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A METHOD OF CLASSIFYING ANALYSES WITH ANY NUMBER OF TERMS

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ABSTRACT

The classification and charting of chemical and mineralogical analyses may be accomplished by the derivation of two or three index numbers that represent uniquely any such analysis. A method for attaining this objective was earlier published by the writer (Mertie, 1961), but for reasons later stated was considered to lack effectiveness. The present paper presents a new method which is believed to overcome the deficiencies of the original method.

The new process is designated as a method of selective weighting, such that for an analysis of a given number of terms every term is so weighted as to preclude the possibility of duplication in the derived index numbers. The weighting is accomplished by arranging the terms of an analysis in all possible combinations, which are determined by the algebraic formula

$${}_{n}V_{r} = \frac{n\cdot(n-1)\cdot(n-2)\cdots(n-r+1)}{r!},$$

where n is the number of terms and r is the number taken at a time. The value of r may be either 3 or 4, giving rise to two sub-methods. The combinations of terms are so written as to produce 3 or 4 columns, each of which is summed for the contained number of terms. The coefficients of the terms in these columns constitute the values that are used in weighting, and these are tabulated. The sums of the columns are so treated as to yield a set of trilinear or quadriplanar coordinates, according to whether three or four columns are used. Finally, by the use of suitable transformation formulae, earlier published by the writer (Mertie, 1964) but repeated for reference, the trilinear or quadriplanar coordinates are converted respectively to 2-dimensional or 3-dimensional cartesian coordinates. The latter constitute the derived index numbers that represent the given analysis.

Numerical examples are given for 7 kinds of igneous rocks, each with 9 terms, and for an oil shale with 18 terms. The tables used in computing the index numbers are designed for analyses with as many as 20 terms; but if needed, these tables, by the use of first and second differences, may readily be extended to cover analyses with any larger number of terms. The index numbers yielded by the method of selective weighting are believed to be unique for analyses with a stated number of terms. It has not been determined whether they are unique for all analyses, regardless of the number of terms.

INTRODUCTION

A method has earlier been presented (Mertie, 1961) for the classification and charting of chemical and mineralogical analyses with 9 terms, by the formulation of matrices and the evaluation of associated determinants. The desired results were obtained by the solution of a cubic equation, whose three roots were regarded as unique indices of the analysis. This method, however, was deficient in three respects. First, it could happen that two of the roots might be complex numbers, which can neither be compared in magnitude nor charted. It is true that a refinement of the process was given that eliminated complex numbers, but this led to additional work. Second, the method could not be applied to an analysis with as few as four terms; and for an analysis of 10 to 16 terms, the work involved rendered the process prohibitive. And third, a question of uniqueness was later raised by two readers of *The American Mineralogist*. The first and second deficiencies were stated by the writer at the time of publication; but if the third proved to be true, the method would be unacceptable.

A different method therefore seemed desirable to overcome these shortcomings. It was kept in mind that the process should be mathematically simple, and that the required calculations should not be onerous. The method to be presented is marked by an entirely new approach, which requires only elementary arithmetical operations, and is based upon formulae whose derivation need not concern the user. It utilizes the possible combinations of the terms of an analysis, taken either 3 or 4 at a time, and is applicable to all analyses having four or more terms. Certain sums derived from these combinations are used to weight the terms of an analysis in such a manner as to preclude any duplications of the derived index numbers. This process may therefore be described as a method of selective weighting.

Method of Selective Weighting

A set of terms may be arranged in combinations according to the algebraic formula

$$_{n}V_{r} = \frac{n\cdot(n-1)\cdot(n-2)\cdots(n-r+1)}{r!},$$

where n represents the number of terms in a set, and r represents the number of terms taken at a time. Thus with 10 terms taken 3 at a time, the number of combinations is

$$_{10}V_{3} = \frac{10\times 9\times 8}{3\times 2} = 120;$$

and taken 4 at a time,

$$_{10}V_{4} = \frac{10\times 9\times 8\times 7}{4\times 3\times 2} = 210.$$

The derivation and application of this principle is described for an analysis of 6 terms, because the demonstration is not lengthy; but the principle is applicable to analyses with any number of terms. Consider an analysis whose terms are designated as A, B, C, D, E and F. Taken 3 at a time, 20 possible combinations are possible, as follows:

A B C	A C E	B C D	B E F
A B D	A C F	B C E	C D E
A B E	A D E	B C F	C D F
A B F	A D F	B D E	C E F
A C D	A E F	B D F	D E F

The sum of the 20 terms of the first vertical columns is

$$X_1 = 10A + 6B + 3C + D.$$

The sums of the 20 terms of the second and third vertical columns are

$$X_2 = 4B + 6C + 6D + 4E$$

and

$$X_3 = C + 3D + 6E + 10F.$$

These three sums are combined to form a set of trilinear coordinates as follows:

$$\alpha = \frac{100\ X_1}{X_1 + X_2 + X_3}, \qquad \beta = \frac{100\ X_2}{X_1 + X_2 + X_3}, \qquad \gamma = \frac{100\ X_3}{X_1 + X_2 + X_3}$$

It thus is apparent that in the derivation of the trilinear coordinates, each term of the analysis is weighted by the coefficients of the terms shown in the three preceding equations.

The 6 terms of the analysis may also be taken 4 at a time, with the following results:

A B C D	A B E F	B C D E
A B C E	A C D E	B C D F
A B C F	A C D F	B C E F
A B D E	A C E F	B D E F
A B D F	A D E F	C D E F

The sums of the 15 terms of the 4 vertical columns are

$$\begin{aligned} Y_1 &= 10A+4B+C \\ Y_2 &= 6B+6C+3D \\ Y_3 &= 3C+6D+6E \\ Y_4 &= D+4E+10F \end{aligned}$$

The quadriplanar coordinates derivable from these sums are

$$\epsilon = \frac{100\ Y_1}{Y_1 + Y_2 + Y_3 + Y_4} \qquad \zeta = \frac{100\ Y_2}{Y_1 + Y_2 + Y_3 + Y_4}$$

$$\eta = \frac{100\ Y_3}{Y_1 + Y_2 + Y_3 + Y_4} \qquad \theta = \frac{100\ Y_4}{Y_1 + Y_2 + Y_3 + Y_4}$$

Thus two alternative methods are feasible, which are called the sub-method of triplets and the sub-method of quadruplets. No formulae are known for the transformation of what might be called pentahyperplanar or pentahyperflat coordinates to 4-dimensional cartesian coordinates.

For this reason no method has been developed for taking the terms of an analysis 5 at a time.

The most difficult task hitherto undertaken by the writer was to find a method whereby unique index numbers could be obtained to represent an analysis of only four terms. Hence the first appraisal of the present method was to apply it to this objective. Four prime numbers, 11, 7, 5, and 3 were taken to represent an analysis of four terms. It is obvious that these numbers may be arranged in 24 permutations; or in other words, 24 analyses could be formulated from these four terms. Each of these 24 analyses was tested by the sub-method of triplets, and no duplications were found in the derived values of α, β, and γ. It is also clear that no such duplications could be produced by any other analysis consisting of four different numbers. Therefore the sub-method of triplets yields unique index numbers for an analysis of four terms, Similarly, both the sub-method of triplets and the sub-method of quadruplets were found to yield unique index numbers for an analysis of five terms. Hence it is inferred that these two sub-methods will yield unique index numbers for analyses with any number of terms.

The trilinear and quadriplanar coordinates thus derived may be further simplified by transforming them respectively into 2-dimensional and 3-dimensional cartesian coordinates. This is accomplished by means of formulae derived and published by the writer (Mertie, 1964) in an earlier publication but repeated herewith. These formulae are as follows:

$$\begin{aligned}
\alpha &= 33.3333 - .8660\,x - .5\,y \\
\beta &= 33.3333 + .8660\,x - .5\,y \\
\gamma &= 33.3333 + 0 + y
\end{aligned}$$

and

$$\begin{aligned}
\epsilon &= 25 + .4714\,x - .8165\,y - .3333\,z \\
\zeta &= 25 + .4714\,x + .8165\,y - .3333\,z \\
\eta &= 25 - .9428\,x + 0 - .3333\,z \\
\theta &= 25 + 0 + 0 + z
\end{aligned}$$

Finally, therefore, all analyses having four or more terms are reducible either to two or three unique index numbers, represented by the derived cartesian coordinates. The choice of the sub-method of triplets or the sub-method of quadruplets is a matter to be decided by the individual worker, depending upon the uses to which these indices are to be applied. For some purposes, the trilinear or quadriplanar coordinates may be used as index numbers, without transforming them into cartesian coordinates.

A further refinement of the sub-methods of triplets and quadruplets is now described. The terms of a chemical analysis may have a numerical range of 9,000:1, or even a higher ratio. The use of such large and very small terms yields index numbers of larger magnitude than is either necessary or desirable. To obviate this condition, all the analyses treated in this paper are altered by using the square roots of their terms instead of the terms themselves. This involves no additional work, as these roots may be read directly from the tables by Barlow (1935). This change diminishes materially all numbers greater than unity, and increases the magnitudes of all numbers less than unity. Thus a ratio of 9,000:1 may be reduced to about 95:1. This, then, is made a standard procedure in all applications. To distinguish these square roots from the original terms of an analysis, the terms have been designated as A, B, C, etc., and the roots as a, b, c, etc. For the stated purpose, cube roots or fifth roots would be even more effective, but their use would add additional algebraic work that seems not to be warranted.

The sub-methods of triplets and quadruplets may be extended upward to cover analyses with any number of terms. For practical reasons, the cut-off in this paper is taken at 20 terms. For an analysis of 7 terms, the number of triplets and quadruplets are both 35. For less than 7 terms, the number of triplets exceeds the number of quadruplets; but for more than 7 terms, the number of quadruplets becomes rapidly larger than the number of triplets. This relationship is shown in the following table.

TABLE OF TRIPLETS AND QUADRUPLETS

Terms	${}_nV_3$	${}_nW_4$	Terms	${}_nV_3$	${}_nW_4$
4	4	—	13	286	715
5	10	5	14	364	1,001
6	20	15	15	455	1,365
7	35	35	16	560	1,820
8	56	70	17	680	2,380
9	84	126	18	816	3,060
10	120	210	19	969	3,876
11	165	330	20	1,140	4,845
12	220	495			

The tabulation of triplets and quadruplets for analyses having as many as 20 terms, and the subsequent additions of the three (or four) resulting columns, would obviously be a laborious undertaking. Enough of these tabulations and additions, however, were made by the writer to discover that the formulae for higher terms can be computed by means of first and second differences. Using the letters a, b, c, etc., indicating the square roots of the terms, the coefficients of the terms in the three or four summations have been tabulated for analyses having 4 to 20 terms. These are presented in the seven following tables.

Combinations of Terms as Triplets, Computation of α

Number of terms	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r
4	3	1																
5	6	3	1															
6	10	6	3	1														
7	15	10	6	3	1													
8	21	15	10	6	3	1												
9	28	21	15	10	6	3	1											
10	36	28	21	15	10	6	3	1										
11	45	36	28	21	15	10	6	3	1									
12	55	45	36	28	21	15	10	6	3	1								
13	66	55	45	36	28	21	15	10	6	3	1							
14	78	66	55	45	36	28	21	15	10	6	3	1						
15	91	78	66	55	45	36	28	21	15	10	6	3	1					
16	105	91	78	66	55	45	36	28	21	15	10	6	3	1				
17	120	105	91	78	66	55	45	36	28	21	15	10	6	3	1			
18	136	120	105	91	78	66	55	45	36	28	21	15	10	6	3	1		
19	153	136	120	105	91	78	66	55	45	36	28	21	15	10	6	3	1	
20	171	153	136	120	105	91	78	66	55	45	36	28	21	15	10	6	3	1
	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r

Certain general relationships are apparent. It will be noted that the sums of the multipliers required to produce successive values of α, β and γ are the same as the tabulated values of ${}_nV_3$; and similarly the sums of

Combinations of Terms as Triplets, Computation of β

Number of terms	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s
4	2	2																
5	3	4	3															
6	4	6	6	4														
7	5	8	9	8	5													
8	6	10	12	12	10	6												
9	7	12	15	16	15	12	7											
10	8	14	18	20	20	18	14	8										
11	9	16	21	24	25	24	21	16	9									
12	10	18	24	28	30	30	28	24	18	10								
13	11	20	27	32	35	36	35	32	27	20	11							
14	12	22	30	36	40	42	42	40	36	30	22	12						
15	13	24	33	40	45	48	49	48	45	40	33	24	13					
16	14	26	36	44	50	54	56	56	54	50	44	36	26	14				
17	15	28	39	48	55	60	63	64	63	60	55	48	39	28	15			
18	16	30	42	52	60	66	70	72	72	70	66	60	52	42	30	16		
19	17	32	45	56	65	72	77	80	81	80	77	72	65	56	45	32	17	
20	18	34	48	60	70	78	84	88	90	90	88	84	78	70	60	48	34	18
	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s

Combinations of Terms as Triplets, Computation of γ

Number of terms	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t
4	1	3																
5	1	3	6															
6	1	3	6	10														
7	1	3	6	10	15													
8	1	3	6	10	15	21												
9	1	3	6	10	15	21	28											
10	1	3	6	10	15	21	28	36										
11	1	3	6	10	15	21	28	36	45									
12	1	3	6	10	15	21	28	36	45	55								
13	1	3	6	10	15	21	28	36	45	55	66							
14	1	3	6	10	15	21	28	36	45	55	66	78						
15	1	3	6	10	15	21	28	36	45	55	66	78	91					
16	1	3	6	10	15	21	28	36	45	55	66	78	91	105				
17	1	3	6	10	15	21	28	36	45	55	66	78	91	105	120			
18	1	3	6	10	15	21	28	36	45	55	66	78	91	105	120	136		
19	1	3	6	10	15	21	28	36	45	55	66	78	91	105	120	136	153	
20	1	3	6	10	15	21	28	36	45	55	66	78	91	105	120	136	153	171
	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t

the multipliers required to produce successive values of ϵ, ζ, η and θ are the same as $_nW_4$. Another relationship is that the multipliers used in obtaining ϵ, and in reverse those of θ, are the same as the successive values of $_{n-1}V_3$ and the preceding numbers in the tabulation of triplets.

Combinations of Terms as Quadruplets, Computation of ϵ

Number of terms	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q
5	4	1															
6	10	4	1														
7	20	10	4	1													
8	35	20	10	4	1												
9	56	35	20	10	4	1											
10	84	56	35	20	10	4	1										
11	120	84	56	35	20	10	4	1									
12	165	120	84	56	35	20	10	4	1								
13	220	165	120	84	56	35	20	10	4	1							
14	286	220	165	120	84	56	35	20	10	4	1						
15	364	286	220	165	120	84	56	35	20	10	4	1					
16	455	364	286	220	165	120	84	56	35	20	10	4	1				
17	560	455	364	286	220	165	120	84	56	35	20	10	4	1			
18	680	560	455	364	286	220	165	120	84	56	35	20	10	4	1		
19	816	680	560	455	364	286	220	165	120	84	56	35	20	10	4	1	
20	969	816	680	560	455	364	286	220	165	120	84	56	35	20	10	4	1
	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q

Combinations of Terms as Quadruplets, Computation of ζ

Number of terms	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r
5	3	2															
6	6	6	3														
7	10	12	9	4													
8	15	20	18	12	5												
9	21	30	30	24	15	6											
10	28	42	45	40	30	18	7										
11	36	56	63	60	50	36	21	8									
12	45	72	84	84	75	60	42	24	9								
13	55	90	108	112	105	90	70	48	27	10							
14	66	110	135	144	140	126	105	80	54	30	11						
15	78	132	165	180	180	168	147	120	90	60	33	12					
16	91	156	198	220	225	216	196	168	135	100	66	36	13				
17	105	182	234	264	275	270	252	224	189	150	110	72	39	14			
18	120	210	273	312	330	330	315	288	252	210	165	120	78	42	15		
19	136	240	315	364	390	396	385	360	324	280	231	180	130	84	45	16	
20	153	272	360	420	455	468	462	440	405	360	308	252	195	140	90	48	17
	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r

A third relationship is that the denominator of the fractions used in deriving the values of α, β and γ is equal to the product of the sum of the roots of the terms, multiplied by the first factor in the table used for obtaining the trilinear coordinates of n terms. Thus for the biotite granite hereafter cited, this denominator is 21.7692×28. Similarly the denominator used in the derivation of ϵ, ζ, η and θ is equal to the product of

Combinations of Terms as Quadruplets, Computation of η

Number of terms	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s
5	2	3															
6	3	6	6														
7	4	9	12	10													
8	5	12	18	20	15												
9	6	15	24	30	30	21											
10	7	18	30	40	45	42	28										
11	8	21	36	50	60	63	56	36									
12	9	24	42	60	75	84	84	72	45								
13	10	27	48	70	90	105	112	108	90	55							
14	11	30	54	80	105	126	140	144	135	110	66						
15	12	33	60	90	120	147	168	180	180	165	132	78					
16	13	36	66	100	135	168	196	216	225	220	198	156	91				
17	14	39	72	110	150	189	224	252	270	275	264	234	182	105			
18	15	42	78	120	165	210	252	288	315	330	330	312	273	210	120		
19	16	45	84	130	180	231	280	324	360	385	396	390	364	315	240	136	
20	17	48	90	140	195	252	308	360	405	440	462	468	455	420	360	272	153
	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s

COMBINATIONS OF TERMS AS QUADRUPLETS, COMPUTATION OF θ

Number of terms	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t
5	1	4															
6	1	4	10														
7	1	4	10	20													
8	1	4	10	20	35												
9	1	4	10	20	35	56											
10	1	4	10	20	35	56	84										
11	1	4	10	20	35	56	84	120									
12	1	4	10	20	35	56	84	120	165								
13	1	4	10	20	35	56	84	120	165	220							
14	1	4	10	20	35	56	84	120	165	220	286						
15	1	4	10	20	35	56	84	120	165	220	286	364					
16	1	4	10	20	35	56	84	120	165	220	286	364	455				
17	1	4	10	20	35	56	84	120	165	220	286	364	455	560			
18	1	4	10	20	35	56	84	120	165	220	286	364	455	560	680		
19	1	4	10	20	35	56	84	120	165	220	286	364	455	560	680	816	
20	1	4	10	20	35	56	84	120	165	220	286	364	455	560	680	816	969
	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t

the roots of the terms multiplied by the first factor in the table used to compute the quadriplanar coordinates, that is, 21.7692$\times$56. By means of these tables, the user of either the sub-method of triplets or the sub-method of quadruplets is relieved of all tabulation of combinations, and is concerned only with the weighting of the terms of an analysis. The reader will be agreeably surprised to find that the derivation of the numerical indices for as many as 20 terms will involve little more work than for half that number of terms.

No duplications are believed to exist in any index numbers derived from analyses with the same number of terms. It may be possible, however, that the index numbers of analyses with n terms may be duplicated approximately by the index numbers of analyses with n+m terms. The word "approximately" is used because the application of square roots in these solutions introduces irrational numbers; therefore the index numbers could not be exactly equal unless n terms of one analysis were equal to n terms of a second analysis of n+m terms, and m terms of the second analysis were rational. But the comparison of the index numbers of all analyses, regardless of the number of their terms, is beyond the intended scope of the method of selective weighting. Only analyses with the same number of terms should be compared with regard to their index numbers. It also is important that the terms of analyses should be tabulated in the same order. For example, if the percentages of Na_2O and K_2O, without being changed, should be reversed in order, a different set of index numbers would be obtained.

Numerical Examples

Eight numerical examples are given. The first three of these are igneous rocks of widely different character, namely, granite, gabbro, and peridotite. The next four are lamprophyres, which are presented to show how effective the method of selective weighting can be for analyses of rocks having nearly the same chemical composition. All these illustrate analyses with 9 terms. Finally, there is shown an analysis of oil shale, which has 18 terms. The calculations of only the first and last of these analyses are given in full. The analyses of the igneous rocks are taken from the volumes on petrography by Johannsen (vol. 2, 1932; vol. 3, 1937; vol. 4, 1938). The analysis of oil shale was made and published by Fahey (1962), but has been slightly modified to total 100 per cent. The complete computations for a biotite granite are given below.

Biotite Granite
Sub-Method of Triplets

	Analysis	Sq. roots	First column	Second column	Third column
SiO_2	71.66	8.4652	237.0256		
Al_2O_3	14.49	3.8066	79.9386	26.6462	
Fe_2O_3	1.46	1.2083	18.1245	14.4996	1.2083
FeO	1.10	1.0488	10.4880	15.7320	3.1464
MgO	0.87	0.9327	5.5962	14.9232	5.5962
CaO	1.97	1.4036	4.2108	21.0540	14.0360
Na_2O	3.06	1.7493	1.7493	20.9916	26.2395
K_2O	4.13	2.0322		14.2254	42.6762
R	1.26	1.1225			31.4300
Totals	100.00	21.7692	357.1330	128.0720	124.3326
			128.0720		
			124.3326		
			609.5376		

$$\alpha = \frac{100 \times 357.1330}{609.5376} = 58.5908 \qquad \beta = \frac{100 \times 128.0720}{609.5376} = 21.0113$$

$$\gamma = \frac{100 \times 124.3326}{609.5376} = 20.3979$$

From the transformation formulae heretofore given, for converting trilinear into cartesian coordinates, we obtain $(x, y) = (-21.70, -12.94)$. These cartesian coordinates constitute two index numbers for this biotite granite.

The solution by the sub-method of quadruplets follows.

BIOTITE GRANITE
Sub-Method of Quadruplets

	Sq. roots	First column	Second column	Third column	Fourth column
SiO_2	8.4652	474.0512			
Al_2O_3	3.8066	133.2301	79.9386		
Fe_2O_3	1.2083	24.1660	36.2490	7.2498	
FeO	1.0488	10.4880	31.4640	15.7320	1.0488
MgO	0.9327	3.7308	22.3848	22.3848	3.7308
CaO	1.4036	1.4036	21.0540	42.1080	14.0360
Na_2O	1.7493		10.4958	52.4790	34.9860
K_2O	2.0322			42.6762	71.1270
R	1.1225				62.8600
Totals	21.7692	647.0706	201.5862	182.6298	187.7886
		201.5862			
		182.6298			
		187.7886			
		1,219.0752			

$$\epsilon = \frac{100 \times 647.0706}{1,219.0752} = 53.0778 \qquad \zeta = \frac{100 \times 201.5862}{1,219.0752} = 16.5360$$

$$\eta = \frac{100 \times 182.6298}{1,219.0752} = 14.9810 \qquad \theta = \frac{100 \times 187.7886}{1,219.0752} = 15.4042$$

From the transformation formulae heretofore given, for converting quadriplanar into cartesian coordinates, we obtain $(x, y, z) = (14.02, -22.38, -9.60)$. These cartesian coordinates constitute three index numbers for this biotite granite.

The analyses of gabbro, peridotite, and the four lamprophyres, followed by their index numbers, are shown below.

GABBRO, PERIDOTITE, AND LAMPROPHYRES

	Gabbro	Peri-dotite	Minette	Ker-santite	Vogesite	Spes-sartite
SiO_2	49.14	41.95	51.13	52.96	50.31	53.07
Al_2O_3	17.45	5.74	14.24	15.66	15.38	15.81
Fe_2O_3	3.75	4.65	3.76	4.25	3.71	3.05
FeO	5.95	6.82	4.39	4.41	5.29	4.83
MgO	6.60	26.97	6.09	5.79	6.33	6.27
CaO	10.59	6.04	6.37	6.01	7.58	7.61
Na_2O	2.58	1.11	2.40	3.28	3.03	3.60
K_2O	1.00	0.63	4.95	3.10	2.74	2.40
R	2.94	6.09	6.67	4.54	5.63	3.36
Totals	100.00	100.00	100.00	100.00	100.00	100.00

	Index numbers of igneous rocks	
	Two index numbers	Three index numbers
Gabbro	(−12.75, −12.45)	(9.85, −11.75, −10.52)
Peridotite	(− 8.84, − 7.65)	(7.46, − 7.60, − 8.61)
Minette	(−12.60, − 7.46)	(8.85, −12.39, − 5.27)
Kersantite	(−13.44, − 9.79)	(9.75, −12.93, − 7.57)
Vogesite	(−12.40, − 8.95)	(8.98, −11.95, − 6.89)
Spessartite	(−13.14, −10.69)	(9.26, −12.82, − 8.74)

A more exacting test than that of the four lamprophyres can be made by adding .01 to the value of SiO_2, and subtracting .01 from the value of some other component, say K_2O. This was done in the cited analysis of biotite granite, and the index numbers of the false analysis were then recomputed by the sub-method of triplets. The results are shown below.

False index numbers, $(x, y)=(-21.7022, -12.9420)$
True index numbers, $(x, y)=(-21.6972, -12.9354)$
Differences, - - - - - - - - - - - .0050, .0066

These differences suffice to change the value of y by one unit in the second decimal place; and the values of x and y, respectively, are changed by 5 and 7 units in the third decimal place. It would be rare indeed to have two analyses so nearly alike as in this demonstration; and therefore for all practical work two decimal places in the index numbers will suffice.

The computation of the index numbers for an oil shale of 18 terms, by the sub-methods of triplets and quadruplets, follows.

OIL SHALE (148, 137)

Sub-method of triplets

	Analysis	Sq. roots	First column	Second column	Third column
SiO_2	15.69	3.9611	538.7096		
Al_2O_3	1.98	1.4071	168.8520	22.5136	
Fe_2O_3	.61	.7810	82.0050	23.4300	.7810
FeO	.95	.9747	88.6977	40.9374	2.9241
MgO	4.74	2.1772	169.8216	113.2144	13.0632
CaO	21.36	4.5217	298.4322	271.3020	45.2170
Na_2O	12.14	3.4843	191.6365	229.9638	52.2645
K_2O	2.07	1.4387	64.7415	100.7090	30.2127
Li_2O	.04	.2000	7.2000	14.4000	5.6000
TiO_2	.10	.3162	8.8536	22.7664	11.3832
P_2O_5	.22	.4690	9.8490	32.8300	21.1050
MnO	.02	.1414	2.1210	9.3324	7.7770
S	.37	.6083	6.0830	36.4980	40.1478
Cl	.30	.5477	3.2862	28.4804	42.7206
CO_2	29.75	5.4544	16.3632	229.0848	496.3504
H_2O-	7.78	2.7893	2.7893	83.6790	292.8765
H_2O+	.81	.9000		14.4000	108.0000
Org. M.	1.07	1.0344			140.6784
Totals	100.00	31.2065	1,659.4414	1,273.5412	1,311.1014
			1,273.5412		
			1,311.1014		
			4,244.0840		

$$\alpha = \frac{100 \times 1,659.4414}{4,244.0840} = 39.1001 \qquad \beta = \frac{100 \times 1,273.5412}{4,244.0840} = 30.0074$$

$$\gamma = \frac{100 \times 1,311.1014}{4,244.0840} = 30.8925$$

From the transformation formulae heretofore given, for converting trilinear into cartesian coordinates, we obtain $(x, y) = (-5.25, -2.44)$. These cartesian coordinates constitute two index numbers for the given oil shale.

Oil Shale (148, 137)
Sub-method of quadruplets

	Sq. roots	First column	Second column	Third column	Fourth column
SiO_2	3.9611	2,693.5480			
Al_2O_3	1.4071	787.9760	168.8520		
Fe_2O_3	.7810	355.3550	164.0100	11.7150	
FeO	.9747	354.7908	266.0931	40.9374	.9747
MgO	2.1772	622.6792	679.2864	169.8216	8.7088
CaO	4.5217	994.7740	1,492.1610	542.6040	45.2170
Na_2O	3.4843	574.9095	1,149.8190	574.9095	69.6860
K_2O	1.4387	172.6440	453.1905	302.1270	50.3545
Li_2O	.2000	16.8000	57.6000	50.4000	11.2000
TiO_2	.3162	17.7072	79.6824	91.0656	26.5608
P_2O_5	.4690	16.4150	98.4900	147.7350	56.2800
MnO	.1414	2.8280	23.3310	46.6620	23.3310
S	.6083	6.0830	72.9960	200.7390	133.8260
Cl	.5477	2.1908	42.7206	170.8824	156.6422
CO_2	5.4544	5.4544	229.0848	1,489.0512	1,985.4016
H_2O^-	2.7893		41.8395	585.7530	1,269.1315
H_2O^+	.9000			108.0000	504.0000
Org. M.	1.0344				703.3920
Totals	31.2065	6,624.1544	5,019.1563	4,532.5027	5,044.7061
		5,019.1563			
		4,532.5027			
		5,044.7061			
		21,220.5200			

$$\epsilon = \frac{100 \times 6,624.1544}{21,220.5200} = 31.2158 \qquad \zeta = \frac{100 \times 5,019.1563}{21,220.5200} = 23.6524$$

$$\eta = \frac{100 \times 4,532.5027}{21,220.5200} = 21.3590 \qquad \theta = \frac{100 \times 5,044.7061}{21,220.5200} = 23.7728$$

From the transformation formulae heretofore given, for converting quadriplanar into cartesian coordinates, we obtain $(x, y, z) = (4.51, -4.26, -1.23)$. These cartesian coordinates constitute three index numbers for the given oil shale.

Summary

A practical method has been presented for classifying analyses with any number of terms. This method, with two sub-methods, is designed to replace an earlier method published by the writer (Mertie, 1961). It is mathematically simple, and the necessary calculations for any analysis can be accomplished on a single sheet of paper. The values of the coefficients used in weighting the terms of an analysis are tabulated for analyses with 4 to 20 terms, thus obviating the only part of the process

that could be considered laborious. The derived index numbers comprise only positive and negative real numbers that may be compared for magnitude, charted, or otherwise utilized. These indices are believed to be unique for all analyses with a fixed number of terms; they may or may not be unique for analyses without regard to their number of terms.

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