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MODAL VARIATIONS WITHIN GRANITIC OUTCROPS

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ABSTRACT

The source and magnitude of the modal variations determined by point counting thin sections from single outcrops of granitic rock were studied to determine the most efficient means of sampling. The rock is equigranular with an average grain size of 2 mm and an I.C. number of 32.

The "average" standard deviation of single thin section modes from an outcrop is 2.6% of the whole for the major minerals compared to the expected variance ("error") for random samples of 1600 points from a homogeneous population of 1.2%. The "average" standard deviation for the major minerals determined from multiple thin sections cut from single hand samples is 2.2% while repeated counts of a single section "average" only 1.3%. Analysis of variance of the modes of four thin sections from each of four hand samples collected only five feet apart from an adamellite outcrop showed that the quartz content of the hand samples was significantly different even though the total standard deviation of the quartz determinations was large (2.4%).

Because these variations are large compared to the differences often sought in studies of plutonic rocks, statistical methods of analysis are needed to design the sampling plan and aid in the interpretation of the results. For example, to determine quartz in the adamellite, it is 2.35 times more efficient to point count one section from each of four hand samples than to count four sections from one hand sample. Thus, by proper design, smaller variations can be detected withoug additional cost of time or money.

INTRODUCTION

A considerable amount of work has been done on the variability of thin section mode determinations of the finer-grained granitic rocks from the levels of operator variations and single thin section reproducibility to the variability of an entire granite mass (Chayes, 1952, 1956). The results of previous work are an indication of the variability to be expected in a modal analysis of similar rocks. However, to properly interpret modal information it is necessary to determine the variations of each rock type studied.

Modal compositions used in the study of over 120 square miles of granitic rock of the Inyo batholith, California-Nevada, were estimates from one thin section for each square mile. Significant differences between the major rock types could be found with no more than ten sections from each unit (Emerson, 1959). Variations within outcrops were found to contain a major part of the pluton variation. This within-outcrop variation can be split into several levels and two of these (sections within hand samples, hand samples within outcrops) were studied.

Volume percentages of the minerals present in the various rock types were estimated by micro point counting. The American Optical Company point counting stage with an east-west point spacing of 0.3 millimeter was used with a microscope magnification of fifty times. Cellophane tape was placed on each end of the thin section to provide a straight edge for the area sampled and to avoid bias from the broken edge of the rock section. The area sampled from each section was nearly constant and averaged 750 square millimeters. Sixteen 100 point traverses were spaced to sample the entire thin section area. Results of previous studies (Larsen and Miller, 1935, p. 723; Chayes, 1956, p. 40) guided the writer in establishing the number of traverses and the total number of points used in these modal analyses.

Eight classes, quartz, stained potash feldspar (for staining technique see Rosenblum, 1956), plagioclase, nonopaque accessories, biotite, chlorite, amphibole and opaque accessories were counted. Fine-grained alteration products of the feldspars were counted as feldspar. Although the albite of some of the perthites was recognizable at the magnification used, it was counted as potash feldspar.

HAND SAMPLE VARIATION

Single sections from hand samples spaced about five feet apart within an outcrop of the Sagehen adamellite, the Leidy adamellite and two outcrops of the Barcroft granodiorite were point counted. These rocks are equigranular, have an average grain size of two millimeters and an identity change number (the number of major mineral identity changes along a 25 mm traverse—Chayes, 1964) of 32. Table 1 summarizes the modal data.

Any one standard deviation from Table 1 is subject to rather large error because it is based on only four thin sections. A method of combining the estimates of variation is needed to obtain a better estimate of this variation. After discussing the assumptions involved, Chayes (1956, p. 67) calculated the square root of the average variance (an "average" standard deviation) of minerals of approximately the same abundance. This combines the variations of separate minerals and provides a better estimate of the variation. "Average" standard deviations calculated in this way for the modal determinations are shown in Table 2. To aid in interpretation, mineral variations discussed will always be based on per cent of the whole and not of the amount present.

These "averages" contain variations due to the technique of estimation as well as the actual mineral content variations. As will be shown later, the within-outcrop variations are dependent on the mineral determined as well as the mineral abundance.

To understand the total variation the variation at the different levels of sampling must be determined. To estimate the magnitude of the varia-

TABLE 1. THIN SECTION MODES FROM SINGLE OUTCROPS

Sect.	Qtz.	K-spar	Plag.	Acc.	Bio.	Chl.	Amph.	Opaque
Sagehen adamellite #4								
Ia	19.4	25.7	46.6	0.4	3.1	1.6	2.5	0.7
IIa	22.8	21.5	47.4	0.7	3.6	0.8	2.4	0.8
IIIa	22.7	23.4	43.6	1.6	3.1	1.5	2.8	1.3
IVa	25.4	21.0	41.7	1.2	5.5	1.9	2.4	0.9
Ave.	22.6	22.9	44.8	1.0	3.8	1.5	2.5	0.9
S.D.	2.5	2.1	2.6	0.5	1.1	0.5	0.2	0.3
Leidy adamellite #18								
I	28.8	30.7	37.1	0.6	1.9	0.2	—	0.7
II	32.4	27.9	36.9	0.7	0.9	0.6	—	0.6
III	31.6	26.4	38.9	0.8	1.5	0.7	—	0.1
IV	33.6	26.6	37.3	0.8	0.9	0.6	—	0.2
Ave.	31.6	27.9	37.6	0.7	1.3	0.5	—	0.4
S.D.	2.0	2.0	0.9	0.1	0.5	0.2	—	0.3
Barcroft granodiorite 11A								
I	10.6	21.4	44.8	0.2	10.4	—	11.6	1.0
II	5.4	18.1	46.1	0.2	12.1	—	16.5	1.6
III	10.1	26.3	39.4	0.5	9.3	—	11.9	2.5
IV	8.4	24.1	39.5	0.2	12.4	—	14.1	1.3
Ave.	8.6	22.5	42.5	0.3	11.0	—	13.5	1.6
S.D.	2.4	3.5	3.5	0.1	1.5	—	2.4	0.6
Barcroft granodiorite #32								
I	12.9	31.8	35.9	0.1	4.9	0.1	11.5	2.8
II	12.4	26.4	40.4	0.3	6.7	0.1	11.7	2.0
III	14.0	23.2	39.4	0.3	12.7	—	8.1	2.3
IV	9.9	33.1	35.3	0.3	3.7	0.9	14.0	2.8
Ave.	12.3	28.6	37.8	0.2	7.0	0.3	11.3	2.5
S.D.	1.7	4.6	2.5	0.1	4.0	0.5	2.4	0.4

tion between thin sections from the same hand sample, four sections were cut from each of four samples taken from the outcrops of Table 1. When the standard deviations of these modes (Table 3) are "averaged" (Table 2) they are seen to contain a major part of the total variation.

To estimate the precision obtained from a thin section, four counts of the same section were made (Table 4). Because of the method of traverse selection, which for each count samples 16 of a possible 200 traverses, repeated traverses of the same path are rare even when duplicate counts are made of the same section. Each count is essentially a random sample of parallel traverses within the same area. These samples, which are selected without replacement from finite traverse population, will yield variances somewhat lower than would be obtained by sampling with replacement.

The "average" standard deviation of replicate counting of a thin section (Table 2) is of the same magnitude as the expected variance ("error") for random samples of 1600 points from homogeneous populations of appropriate compositions (Chayes 1956, p. 39). More points must be counted in order to improve the estimate of the minerals that constitute less than five per cent of the rock. There were a sufficient number of points counted to show that the more abundant minerals have more variation than can be attributed to a single thin section. In addition, it appears that one hand sample does not adequately represent an outcrop of granitic rock.

TABLE 2. "AVERAGE" STANDARD DEVIATIONS OF SINGLE THIN SECTION MODES

Level of Variation	Mineral abundance			
	50 to 20%	20 to 5%	5 to 1%	1 to 0%
Outcrop (Table 1)	2.6	2.4	0.6	0.3
Hand Sample (Table 3)	2.2	2.0	0.4	0.2
Thin Section (Table 4)	1.3	0.9	0.6	0.2
"Error" (at maximum mineral abundance)	1.2	1.0	0.5	0.2

ANALYSIS OF VARIANCE

To estimate more precisely the magnitude of the variation between thin sections from one hand sample and between hand samples from a single outcrop, four samples about five feet apart were collected from one outcrop of the Sagehen adamellite. Four thin sections from each sample were point counted (Table 5). It is interesting to note that the maximum analytical error in the point count determination of the major compo-

Table 3. Thin Section Modes from Single Hand Samples

Sect.	Qtz.	K-spar	Plag.	Acc.	Bio.	Chl.	Amph.	Opaque
Sagehen adamellite Hand sample #4(I)								
a	19.4	25.7	46.6	0.4	3.1	1.6	2.5	0.7
b	20.1	24.5	46.5	1.3	4.0	0.4	2.2	1.0
c	22.3	21.5	46.1	0.8	3.9	0.9	3.0	1.5
d	21.4	27.1	43.0	0.8	4.7	0.8	1.9	0.3
Ave.	20.8	24.7	45.6	0.8	3.9	0.9	2.4	0.9
S.D.	1.3	2.4	1.7	0.4	0.7	0.5	0.5	0.5
Leidy adamellite Hand sample #18(I)								
a	28.8	30.7	37.1	0.6	1.9	0.5	—	0.7
b	28.1	30.5	38.4	0.4	1.6	0.4	—	0.6
c	30.9	24.1	41.5	0.8	1.4	0.6	—	0.7
d	27.4	29.6	39.2	0.8	2.0	0.7	—	0.3
Ave.	28.8	28.7	39.1	0.6	1.7	0.5	—	0.6
S.D.	1.5	3.1	1.8	0.2	0.3	0.1	—	0.2
Barcroft granodiorite Hand sample 11A(I)								
a	10.6	21.4	44.8	0.2	10.4	—	11.6	1.0
b	11.8	17.7	43.4	0.1	15.7	—	10.2	1.1
c	11.9	22.1	40.7	0.3	15.2	—	8.4	1.4
d	11.6	21.7	39.9	0.2	14.1	—	11.2	1.3
Ave.	11.5	20.7	42.2	0.2	13.8	—	10.4	1.2
S.D.	0.6	2.0	2.3	0.1	2.4	—	1.4	0.2
Barcroft granodiorite Hand sample #32(I)								
a	12.9	31.8	35.9	0.1	4.9	0.1	11.5	2.8
b	13.2	29.6	37.6	0.2	6.6	0.2	10.7	1.9
c	14.0	28.6	38.2	0.4	8.9	0.1	7.6	2.2
d	14.6	22.3	40.6	0.6	13.8	0.2	6.1	1.8
Ave.	13.7	28.1	38.1	0.3	8.5	0.1	9.0	2.2
S.D.	0.8	4.1	1.9	0.2	3.9	0.0	2.6	0.4

TABLE 4. QUADRUPLICATE POINT COUNTS FROM SINGLE THIN SECTIONS

Count	Qtz.	K-Spar	Plag.	Acc.	Bio.	Chl.	Amph.	Opaque
Sagehen adamellite #4 (Ia)								
(1)	19.4	25.7	46.6	0.4	3.1	1.6	2.5	0.7
(2)	22.2	26.2	42.0	0.3	4.1	1.2	2.9	1.0
(3)	19.8	26.9	46.9	0.7	2.2	1.3	1.6	0.6
(4)	20.0	27.1	45.6	0.6	2.8	0.7	2.3	0.9
Ave.	20.4	26.5	45.3	0.5	3.0	1.2	2.3	0.8
S.D.	1.3	0.6	2.2	0.2	0.8	0.4	0.5	0.2
Sagehen adamellite #6								
(1)	26.7	22.7	45.2	0.6	3.0	1.6	0.1	0.1
(2)	25.8	19.9	49.8	0.5	1.2	2.7	—	0.1
(3)	25.4	22.0	48.3	0.9	1.4	1.9	—	0.1
(4)	26.5	20.6	46.1	1.3	3.3	2.1	—	0.1
Ave.	26.1	21.3	47.4	0.8	2.2	2.1	0.0	0.1
S.D.	0.5	1.3	2.1	0.4	1.1	0.5	0.0	0.0
Barcroft granodiorite 11A (III)								
(1)	10.1	26.3	39.4	0.5	9.3	—	11.9	2.5
(2)	9.8	27.2	39.2	0.5	8.8	—	12.4	2.1
(3)	10.5	28.1	39.0	0.3	9.3	—	11.3	1.5
(4)	11.7	24.0	39.7	0.6	10.6	—	11.3	2.1
Ave.	10.5	26.4	39.3	0.5	9.5	—	11.7	2.0
S.D.	0.8	1.8	0.3	0.1	0.8	—	0.5	0.3
Barcroft granodiorite #32 (Ia)								
(1)	12.9	31.8	35.9	0.1	4.9	0.1	11.5	2.8
(2)	9.8	34.1	37.4	0.1	5.9	0.1	10.7	1.9
(3)	10.1	31.2	38.8	0.3	7.0	0.3	10.1	2.2
(4)	9.5	31.6	37.6	0.5	6.7	—	11.8	2.3
Ave.	10.5	32.2	37.5	0.3	6.1	0.1	11.0	2.3
S.D.	1.6	1.3	1.2	0.2	0.9	0.1	0.8	0.4

nents is not more than the 2.5% predicted by Chayes (1956, p. 83) for sections of this area and grain size. For comparison with the grand total standard deviation, the last line of Table 5 is the "error expected for random samples of 1600 points from homogeneous populations."

Table 5. Hand Sample Thin Section Modes from One Outcrop of the Sagehen Adamellite

Sect.	Qtz.	K-spar	Plag.	Acc.	Bio.	Chl.	Amph.	Opaque
Hand sample #4 (I)								
a	19.4	25.7	46.6	0.4	3.1	1.6	2.5	0.7
b	20.1	24.5	46.5	1.3	4.0	0.4	2.2	1.0
c	22.3	21.5	46.1	0.8	3.9	0.9	3.0	1.5
d	21.4	27.1	43.0	0.8	4.7	0.8	1.8	0.3
Ave.	20.8	24.7	45.6	0.8	3.9	0.9	2.4	0.9
S.D.	1.3	2.4	1.7	0.4	0.7	0.5	0.5	0.5
Hand sample #4 (II)								
a	22.8	21.5	47.4	0.7	3.6	0.8	2.4	0.8
b	22.0	23.7	44.1	1.3	3.8	0.8	3.3	1.0
c	27.0	22.2	44.5	1.2	2.1	0.9	1.5	0.6
d	26.7	25.0	40.0	0.6	4.7	0.9	1.4	0.7
Ave.	24.6	23.1	44.0	1.0	3.6	0.8	2.1	0.8
S.D.	2.6	1.6	3.0	0.4	1.1	0.1	0.9	0.2
Hand sample #4 (III)								
a	22.7	23.4	43.6	1.6	3.1	1.5	2.8	1.3
b	25.1	23.2	43.7	1.1	3.2	0.7	2.5	0.5
c	27.8	24.9	40.1	0.6	2.1	1.0	2.7	0.8
d	24.6	24.7	43.6	0.9	2.1	1.2	2.2	0.7
Ave.	25.0	24.0	42.8	1.0	2.6	1.1	2.6	0.8
S.D.	2.1	0.9	1.8	0.4	0.6	0.3	0.3	0.4
Hand sample #4 (IV)								
a	25.4	21.0	41.7	1.2	5.5	1.9	2.4	0.9
b	22.9	25.9	42.7	0.8	2.8	1.7	2.4	0.8
c	23.7	28.2	40.4	0.5	1.8	2.2	2.9	0.3
d	24.6	23.8	45.5	0.3	3.1	0.4	1.7	0.6
Ave.	24.1	24.7	42.6	0.7	3.3	1.6	2.4	0.6
S.D.	1.1	3.1	2.2	0.4	1.6	0.8	0.5	0.3
Grand total								
Ave.	23.6	24.1	43.7	0.9	3.4	1.1	2.4	0.8
S.D.	2.4	2.0	2.4	0.4	1.0	0.5	0.5	0.3
Expected "Error"	1.1	1.1	1.2	0.2	0.4	0.3	0.4	0.2

When n is large and the product np, the number of points within a sample (n) times the proportion of the mineral in the sample (p), is larger than 10 the binomial distribution is very close to a normal distribution and analysis of variance is appropriate without a transformation of the data. The data (Table 5) were treated by a random effects model of the analysis of variance (Snedecor, 1956, p. 259).

Quartz is found to be significantly different between hand samples (Table 6). This difference appears to be due to sample #4(I) which averages almost four per cent less quartz than the other three samples. The other minerals show no significant differences between hand samples.

TABLE 6. ANALYSIS OF VARIANCE OF WITHIN OUTCROP VARIATION STUDY (DATA FROM TABLE 5)

Source of variation	d.f.	Quartz		Potash feldspar		Plagioclase		Nonopaque accessories	
		M.S.	F.	M.S.	F.	M.S.	F.	M.S.	F.
Hand Sample	3	15.04	4.30[1]	2.33	0.51	7.57	1.51	1.20	1.08
Thin Section	12	3.50		4.57		5.00		1.11	
Total	15	5.81		4.12		5.51		1.12	
		Biotite		Chlorite		Amphibole		Opaque accessories	
		M.S.	F.	M.S.	F.	M.S.	F.	M.S.	F.
Hand Sample	3	1.20	1.08	0.39	1.56	0.11	0.33	0.04	0.33
Thin Section	12	1.11		0.25		0.33		0.12	
Total	15	1.12		0.28		0.29		0.10	

[1] Significant at the 5 per cent level.

The cost, in time and money, of making and point counting thin sections is high. To be most efficient, it is important to obtain the maximum amount of information from the minimum number of thin sections. The analysis of variance provides the information necessary to determine the magnitude of the variations at various levels in the analysis. From this information the best sampling plan can be designed. Among the major minerals of this study, quartz shows the greatest variation. The following plan is based on its determination.

If the magnitude of the variation sought is known, you can determine the number of thin sections needed. However, in petrology the limiting factor is often the number of thin sections that can be studied in the time available. Also, the magnitude of the variation is seldom known. If a fixed number of thin sections (*e.g.* 4) are to be used at each outcrop, the problem is to determine what sections should be made and counted. From the analysis of variance, the section mean square (3.50 for quartz in Table 6) is an estimate of the component of variance due to variations between thin sections from a hand sample. This estimate is designated s^2. The sample mean square (15.04) is an estimate of the section mean square plus four times the component of variance due to variations between hand samples $(s^2+4s_H^2)$. For quartz, the estimate of the component of variance due to hand samples, $(s_H^2)=(15.04-3.50)/4=2.88$. Snedecor (1956, p. 261) shows that the mean square of the hand sample mean, $s_{\bar{H}}^2=[s^2+(s_H^2)]/an$; where $a=$ number of hand samples and $n=$ number of sections per hand sample. To be most efficient, $s_{\bar{H}}^2$ should be minimized. If one hand sample is used, $a=1$, $n=4$ and $s_{\bar{H}}^2=3.76$. If, on the other hand, four hand samples are used with only one section from each one, $a=4$, $n=1$ and $s_{\bar{H}}^2=1.60$. The second plan when compared to the first is said to be 2.35 times or 235% more efficient. By the same reasoning, two thin sections from each of two hand samples would be 1.64 times or 165% more efficient than taking four sections from one sample.

SUMMARY

One of the biggest problems in quantitative petrology is obtaining the data necessary to properly design a sampling plan. This study gives some idea of the magnitude and location of the variations found by point counting plutonic rocks. Each mineral from each rock type presents a separate sampling problem. If the fluctuation or "error" of the modal analyses used in a specific problem are of the same order of magnitude as the differences sought, then detailed statistical methods of analysis are needed to design efficient sampling plans and interpret the resulting information.

In this example, the between-thin-section results show that a large part of the total variation is at this level. More or larger sections are needed to detect differences between outcrops. The between-hand-sample variation is clearly large enough that more than one sample should be taken from each outcrop to detect differences between outcrops. To determine within-pluton variations of the Inyo granitic rocks more than one section and more than one hand sample should be used. Both of these conditions can best be met by examining a thin section from two or more hand samples taken from each outcrop studied.

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