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EXTINCTION AND 2V: A SIMPLE STEREOGRAPHIC SOLUTION OF GENERAL APPLICATION

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ABSTRACT

A new stereographic method of 2V determination, usable with extinction data obtained via any type of rotation apparatus, is presented. It is based on very simple geometrical considerations and, provided the orientations of the three principal axes of the indicatrix, however obtained, are known, employs a minimum of extinction data—the extinction angle associated with a single carefully selected general wave-normal. The sensitivity of this single extinction measurement, in terms of 2V determination by this method, varies considerably, and depends on both the orientation of the wave-normal and the value of 2V. This is illustrated by two contoured stereograms.

INTRODUCTION

There have recently been presented several purely graphical methods for the determination of 2V on the basis of varying numbers of extinction measurements plotted in stereographic projection (Wilcox, 1960; Tocher, 1962, 1964a; Joel, 1964). These are all connected, explicitly or implicitly, with coplanar extinction curves (Joel and Tocher, 1964) in so far as all the wave-normals involved are coplanar and at right angles to a single axis, P_o, about which the crystal is rotated. Moreover, in order that the necessary great circles containing either the wave-normals (Tocher, 1962, 1964a; Joel, 1964) or the axis of rotation (Wilcox, 1960, 1961) may, in each case, be constructed or utilised with the least manipulation of the net, plotting has been done with P_o plotted in the center or on the primitive circle respectively. Most of these methods are, of course, adaptable for use with conical extinction curves (Joel and Tocher, 1964), but the siting of the associated wave-normals in general positions in the stereogram, even when P_o is plotted centrally or on the primitive circle, makes the graphical constructions tedious and time-consuming, particularly with the methods of Tocher (1962, 1964a) and Joel (1964).

In addition to the above graphical methods, others involving coplanar and/or conical extinction curves and n_o curves (Joel, 1963) have been more recently advanced (Tocher, 1964b; Villarroel and Joel, 1965): these make deliberate use of the special case where the wave-normal locus contains an optic axis but, in each case, the number of experimental observations required after location of the optic axial plane may be considerable.

The graphical method now presented, although based on more general considerations, has the advantage of fairly rapid application: after the indicatrix axes and the optic axial plane have been located, the comparatively short graphical construction required is based on only one extinction measurement and can be applied with equal facility for general wave-normals in all parts of the stereogram.

The Basis of the Method

Consider any wave-normal, N (Fig. 1), in a general position with respect to the principal planes, βB_1, βB_2, and B_1B_2,[1] of a biaxial indicatrix. The associated wave-front, F, containing vibrations P, P′, will cut the βB_1 and βB_2 planes in points S and T respectively. It will also cut the circular sections, βa_1 and βa_2, at the points a_1 and a_2, one in the acute, the other in the obtuse angle between S and T.

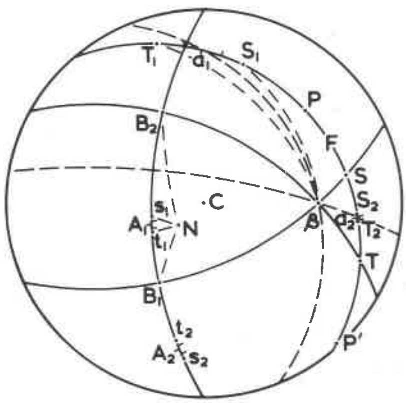


Fig. 1. Stereogram, center C, illustrating both the geometrical basis of the method and the final determination of the optic axes, A_1, A_2, and associated circular sections, βa_1, βa_2, by interpolation.

Since, by the geometry of the indicatrix, the circular sections are symmetrically disposed about β to the planes βB_1 and βB_2, *i.e.* since angles $a_1\beta S$ and $S\beta a_2$ are equal, then, in the wave-front, S must lie between a_1 and a_2. Moreover, P must bisect the angle a_1a_2. Thus, since both P and S must lie between a_1 and a_2, no circular section can cut the wave-front between P and S. But the circular sections must cut the wave-front at points on either side of and equidistant from vibration P. Therefore, over an angular distance of PS on either side of P, *i.e.* between points S and S_1 where $SP = PS_1$, no circular section can cut the wave-front. Similar considerations apply to points T and T_1, equidistant from and on either

[1] B_1 and B_2, the two bisectrices, are not differentiated as α and γ or *vice versa* in this paper, for the value of 2V about both bisectrices is determinable without a knowledge of the associated refractive indices. These, if required for sign determination, must be determined by an entirely different investigation.

side of vibration P′. It may be noted here that these considerations are confined to two octants[2] of the indicatrix diametrically opposed across β and, in consequence, are concerned with restricting the circular section traversing these octants to within an angle defined by planes βS_1 and βT_1: this circular section can cut the wave-front only between points S_1 and T_1.

Attention may now be directed towards the triangles polar to $\Delta S\beta S_1$, $\Delta S_1\beta T_1$, and $\Delta T_1\beta T$. These, ΔB_2Ns_1, Δs_1Nt_1, and Δt_1NB_1 respectively, are defined by adjacent parts, B_2s_1, s_1t_1, and t_1B_1, of the optic axial plane, on the one hand, and by the great circles, passing through N, whose poles are S, S_1, T_1 and T, on the other. Since, of the three angles $S\beta S_1$, $S_1\beta T_1$, and $T_1\beta T$, the circular section under consideration can lie only in angle $S_1\beta T_1$ it is clear that of the three parts of the optic axial plane, B_2s_1, s_1t_1, and t_1B_1, in the associated polar triangles, the corresponding optic axis can lie only between s_1 and t_1.

The second optic axis can therefore lie only between s_2 and t_2, where these are symmetrically disposed to s_1 and t_1 respectively across both bisectrices; and the associated circular section can cut the wave-front only between S_2 and T_2, where these are the points of intersection of the great circles polar to s_2 and t_2 respectively with the wave-front.

Practical Application

As a preliminary to the application of this graphical method, it is essential that the orientations of the principal axes and planes of the indicatrix be known as accurately as possible. These are determinable in a variety of ways depending upon the apparatus employed: by coplanar extinction curves, using the simple or modified (Villarroel and Joel, 1965) spindle stage (Joel and Garaycochea, 1957; Tocher, 1962; Garaycochea and Wittke, 1964) or the universal stage (Joel and Muir, 1958a); or by conical extinction curves using the inclined spindle stage or universal stage (Joel and Tocher, 1964). Orthoscopic methods with the universal stage are, in general, insufficiently accurate in this connection and should only be used for approximation purposes prior to the exact location of the principal axes by extinction methods.

On the basis of the extinction angle associated with any general wave-normal, the construction may now proceed. If using the simple spindle stage, the wave-normal will have to lie in the coplanar wave-normal locus used for determining the indicatrix; but with more versatile apparatus like the modified spindle stage, the inclined spindle stage, or the universal stage, it may be chosen in any suitable orientation within the

[2] The octants in question are, of course, those outlined by the three principal planes, βB_1, βB_2 and B_1B_2, of the indicatrix.

range of the instrument regardless of the wave-normal locus or loci used in determining the indicatrix.

Wave-normal N, wave-front F, and vibrations P and P′ having been plotted (Figs. 1, 2), points S and T can be identified and points S_1 and T_1 located. Next, points s_1 and t_1 are plotted where the great circles polar to S_1 and T_1 respectively cross the optic axial plane (although the great circles themselves need not be plotted it is worth checking that both pass through N). These in turn give rise, respectively, to s_2 and t_2, symmetrically disposed to the former across both bisectrices. Finally, points S_2 and T_2, where the great circles polar to s_2 and t_2 respectively cut the wave-front, are located. This ends the first stage of the determination and from here the final location of the optic axes and circular sections may be carried out by one of two methods: by interpolation or by progression.

By interpolation. Those portions of the optic axial plane, s_1t_1 and s_2t_2, within which the optic axes must lie, are of equal angular width, but, in general, the corresponding portions of the wave-front, S_1T_1 and S_2T_2 respectively, within which the latter must be cut by the circular sections, are of unequal angular width. This means that, for trial positions of the optic axes symmetrically disposed about the bisectrices in the arcs s_1t_1 and s_2t_2, the corresponding trial positions a_1 and a_2 (the points where the circular sections cut the wave-front in arcs S_1T_1 and S_2T_2 respectively) will, in general, be asymmetrically disposed about vibrations P and P′. Only one such trial pair of a_1 and a_2 points will have the necessary symmetrical disposition to vibrations P and P′. These are the true a_1 and a_2 positions, and the corresponding trial positions within the arcs s_1t_1 and s_2t_2 in the optic axial plane are the positions of the optic axes A_1 and A_2 respectively. Figure 1 shows several such trial positions for A_1, A_2 and a_1, a_2: the finally determined true positions are, in each case, indicated.

By progression. At the end of the first stage it was shown that, of the two circular sections, one must cut the wave-front between S_1 and T_1, the other between S_2 and T_2. However, although these two arcs, S_1T_1 and S_2T_2, subtend equal angles at β, they are not, within the wave-front, symmetrically disposed to the vibrations P and P′. In fact, the portions of the wave-front free of intersection by circular sections, originally shown to be the arcs SPS_1 and $TP'T_1$, are now extended to embrace the arcs S_1PS_2 and $T_1P'T_2$. But whereas the original arcs were determined on the premise that they must be bisected by P and P′ respectively, this is no longer the case: $S_2P > PS_1$ and $T_2P' > P'T_1$. Thus, to restore the condition of the original premise, arcs PS_1 and $P'T_1$ must be extended to points S_1' and T_1' respectively (Fig. 2) such that $S_2P = PS_1'$ and $T_2P' = P'T_1'$.

Reduction of the arc within which the wave-front may be intersected by a circular section from S_1T_1 to $S_1'T_1'$ now renders invalid the arc s_1t_1 within which an optic axis may fall. This must now be reduced to arc $s_1't_1'$ by the great circles polar to S_1' and T_1'. Next follows a reduction of arc s_2t_2 to $s_2't_2'$ so that $s_1't_1'$ and $s_2't_2'$ are equal and symmetrically disposed about both bisectrices. And this, of course, necessitates a corresponding reduction in arc S_2T_2 to $S_2'T_2'$ by the great circles polar to s_2' and t_2'.

At this point, the end of the second cycle of the progressive construction, the portions of the wave-front free of intersection by circular sections are $S_1'PS_2'$ and $T_1'P'T_2'$, neither of which is bisected by the

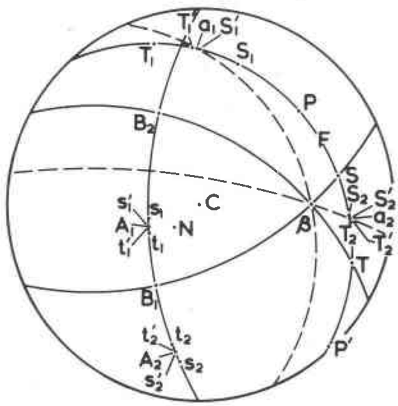


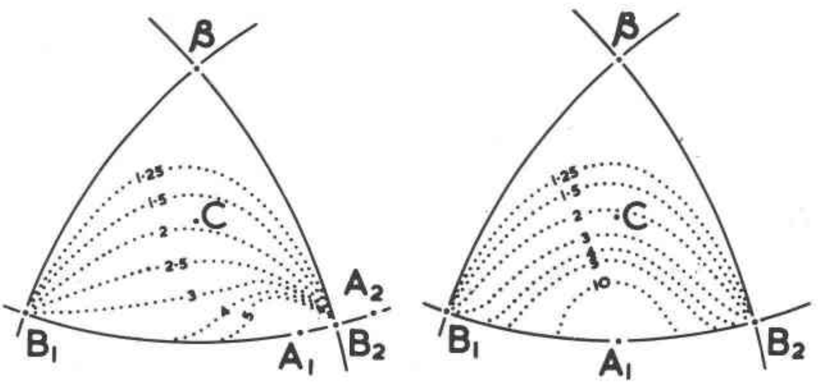
FIG. 2. Stereogram, center C, illustrating the final determination of the optic axes and associated circular sections by progression.

relevant vibration. A third cycle and, if necessary, subsequent cycles will reduce still further the arcs within which the circular sections may cut the wave-front and those within which the optic axes may lie. It is clear, in fact, that as the process is continued, the several arcs progressively decrease in size and converge on the points a_1, a_2 and the optic axes A_1, A_2 respectively.

In the more sensitive cases (see below) two complete cycles are sufficient to limit each optic axis to within a $\frac{1}{4}°$ arc of the optic axial plane (Fig. 2) and to effect virtual coincidence of points S_2' and T_2' on the wave-front—this latter coincidence serves, of course, to define point a_2 where one circular section cuts the wave-front. Point a_1 is now readily determinable between S_1' and T_1': it is symmetrically disposed with respect to a_2 about both P and P'. The optic axes are now determinable with a high degree of accuracy as the poles of planes βa_1 and βa_2. The accuracy of the final determination is, of course, dependent upon many factors not the least of which is plotting accuracy. However, given a good net of at least 20 cms diameter, plotting of points and planes can normally be done to within $\frac{1}{2}°$ of their correct positions. Of course, even this minimum tolerance can, with practice, with greater care, or with the use of a larger net be improved upon.

Sensitivity

The sensitivity of this method, as of all other methods of 2V determination, varies with conditions. It depends upon two factors: the angle 2V, and the orientation of the chosen wave-normal N with respect to the optic axes and the principal planes of the indicatrix.



Figs. 3 and 4. Partial stereograms, center C, depicting one octant of the indicatrix in the cases where $2V=20°$ (Fig. 3) and $2V=80°$ (Fig. 4). The contours show the variation in value of the sensitivity ratio, S_1T_1/S_2T_2, for general wave-normals.

However, while the sensitivity of previously advanced methods has had, perforce, to be described purely qualitatively, that of this method can be fairly simply presented in a more quantitative manner. A simple numerical indicator of the sensitivity in the case of any chosen wave-normal can be obtained by comparing the angular lengths of the two arcs, S_1T_1 and S_2T_2, at the end of the first complete graphical cycle: the greater the ratio S_1T_1/S_2T_2, the greater the sensitivity for the wave-normal concerned.

In each of Figs. 3 and 4, one octant of the indicatrix has been contoured to show how the value of the ratio S_1T_1/S_2T_2 varies for different wave-normals: Figure 3 is representative of small 2V (20°); Fig. 4 of large 2V (80°). In all cases the sensitivity decreases towards the minimum ($S_1T_1/S_2T_2=1$) as the wave-normal approaches the βB_1 and βB_2 planes, and towards the maximum ($S_1T_1/S_2T_2=\infty$) as the wave-normal approaches an optic axis. For both of these special cases (see below) the problem is insoluble by this method. It is of interest and of great value, however, that, for most positions of N within the βB_1B_2 triangle, the sensitivity of the method reaches its maximum when $2V=90°$—the case where, for the direct determination of 2V, the universal stage is, in normal random thin section work, least often usable and, even when usable, is at its least accurate (Johnston, 1953; Wyllie, 1959; Munro, 1963). Exceptions to this general rule, of course, exist for wave-normals near the optic axial plane: here, the sensitivity normally shows an increase as an optic axis is approached, regardless of the value of 2V.

Special Cases

N on the βB_1 or βB_2 plane. In this case, vibration P coincides with point S on the same principal plane as and 90° from N, while vibration P′ and point T both coincide with the bisectrix polar to that plane. Since the method is based on the non-coincidence of P and P′ with S and T respectively, it can not provide a solution in this case.

N on the optic axial plane. Here, the wave-front passes through β: in general, vibration P and both of the points S and T coincide with β; and vibration P′ falls on the optic axial plane 90° from N. As above, due to the coincidence of P with S and T, the method can provide no solution.

Refinements

Using this method alone, the most obvious refinement consists of ensuring that a wave-normal of high sensitivity is used in the final determination of 2V. Since the sensitivity of a general wave-normal in any given position depends upon the value of 2V, an approximation to the value of 2V should first be made, using any convenient general wave-normal of reasonable sensitivity. Only then can an idea of the sensitivity distribution of the available wave-normals be obtained and a suitable choice made for the final determination. Comparison of Figs. 3 and 4 shows, for example, that curves of low sensitivity, up to about 2.0, are more or less coincident for all values of 2V, and that only thereafter does the effect of 2V become very apparent.

However, although the theoretical sensitivity approaches infinity as the wave-normal approaches an optic axis, it must, at the same time, be remembered that close to an optic axis there is also a rapid decrease in the sharpness of the extinction position: a suitable balance must therefore be struck between these two phenomena in order to achieve simultaneously high sensitivity and an acceptable degree of sharpness in the extinction position. If 2V is large, there is little difficulty for, even as far as 25° in any direction from an optic axis, the sensitivity ratio has a value as high as 4.0 (Fig. 4). If 2V is small, however, the sensitivity distribution is radically different: comparable sensitivity (4.0) with acceptable sharpness in extinction (N about 20° from an optic axis) is best achieved within about 10° of the optic axial plane and on the obtuse bisectrix side of an optic axis, where the sensitivity gradient is low (Fig. 3).

With the universal stage, a very wide choice of wave-normals is normally available and a satisfactory solution can usually be obtained. With the various types of spindle stage, the choice of wave-normals is much more restricted, but reference to Figs. 3 and 4 will normally permit one to make a good final choice from those available.

Other Uses

Although this method is obviously at its most useful when neither optic axis is directly determinable, it is clear that it may be used with profit in other circumstances. It may, for example, be used in those cases where an optic axis, although directly observable with the universal stage, is near the limit of tilt. It may also be used to supplement the Biot-Fresnel construction advocated for use by Joel and Muir (1958b) in the case where one optic axis is directly observable with accuracy: the two methods employ the same data.

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