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THE GOLDSCHMIDT TWO-CIRCLE METHOD. CALCULATIONS IN THE ISOMETRIC SYSTEM

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Forms and Symbols[1]

In the gnomonic projection the center of the projection is occupied by the pole of the top face of the cube; direction lines towards the front and side faces give the coördinates. In figure 24 the poles of faces of one each of the seven holohedral forms are shown, complete for one octant. The cube face (010) is the zero meridian ($\varphi = v_0 = 0$).

Cube:

Index	Gdt.	φ	ρ
(001)	0	—	0 00
(010)	0∞	0 00	90 00
(100)	$\infty 0$	90 00	90 00

[1] The derivation of the Goldschmidt symbol from that of Miller has been explained in general terms on a previous page (Gnom. Proj., p. 72). The index symbol is divided thru by its last term, making the last term unity, which is omitted. The two resulting numbers, whole or fractional, constitute the Goldschmidt symbol. Some confusion is apt to result in the case of index symbols having the last term 0 and a few examples may help to understand the usage.

(123)	$\frac{1}{3}\ \frac{2}{3}\ \frac{3}{3}$	$\frac{1}{3}\ \frac{2}{3}$	(1)	$\frac{1}{3}\ \frac{2}{3}$
(112)	$\frac{1}{2}\ \frac{1}{2}\ \frac{2}{2}$	$\frac{1}{2}\ \frac{1}{2}$	(1)	$\frac{1}{2}$
(100)	$\frac{1}{0}\ \frac{0}{0}\ \frac{0}{0}$	$\infty\ 0$	(1)	$\infty\ 0$
(110)	$\frac{1}{0}\ \frac{1}{0}\ \frac{0}{0}$	$\infty\ \infty$	(1)	∞
(120)	$\frac{1}{0}\ \frac{2}{0}\ \frac{0}{0}$	$\infty\ 2\infty$	(1)	$\infty\ 2$
(230)	$\frac{2}{0}\ \frac{3}{0}\ \frac{0}{0}$	$2\infty\ 3\infty$	(1)	$\infty\ \frac{3}{2}$
(210)	$\frac{2}{0}\ \frac{1}{0}\ \frac{0}{0}$	$2\infty\ \infty$	(1)	$2\ \infty$

The meaning of the two-place symbol, pq, is this: a distance p is measured in the X coördinate direction, and from the point so reached a distance q is measured parallel to the Y coördinate direction. The point so established is the face-pole of pq. In systems other than the isometric the distances measured are pp_0 and qq_0 and the starting point is the coördinate center, which may not coincide with the center of the projection.

For prism faces the symbol means that the face-pole is infinitely distant along a line thru the coördinate center and a face-pole whose coördinates are given in the symbol. In the cases given above the face-poles are respectively 10, 11, 12, $1\frac{3}{2}$, 21.

Where the two figures of the symbol are identical, one only is written.

OCTAHEDRON:			
(111)	1	45 00	54 44
DODECAHEDRON:			
(011)	01	0 00	45 00
(101)	10	90 00	45 00
(110)	∞	45 00	90 00

Four faces of the dodecahedron lie at the ends of the coördinate diameters and therefore on the unit circle. Four others lie at infinity along the diagonals. The four octahedron faces lie along the diagonals at the intersection points of lines tangent to the unit circle at the dodecahedron poles.

These three forms are invariable in their position in the projection. The remaining forms vary with the particular values of their indices but the general position of their poles is always as described below. The position of poles in a single octant only is described, as shown in the figure.

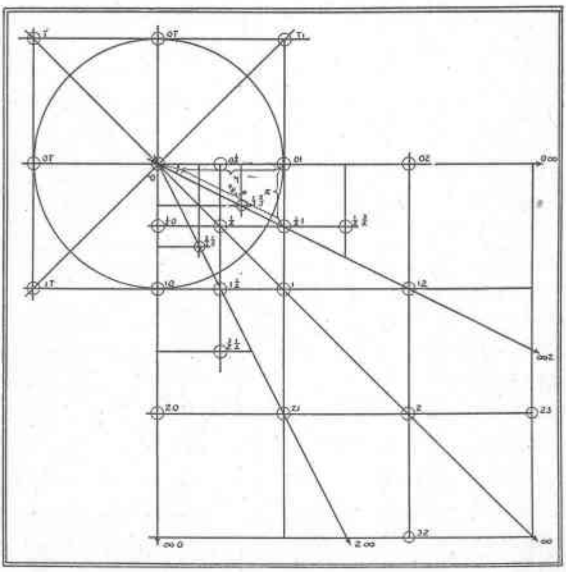

FIG. 24.

TRISOCTAHEDRON (triangle-faced trisoctahedron):

Index	Gdt.	φ	ρ
(122)	$\frac{1}{2}$ 1	26 34	48 11
(212)	1 $\frac{1}{2}$	63 26	48 11
(221)	2	45 00	70 31

The first two faces have φ complementary and the same ρ and one of them only (the first) is given in the angle-table. ρ of the third face is always greater than ρ of the first two.

TRAPEZOHEDRON (quadrilateral-faced trisoctahedron):

Index	Gdt.	φ	ρ
(112)	$\frac{1}{2}$	45 00	35 16
(121)	12	26 34	65 54
(211)	21	63 26	65 54

The last two faces have φ complementary and the same ρ and one of them only (the second) is given in the angle-table. ρ of the first face is always less than ρ of the last two.

TETRAHEXAHEDRON:

Index	Gdt.	φ	ρ
(012)	$0\frac{1}{2}$	0 00	26 34
(102)	$\frac{1}{2}$ 0	90 00	26 34
(021)	0 2	0 00	63 26
(201)	2 0	90 00	63 26
(120)	$\infty 2$	26 34	90 00
(210)	2∞	63 26	90 00

Each pair of faces with the same ρ have φ complementary; one of each pair (the first) is therefore necessary to define the form in the angle-table. It will be noticed that ρ of the first pair is complementary to ρ of the second pair and equal to φ (or its complement) of the third pair.

HEXOCTAHEDRON:

Index	Gdt.	φ	ρ
(123)	$\frac{1}{3}\ \frac{2}{3}$	26 34	36 42
(213)	$\frac{2}{3}\ \frac{1}{3}$	63 26	36 42
(132)	$\frac{1}{2}\ \frac{3}{2}$	18 26	57 41
(312)	$\frac{3}{2}\ \frac{1}{2}$	71 34	57 41
(231)	2 3	33 41	74 30
(321)	3 2	56 19	74 30

Each pair of faces with the same ρ have φ complementary; one of each pair (the first) is therefore neccessary to define the form in the angle-table.

The positions of faces of these several forms have been considered in the first octant. All forms will have corresponding faces in the other octants with the same values of ρ and with φ values as follows: In the second octant (counting clockwise) $\varphi^2 = 180° - \varphi^1$; in the third octant $\varphi^3 = -(180° - \varphi^1)$; in the fourth octant $\varphi^4 = -\varphi^1$.

In the angle-tables the forms are tabulated alike for all the various symmetry groups of the system. The symmetry is stated at the head of the table and it is left to the reader to decide which faces of a given form should be present for a given symmetry.

Calculation of Symbol from Measured Angles, φ and ρ

In Gnom. Proj., page 69, are given the following equations:

$$x = \sin\varphi \tan\rho = pp_0$$
$$y = \cos\varphi \tan\rho = qp_0$$

Since in the isometric system $p_0 = 1$, $x = p$ and $y = q$.

The values of x and y calculated from the measured angles will therefore give directly the symbol pq. The numbers obtained from log x and log y will be approximately either whole numbers or decimal equivalents of simple fractions. On page 24 of Goldschmidt's *Winkeltabellen* is given a table of decimal equivalents of the commoner fractions, which is often of use when the fraction to be identified is complex.

The symbols of most of the common isometric forms can be determined graphically as soon as the measurements are plotted in gnomonic projection.

Calculation of Angles from Symbol

$$pq \quad (p < q) \quad \tan\varphi = \frac{p}{q} \quad \tan\rho = \sqrt{p^2 + q^2} = \frac{p}{\sin\varphi} = \frac{q}{\cos\varphi}$$

(See figure 24 where the values φ and $\tan\rho$ are indicated for the form $\frac{1}{2}$ 1).

$\frac{p}{n}\frac{q}{n}$ (p and q fractional) $\tan\varphi = \frac{p}{q}$ $\qquad \tan\rho = \frac{1}{n}\sqrt{p^2+q^2}$

p $(p = q)$ $\qquad \tan\varphi = 1;\ \varphi = 45°$ $\qquad \tan\rho = p\sqrt{2}$

$0q$ $\qquad \tan\varphi = 0;\ \varphi = 0$ $\qquad \tan\rho = q$

∞q $\qquad \tan\varphi = \frac{1}{q}$ $\qquad \tan\rho = \infty; \rho = 90°$

In the Winkeltabellen, page 25, there is a table containing values of φ for all commonly occurring values of p and q. It can be used either to find φ for a given symbol (ratio of $p : q$) or to find the symbol for a measured φ angle.

On pages 22 and 23 are tables giving log tan ρ for all values of p and q from 1 to 10 and for fractional values (n = 1 to 10).

To illustrate the use of these tables, find the three values of φ and ρ for the form (123). (See above, hexoctahedron, page 114).

$\frac{p}{n}\ \frac{q}{n}$	p : q	φ (p. 25)	$\log \frac{1}{n} \sqrt{p_2 + q_2}$ (p. 22)	log tan ρ	ρ
$\frac{1}{3}\ \frac{2}{3}$	1 : 2	26° 34′	$\log \frac{1}{3} \sqrt{5}$	9.87236	36° 42′
$\frac{1}{2}\ \frac{3}{2}$	1 : 3	18 26	$\log \frac{1}{2} \sqrt{10}$	0.19897	57 41
2 3	2 : 3	33 41	$\log \sqrt{13}$	0.55697	74 30

ILLUSTRATION OF THE ISOMETRIC SYSTEM.—PYRITE FROM FALLS OF FRENCH CREEK, PA.

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Pyrite is an abundant mineral at the Falls of French Creek iron mine, and the remarkable elongated octahedral crystals formerly found there have been studied in detail by Penfield.[1] These occurred embedded in calcite; and the majority of the crystals found in that association are simple octahedral in habit, with but rarely small faces of other forms. The crystals which occur implanted on the crusts of magnetite are, however, of an entirely different habit. In them the cube is dominant, but there are also prominent faces of a rather flat diploid, and smaller ones of many other modifying forms. The cube faces are usually curved, often decidedly wavy, and more or less dulled by etching. Reaching a maximum diameter of a centimeter or more, and occurring in groupings often thickly scattered over the brilliant black magnetite, these pyrites yield striking mineral specimens, and they seem worthy of description, even though they show no new nor even unusual forms for the mineral.

Two of these crystals[2] were mounted and studied on the Goldschmidt two-circle goniometer as described in preceding papers in this series. They were oriented by means of the smoothest cube faces which could be found on each. The most brilliant faces present prove to be the quadrilateral-faced trisoctahedron or trapezohedron, (211); the dominant diploid, which proves to be (421), comes next in order of brilliance, closely followed by the cube (100). The faces of the remaining forms are relatively dull, curved, or otherwise imperfect.

[1] *Am. J. Sci.* [3], **37**, 209, 1889.
[2] Kindly loaned by Mr. S. G. Gordon.