qp_0)^2 + (\tfrac{1}{2}qp_0\sqrt{3})^2 = p_0^2(p^2 + pq + q^2),$$

whence

$$\tan \varphi_1 = \frac{\sqrt{3}}{2n + 1} \text{ where } n = \frac{p}{q}; \quad \tan \rho = p_0\sqrt{(p^2 + pq + q^2)}.$$

Since φ_1 is independent of p_0 the φ angles are alike for all hexagonal forms with like ratio of p to q. The values may be found for most cases from the table of page 25, *Winkeltabellen*. In the same way log tan ρ may be found for most forms from the tables of pages 22 and 23.

For example, to find φ and ρ for the scalenohedron $(21\bar{3}4)$ of calcite:

$$21\bar{3}4 = \tfrac{2}{4}\,\tfrac{1}{4}\,(G_1) = \tfrac{4}{4}\,\tfrac{1}{4}\,(G_2).$$

Winkeltabellen, p. 25: $p : q = 1 : 4$, $\varphi = 10° 53'$.

Winkeltabellen, p. 22: $\tan \rho = p_0\tfrac{1}{4}\sqrt{16 + 4 + 1} = p_0\tfrac{1}{4}\sqrt{21}$

$$\lg \tfrac{1}{4}\sqrt{21} = 0.05905$$

calcite $p_0 = .5695$ $\lg p_0 = 9.75552$

$$\lg \tan \rho = 9.81457 \qquad \rho = 33° 07'$$

ILLUSTRATION OF THE HEXAGONAL SYSTEM. HEMATITE FROM NEW MEXICO.[1]

WILLIAM F. FOSHAG

U. S. National Museum

A specimen of hematite in the U. S. National Museum (Mus. No. 93761) from the western part of the San Augustine Plain, Socorro Co., New Mexico has been found to show some unusual features and seems worthy of a short description. The specimen consisted of a somewhat cellular quartz in which are embedded single hematite crystals of excellent development and lustrous faces. The hematite includes quartz and the two minerals were no doubt formed at the same time.

The crystals are thick tabular in habit and, due to the equal development of the + 1 and −1 rhombohedrons, have a hexagonal aspect. The trigonal character of the crystals is brought out, however, by concentric triangular markings on the base of

[1] Published with the permission of the Secretary of the Smithsonian Institution.