

PIEZOBIREFRINGENCE IN SILICON*

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ABSTRACT

A stress-birefringence investigation has been carried out on high purity silicon at a wavelength of 1.11μ . Values have been obtained for the piezobirefringence constants ($q_{1111}-q_{1122}$) and $2q_{1212}$. The linearity of the stress-optical relationship in silicon has been established up to stresses of approximately 450 kilograms/cm². A qualitative indication has been obtained of the directions of change in the index of refraction of silicon under a stress applied parallel to [100]. Various transmission curves are given for World War II surplus infrared image converter tubes with Si, Si+H₂O, and Si+ Corning No. 5850 filters, respectively. An observation is noted on orders of interference resulting from the use of non-monochromatic infrared light analogous to the orders of interference colors commonly observed with visible white light.

INTRODUCTION

Thorough reviews of the field of study concerned with the stress-optical behavior of crystalline materials have been given by both Poindexter (1) and Giardini (2). The present paper reports on the stress-birefringence of high purity silicon at a wavelength of 1.11μ .

The silicon used in this investigation was obtained from the Army Signal Corps Engineering Laboratories, Fort Monmouth, New Jersey. The boule from which specimens were cut was elongated (pulled) parallel to [111]. Some spinel-type twinning was present in the boule; however, specimens were taken from areas free of twinning. The latter was determined both by visual inspection of specimen surface and by x-ray diffraction. No quantitative information is available on the purity of the silicon other than it is described as "high purity" material.

Since silicon possesses $m3m$ symmetry, only constants q_{1111} , q_{1122} and q_{1212} are required for a complete description of stress-optical behavior (3, 4). Although definitions and derivations of these constants can be found in either reference (1) or (2) given above, they will be redefined here for the sake of completeness.

The stress-optical constant q_{1111} describes the relationship between a homogeneous stress directed along the crystallographic axis $X_1\bar{X}_1$ and the resulting retardation of light vibrating parallel to and traveling perpendicular to $X_1\bar{X}_1$. The stress-optical constant q_{1122} defines the relationship between a homogeneous stress directed along $X_1\bar{X}_1$ and the induced retardation of light vibrating parallel to $X_2\bar{X}_2$ and traveling parallel to $X_3\bar{X}_3$. The value ($q_{1111}-q_{1122}$) constitutes the stress-optical difference or piezobirefringence constant. The constant $2q_{1212}$ describes the relationship

* Contribution from the Department of Mineralogy, University of Michigan, No. 216. This research was supported financially by the Army Signal Corps Engineering Laboratories, Fort Monmouth, New Jersey.

between a homogeneous stress directed along X_{12} (along $2/m$ for symmetry class $m3m$) and the resulting difference in retardation of light vibrating parallel to $X_1'\bar{X}_1'^*$ and $X_2'\bar{X}_2'^*$,* and traveling parallel to $X_3\bar{X}_3$. Constant $2q_{1212}$, therefore, constitutes a piezobirefringence constant for crystals possessing $m3m$ symmetry.

Values for both piezobirefringence constants ($q_{1111}-q_{1122}$) and $2q_{1212}$, at a wavelength of 1.11μ have been determined and will be presented later in this report.

SPECIMEN PREPARATION

Polished oriented rectangular parallelepiped specimens are generally preferred for piezobirefringence investigation. The equipment and procedures which were used for orienting and cutting silicon are described in this issue on page 370.

For the measurement of the difference constant ($q_{1111}-q_{1122}$), a parallelepiped was prepared having all crystallographic cube faces, and dimensions $0.312 \text{ cm.} \times 0.4145 \text{ cm.} \times 0.518 \text{ cm.}$ For the determination of $2q_{1212}$, a parallelepiped was cut having a zone of four dodecahedral faces terminated by a pair of parallel cube faces. The dimensions of the cube faces are $0.648 \text{ cm.} \times 0.574 \text{ cm.}$ and that common to the dodecahedral faces is 0.284 cm. The crystallographic orientation of both specimens is accurate to within 15 minutes of arc. The parallelism of opposing sides of the parallelepipeds are correct to within 1 minute of arc.

In order to investigate the absolute change in index of refraction as a function of stress applied parallel to $[100]$, a silicon specimen was prepared in the form of a prism frustum. The right trapezium faces of the prism were crystallographic (100) faces with the following dimensions: base = 0.890 cm. , altitude = 0.393 cm. , second altitude = 0.160 cm. , hypotenuse = 0.920 cm. The included prism angle, as determined by optical goniometry, is $14^\circ 36.5'$. The frustum is preferred to a right triangular type of prism because of its superior structural configuration.

Polishing of the oriented specimens was carried out with the following abrasive media in the order listed: 1) 350 grit SiC paper mounted on plate glass, 2) 500 grit SiC paper mounted on plate glass, 3) 8-25 micron diamond powder suspended in cold cream and spread on a sheet of index card paper which in turn is mounted on plate glass, 4) same as 3) but with 4-8 micron diamond powder, 5) same as 3) but with $\frac{1}{2}$ -3 micron diamond powder. Each stage requires roughly about 1-2 minutes of polishing with a light hand pressure. Both the specimen and hands must be thoroughly cleansed before proceeding from one stage to the next.

* Directions $X_1'\bar{X}_1'$ and $X_2'\bar{X}_2'$ are respectively parallel to and normal to X_{12} and in the plane of crystallographic directions $X_1\bar{X}_1$ and $X_2\bar{X}_2$.

The above procedure has been found to produce a good quality optical surface on silicon.

INSTRUMENTATION

A schematic view of the experimental apparatus used to determine the values of the piezobirefringence constants of silicon is given in Fig. 1. The light source consists of a General Electric 6 volt, 54 watt vertical tungsten ribbon lamp. A wide aperture lens system was used to collimate the light into a parallel beam and direct it through a nicol prism polarizer. The vibration direction of the polarizer is oriented 45° from verti-

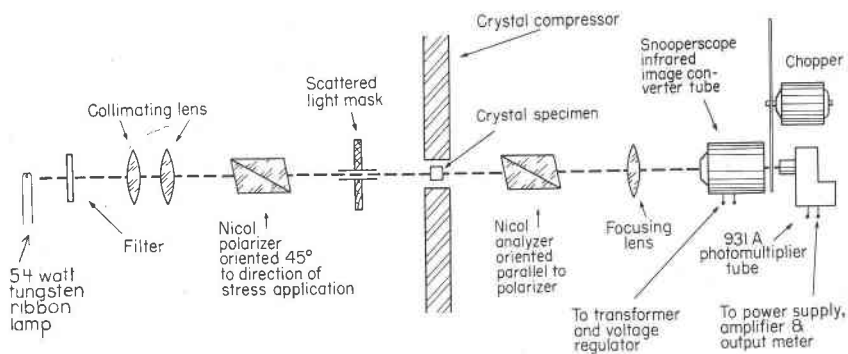


FIG. 1. A schematic diagram of the experimental apparatus used to determine the piezobirefringence constants of silicon.

cal, the latter being the direction of stress application to the crystal. A mask, provided with an aperture proper to the crystal dimensions, is placed between the polarizer and crystal specimen in order to eliminate all light except that passing through the specimen.

The apparatus used to apply stress to the silicon is a second class lever type of crystal compressor. The unit is similar to one previously described (2); however, the following modifications have been made: *a*) The pressure ram of the earlier model has been replaced by opposed vertical sections of $\frac{3}{4}$ inch diameter steel drill rod machined square at the ends and equipped with flat and parallel hardened steel tips. *b*) The cold-rolled steel plates formerly used as pressure ram guides have been replaced by a $1\frac{3}{8}$ inch thick normalized cast-iron block provided with a vertical guide hole for the drill rod plungers and an intersecting horizontal aperture for light transmission through the crystal. The pressure ram guide block is mounted vertically on a similar $1\frac{3}{8}$ inch thick cast iron base block. These improvements have been found to provide more easily reproducible conditions of stress distribution on crystal surfaces.

A nicol prism analyzer, oriented parallel to the polarizer, is placed on the opposite side of the crystal from the polarizer. A lens is used to focus the image of the crystal upon the cathode surface of a World War II war surplus British "snooperscope" infrared image converter tube, model CV-147. The brightness of the converted infrared image was measured by a 931A photomultiplier tube. The output signal from the photomultiplier tube was fed to a four-stage all-triode audio frequency amplifier tuned to 750 cps. The light chopper was placed between the infrared image converter and photomultiplier tubes in order to eliminate the effect of any image persistence by the active material of the converter tube. Changes in stress-induced retardation by the crystal specimen as a function of applied stress were read visually on a Triplett model 420-PL D.C. milliammeter. All electrical power used was drawn through a Sola CVH constant voltage transformer.

An infrared image converter tube was used in preference to a lead sulfide detector in order to permit visual inspection of the crystal specimen for optical defects and homogeneity of stress distribution. For some applications, however, the lead sulfide detector is a more advantageous instrument. The need for a phototube is then eliminated, the signal to noise ratio is better and power requirements are very simple.

Calibration of the "snooperscope" tube with respect to conversion range and band pass was carried out with a Leiss double prism-single pass spectrometer with a motor driven wavelength selector.* A zirconium point source lamp was used as a light source. The spectrometer itself was calibrated by means of a mercury vapor lamp spectrum. In order to establish the effective absorption cutoff of silicon and establish the conversion range of the image converter tube (operated at approximately 3000 volts A.C.), light from the spectrometer was transmitted through a 0.15 cm. thick polished silicon window. The converted light was observed with a 1P28 photomultiplier tube and recorded on a synchronized chart recorder.

In order to restrict the band pass, a 0.409 cm. thick Corning No. 5850 filter was inserted into the "snooperscope"-silicon system. Later, the No. 5850 filter was replaced by a distilled water filter (water thickness = 2.70 cm.). The observed transmission curves for (A) the Leiss spectrometer with the zirconium point light source, (B and C) the "snooperscope" image converter tube with a 0.15 cm. thick polished silicon window, (D) the distilled water filter, (E) the image converter tube with silicon window and water filter, (F) the Corning No. 5850 filter, and (G) the image converter tube with the silicon window and the Corning No. 5850 filter, are given for the wavelength region of 0.7-1.4 microns in Fig. 2.

* We wish to express our thanks to the University of Michigan Willow Run Laboratories, Infrared Section, and to J. Baker for operating the spectrometer.

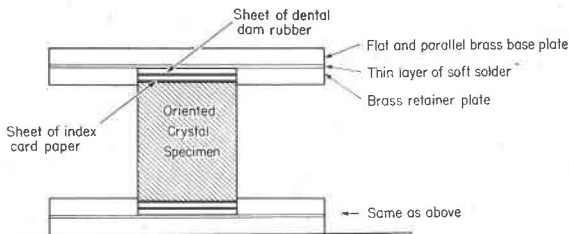


FIG. 2. A drawing of the "retained" resilient pressure pads used to provide good stress homogeneity on crystal surfaces.

An Eastman Kodak lead sulfide detector with a 1 cm.² window was used to measure "intensities" for curves A, D, and F, and a 1P28 photomultiplier tube was used for curves B, C, E, and G listed above. Transmission maxima were adjusted to a common value by means of the detector output signal amplifier. This procedure was maintained during piezobirefringence investigations.

The observed values of band pass and average wavelength at one-half "intensity" are given in Table 1. These values have been found to be reproducible to ± 0.005 micron. Therefore, data given in Table 1 are considered as correct to the second decimal place.

TABLE 1. BAND PASS AND AVERAGE WAVELENGTH (IN MICRONS) OF THE "SNOOPERSCOPE" INFRARED IMAGE CONVERTER TUBE (OPERATED AT 3000 VOLTS A.C. WITH FILTERS AS INDICATED)

	0.15 cm. thick polished silicon filter	0.15 cm. silicon +2.70 cm. water filter	0.15 cm. silicon +Corning No. 5850 filter
Band pass at $\frac{1}{2}$ "intensity"	0.14	0.08	0.10
Av. wavelength at $\frac{1}{2}$ "intensity"	1.140	1.113	1.110

Briggs (6), Fan (7), and more recently, Salzberg and Villa (8) have determined the index of refraction of silicon in the wavelength region covered in this work. Because of the absorption cutoff, a significant dispersion exists in this part of the spectrum. For example, from the data of Briggs (6), the index of silicon at a wavelength of 1.05 microns is 3.565, whereas at 1.40 microns it is 3.499. Because of this appreciable dispersion, a small error in establishing the correct value of wavelength can cause an appreciable error in computing the values of the piezobirefringence constants. It will be shown later that the piezobirefringence constant is proportional to the wavelength and inversely proportional to the

cube of the index of refraction. At present, however, there is no theoretical explanation available by which refractive index dispersion may be related to possible piezobirefringent dispersion.

It has been shown (1, 2) that the application of a homogeneous stress to crystal surfaces is one of the most serious experimental problems encountered in stress-optical studies. In order to minimize this source of error, "retained" resilient pressure pads similar to those which Poindexter (1) developed and used successfully in his work on diamond were constructed. The pads are placed on the specimen pressure surfaces in order to minimize the effect of mechanical deficiencies in contact surfaces, and directional error in stress application. Each pad consists of a section of flat and parallel 1/32 inch thick brass plate tinned on one side with soft solder to a thickness of approximately 0.020 inch. Four similar but smaller pre-cut brass plates are then soldered to the base plate in such a way as to create a depression to fit the crystal specimen. For silicon and materials of similar thermal behavior, the finished oriented parallelepiped specimen itself may be used as a soldering template. Excess solder can be scraped from the depression by a sharp pointed jeweler's file or pocket knife. A thin sheet of dental dam rubber, cut to the proper size, is then inserted into the depression, followed by a sheet of index card paper of proper dimensions. The primary purpose of the paper is to act as a dam to prevent extrusion of the rubber between the crystal and brass retainer walls. A schematic diagram of the pressure pads mounted on a crystal is given in Fig. 3.

The retained Bridgman type of packing described above has been found to provide a very uniform distribution of stress on specimen surfaces up to pressures of approximately 700 kg./cm.². The homogeneity of stress distribution can be established qualitatively by visual inspection (infrared image converter tube) of the uniformity of polarized light transmission through the stressed crystal. Quantitative verification was carried out by measurement of the half wavelength positions of stress-induced light retardation.

SOURCES OF ERROR

The following sources of error are recognized in this study of the piezobirefringence in silicon:

- 1) Non-homogeneous stress distribution
- 2) Misalignment in polar orientation
- 3) Misalignment in crystal orientation
- 4) Spectral characteristics of the optical system
- 5) Non-monochromaticity of the light used
- 6) Error in the determination of the light wavelength
- 7) Error in the refractive index of silicon

- 8) Light beam non-parallelism
- 9) Fluctuations in line voltage
- 10) Drift and noise in the electronic circuit
- 11) Measurement of crystal dimensions

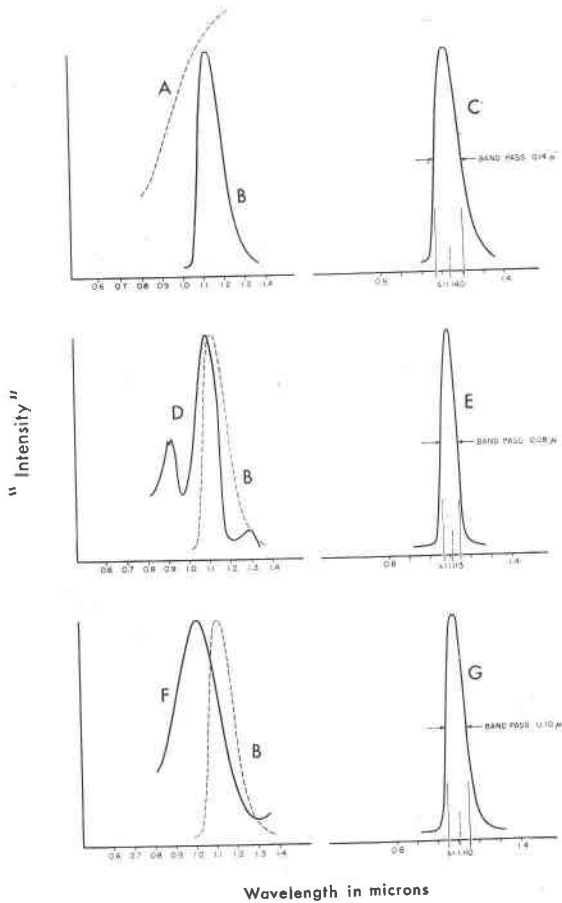


FIG. 3. Calibration curves for British World War II war surplus infrared image converter tubes, model CV-147, operated at 3000 volts A.C. with various filters. See text for details.

- 12) Inaccuracy in weight calibration
- 13) Inaccuracies in the calibration of the mechanical advantage of the lever arm
- 14) Frictional losses in the mechanical system
- 15) Personal error

The relative non-uniformity of stress distribution in this work has been found to be negligible. Values of $n\lambda/2$ intervals of stress-induced retardation for a given experiment are found to be reproducible to $\pm 1\%$. Independent experimental runs are also found to be reproducible to $\pm 1\%$. Errors in polar and crystal orientation are less than ± 15 minutes of arc, and therefore negligible. A thorough discussion on the effects of and corrections for poor stress distribution, polar misorientation and crystal misorientation in piezobirefringence investigation can be found in reference (9).

The determination of average wavelength, band pass and index of refraction of silicon are believed to be correct to $\pm 0.5\%$. The divergence of the light beam in air is less than 1 degree of arc, and therefore negligible. Measurements of crystal dimensions are correct to within ± 0.0005 cm. or approximately $\pm 0.2\%$. Weight calibration is correct to within $\pm 0.05\%$. The effect of fluctuations in line voltage was minimized by drawing all current through a Sola CVH constant voltage transformer.

All other sources of error listed above are variable and not amenable to quantitative evaluation. Their composite effect on the accuracy of the piezobirefringence constant is difficult to evaluate. However, the reported values are believed to be correct to within $\pm 2.5\%$.

EXPERIMENTAL RESULTS

Values for the piezobirefringence constants of crystalline materials may be calculated on the basis of the empirical mathematical stress-optical theory developed by F. Pockels (3, 4). Pockels' theorem may be expressed as

$$q = \frac{2h\lambda}{n_0^3Td}$$

For silicon, q represents the piezobirefringence constants ($(q_{1111}-q_{1122})$ and $2q_{1212}$); h = the amount of stress-induced retardation in units of wavelength; λ = the wavelength in air of the light expressed in centimeters; n_0 = the index of refraction for the unstressed material; T = the applied homogeneous stress expressed in dynes/cm.²; d = the path length through the crystal in centimeters. The wavelength of radiation used in this work has been measured as 1.11 microns with a band pass of 0.08 micron at $\frac{1}{2}$ "intensity" (silicon + water filters).

The experimentally measured value of the piezobirefringence constant ($q_{1111}-q_{1122}$) of silicon at 26° C. $\pm 1^\circ$ for a wavelength of 1.11 $\mu = -14.4 \times 10^{-14}$ cm.²/dyne. The reported value of ($q_{1111}-q_{1122}$) has been obtained from the average of three independent determinations. The value of each determination, in turn, is obtained from the average of eight consecutive orders of $\lambda/2$ stress-induced retardation. The reproducibility between the

values of the consecutive orders of $\lambda/2$ retardation in each experimental run and between each of the three independent experiments both amount to $\pm 1.0\%$.

The experimentally determined value of piezobirefringence constant $2q_{1212}$ at $26^\circ \text{C} \pm 1^\circ$ for radiation of 1.11μ wavelength has been found to be $-10.0 \times 10^{-14} \text{ cm.}^2/\text{dyne}$. This value represents the average of three independent determinations. Each determination in turn consists of the average obtained from five consecutive orders of $\lambda/2$ retardation. The reproducibility between the consecutive orders of $\lambda/2$ retardation and between each of the three independent determinations both amounted to $\pm 1.0\%$.

The piezobirefringent behavior of silicon at increasing values of stress is illustrated by Figs. 4 and 5. Figure 4 shows the linearity of the stress-optical relationship in silicon with an increasing homogeneous stress applied parallel to $X_1\bar{X}_1$ and light vibrating parallel to $X_1\bar{X}_1$ and traveling normal to $X_1\bar{X}_1$. Figure 5 illustrates the linearity of the stress-optical behavior of silicon for light vibrating parallel to X_{12} and traveling parallel to $X_3\bar{X}_3$ under an increasing stress directed along X_{12} .

Figure 6, a plot of "intensity" versus stress-induced retardation, illustrates the effect of a relatively wide band pass in the optical system of the experimental apparatus. The data were obtained by using unfiltered light from a 108 watt General Electric tungsten spiral filament lamp. Although the positions of the maxima and minima of transmitted intensity fall at uniform intervals, their respective amplitudes can be seen to decrease and increase in a nearly linear fashion. The fact that the positions of $n\lambda/2$ retardation occur at regular intervals indicates good stress distribution upon the pressure surfaces of the crystal. With poor stress distribution, the positions of $n\lambda/2$ retardation would be expected to occur at consecutively closer intervals due to an increasing non-homogeneity of the stress as the pressure is increased. A theoretical discussion of this phenomenon has been given (9).

The effect of decreasing the band pass has been experimentally observed by the insertion of a Corning No. 5850 filter (0.11μ band pass at $\frac{1}{2}I$) and a water cell filter (0.08μ band pass at $\frac{1}{2}I$), respectively, into the optical system. In both cases, the intensity at positions of $n\lambda/2$ retardation was observed to become more constant with a more restricted band pass. It appears, therefore, that the observed decrease in intensity contrast between increasing orders of $\lambda/2$ retardation as illustrated in Fig. 6 is due to an appreciable extent to the non-monochromaticity of the light used. The observed relationship appears to be analogous to the orders of interference colors commonly observed with the use of white visible light.

An attempt was made to determine the absolute values of the stress-

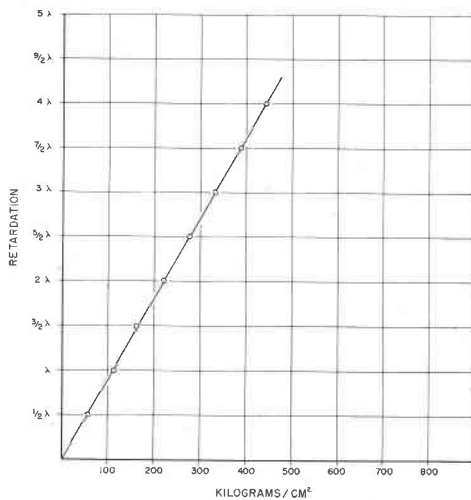


FIG. 4. The observed linearity of the stress-optical relationship in silicon described by the piezobirefringence constant ($q_{1111} - q_{1122}$).

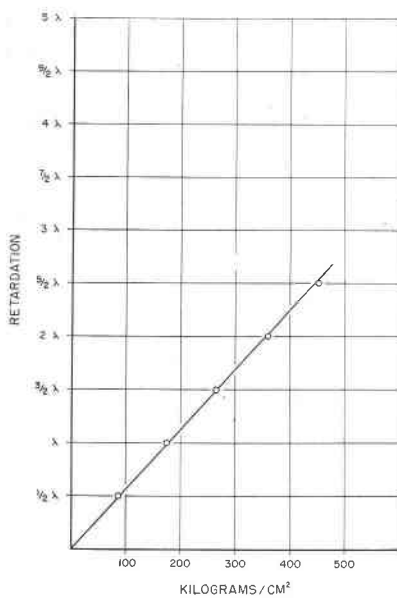


FIG. 5. The observed linearity of the stress-optical relationship in silicon described by the piezobirefringence constant $2q_{1212}$.

optical constants q_{1111} and q_{1122} . However, no quantitative success was achieved. The prism-deviation method described for SrTiO_3 (2) was used with the additional requirements of the infrared image converter tube. A description of the prism used is given in the section on Specimen Preparation.

The collimated light beam from a 0.025 cm. wide vertical slit was di-

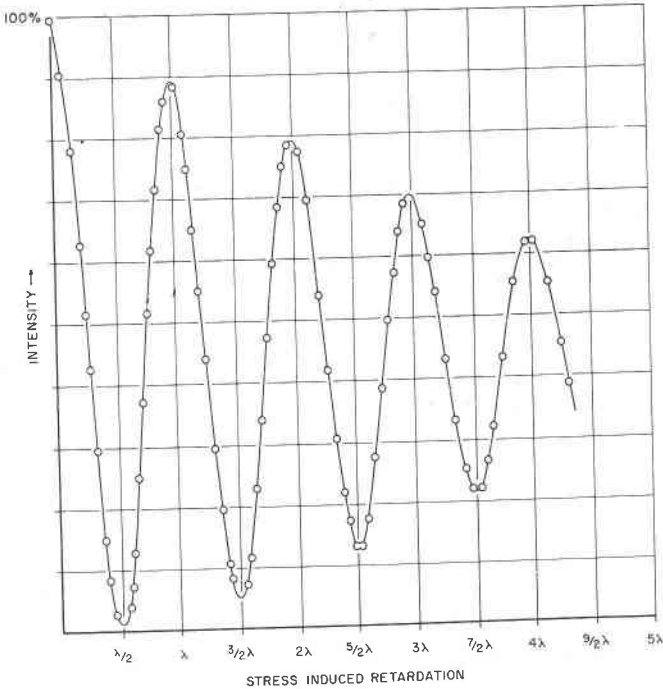


FIG. 6. The observed relationship between the relative transmission intensities and orders of stress-induced retardation for a uniformly stressed crystal when using poorly monochromatized infrared light.

rected to the prism at normal incidence. This orientation was established by observing the back reflection of the incident light ray. Stress was applied to the (100) right trapezium prism faces. A nicol polarizer, oriented with the vibration direction parallel to the direction of compression, was used in the attempt to evaluate q_{1111} . The vibration direction was oriented normal to the direction of stress for evaluation of q_{1122} .

A prism to detector distance of 3.25 meters was used and stresses up to approximately 600 kg./cm.² were applied to the crystal. This type of measurement is an extremely difficult one. The magnitude of the meas-

urements is approximately of the same order as the sensitivity of the measuring apparatus. A detailed discussion of the sources and magnitude of error involved can be found in reference (2). Additional difficulties encountered are the following: 1) wide band pass, 2) poor resolution of the image converter tube, 3) working with light that cannot be seen.

The experiment was repeated 15 times. Due to the serious inconsistencies in the results, no further attempts were made. However, the averaged values indicate the following approximate ratio of change in the index of refraction:

$$\Delta n_{11}:\Delta n_{22} \approx (-)3:(+)2.$$

The minus sign indicates the index of refraction parallel to the direction of stress decreases, whereas that normal to the direction of stress (positive) increases. Indications are, therefore, that silicon becomes a negative uniaxial crystal under a compression stress directed along [100]. If one tentatively accepts the above approximate ratios for Δn_{11} and Δn_{22} , indications of the values of the stress-optical constants q_{1111} and q_{1122} may be obtained from the stress-optical difference constant ($q_{1111}-q_{1122}$), namely, $q_{1111} \approx -9 \times 10^{-14}$ cm.²/dyne and $q_{1122} \approx 6 \times 10^{-14}$ cm.²/dyne.

DISCUSSION OF RESULTS

Since silicon and diamond both possess $m3m$ symmetry, have similar structures, and are both elemental materials, it should be of interest to compare their respective piezobirefringent behavior. It is desirable, however, that the following differences in physical properties and experimental conditions be tabulated first:

- 1) Diamond is transparent from the ultraviolet region of the spectrum to the near infrared.
- 2) Diamond possesses an absorption cutoff in the ultraviolet region of the spectrum at approximately 3000 Å (10, 11, 12).
- 3) Silicon is opaque to visible light.
- 4) The silicon absorption cutoff is at approximately 10500 Å.
- 5) The physical strength, therefore, bond strength in diamond, is many times that of silicon.
- 6) Comparable stress-optical work on diamond to date has been carried out at approximately 5400 Å, which is far removed from the absorption edge.
- 7) The present work on silicon has been carried out in the vicinity of the absorption cutoff, namely at 11100 Å.

The values of piezobirefringence constants observed by various workers on diamond are given in Table 2, along with the observed values for silicon reported in this paper.

As can be seen, the agreement of the values reported by different work-

ers on diamond is poor. The most recent work by Poindexter (1), however, is probably the most accurate.

As is to be expected, the numerical values of the respective constants of diamond and silicon differ. Of more pointed interest, however, is the piezobirefringent anisotropy, that is, the difference between $(q_{1111} - q_{1122})$ and q_{1212} . Although Ramachandran's values indicate a significant anisotropy, those reported by Grodzinski and Poindexter show only a small difference in value for the two piezobirefringence constants. It should be

TABLE 2. TABULATION OF PIEZOBIREFRINGENCE MEASUREMENTS OF DIAMOND BY DIFFERENT INVESTIGATORS AND OF SILICON

Material	Worker	Wavelength	$(q_{1111} - q_{1122})^*$	$2q_{1212}^*$
Diamond	Wertheim (13)	?	-3.3-	
Diamond	Ramachandran (14)	5461 Å	- 7.2	- 2.8
Diamond	Grodzinski (15) and Fisher	(Not reported)	- 3.85	- 3.78
Diamond	Poindexter (1)	5400 Å	- 3.04	- 2.98
Silicon	(This paper)	11100 Å	-14.4	-10.0

* Expressed in units of $\times 10^{-14}$ cm.²/dyne.

noted, however, that the magnitude of $(q_{1111} - q_{1122})$ reported by all workers on diamond (except Wertheim) is consistently greater than $2q_{1212}$. This agrees with the presently observed values for silicon. Silicon, however, shows a marked piezobirefringent anisotropy.

The fact that silicon has been studied very close to the absorption cut-off may be related to the observed difference in anisotropy between diamond and silicon. Most past work on diamond has been carried out at an average wavelength of 5400 Å. Poindexter has investigated the piezobirefringent dispersion of $2q_{1212}$ as a function of wavelength from 4400 Å to 7700 Å. His results show a slight dispersion amounting to approximately 4% in this wavelength range. No study has been made on the dispersion of $(q_{1111} - q_{1122})$.

Diamond may possess a piezobirefringent anisotropy at the absorption edge in the ultraviolet region of the spectrum which is comparable to that observed in silicon at 1.11 μ . The wavelength relationship of both piezobirefringence constants of diamond should be investigated to approximately 3000 Å. Similarly, silicon should be studied further into the infrared.

CONCLUSIONS

The following values have been determined for the piezobirefringence

constants of "high purity" silicon at a wavelength of 1.11μ and a temperature of $26^\circ \text{C.} \pm 1^\circ$:

$$\begin{aligned} (q_{1111} - q_{1122}) &= -14.4 \times 10^{-14} \text{ cm.}^2/\text{dyne} \\ 2q_{1212} &= -10.0 \times 10^{-14} \text{ cm.}^2/\text{dyne} \end{aligned}$$

The linearity of the stress-induced retardation described by both constants has been established up to pressures of approximately 450 kg./cm.² The observed significant piezobirefringent anisotropy of silicon in the vicinity of its absorption cutoff indicates the possibility of greater anisotropy for diamond at its absorption cutoff ($\sim 3000 \text{ \AA}$). Qualitative observations indicate that silicon becomes a uniaxial negative crystal under a directional stress parallel to [100].

ACKNOWLEDGMENTS

The writer wishes to express his gratitude to M. Conrad, J. Mandarino and C. Salotti for their generous assistance in performing the experimental work, and to Prof. R. M. Denning for many helpful discussions and critically reading the manuscript.

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Manuscript received July 10, 1957