INTERTRACIAL ANGLES AS INDICATORS OF OPTICAL AND DIMENSIONAL ORIENTATION OF SOME MONOCLINIC CRYSTALS IN RANDOM SECTIONS

E. den Tex, *The University of Melbourne, Victoria, Australia.*

Abstract

A Universal Stage method is presented to construct the location of dominant zone axes in random sections of crystals from the orientation of the indicatrix and the values of apparent dihedral angles (hereinafter called intertracial angles) as compared with corresponding true angles. It applies to monoclinic crystals with known axial elements, identifiable faces or cleavages and known crystallographic orientation of the optic axial plane; such as the majority of pyroxenes and hornblendes. Applicability, advantages and limitations of the method are discussed. It appears suitable for petrofabric analysis, giving sufficiently accurate results for all orientations. Special reference is made to aegirine-augite in a pseudomassive alkaline trachyte.

Introduction

The optical orientation, more specifically the extinction angle $Z\wedge c$ of monoclinic pyroxenes and amphiboles having the optic axes situated in (010), has been the subject of a number of papers during the last twenty years.

Strictly speaking, direct observation of the extinction angle in 010 is limited to sections parallel to *c* with inclinations of 010 at less than 40° to the section, so as to enable its correct adjustment perpendicular to the microscope axis on the universal stage. If the latter operation is impossible, $Z\wedge c$ may be read from the stereographic projection. In all cases where *c* is not parallel to the section, the traces of *hk*0 planes in the section are not parallel to *c* and should therefore be discarded as a direct reference for $Z\wedge c$ in 010.

The above limitations are responsible for the methods developed for indirect computation of the extinction angle from universal stage measurements. In a review, Turner (1942) has indicated the limitations and the range of error of the better known procedures, at the same time suggesting variations to overcome some of the former (1940) and reduce some of the latter obstacles (1942). Special attention was given to errors resulting from vertical adjustment of crystallographic reference planes, due to imperfect development, low inclination to the section, difference in R.I. between section and hemispheres and its impact on aberration of the rays in inclined positions of the section. Preference was shown by Turner for the use of selected sections with Nemoto's (1938) procedure of computing crystallographic references from postulated symmetry relations between the indicatrices of twinned crystals.

From the crystallographic point of view, application of universal stage methods to identification of crystal faces and to measurement of interfacial angles in the stereographic projection was presented by Nieland (1932) and by Gilbert and Turner (1949) respectively, whilst Donnay and O'Brien (1954) described a microscopic procedure for the determination of axial elements from apparent interedge angles as well as for crystal identification from interzonal or apparent interzonal angles in independent sections. However, the latter treatment applies to oriented sections only and does not consider the optical orientation of the crystal.

To determine lineations and foliations of hornblende crystals in schists and gneisses Schmidt (1928) and Wenk (1936) used the intersection of *hk*0 planes as a means of locating *c*, a method improved by Burri (1931) through construction of [001] as pole of a great circle containing the poles of *hk*0 planes. Thus universal stage stereography was applied to the field of dimensional orientation as treated statistically by petrofabric analysis. Schmidt also drew the attention to the lack of accuracy in his method for sections subparallel to *c* where often only one (110) cleavage is present at low inclination to the section (*op. cit.*).

In the course of a structural analysis of a number of alkali-trachytic domes in the Macedon district, near Melbourne, it proved desirable to the present author to microscopically determine a possible lineation of aegirine-augite "phenocrysts" with dominant [001], which were too small to be effectively measured in the hand specimen.

For this purpose the optical orientation of each individual crystal was determined from oriented thin sections by means of the universal stage. The axis (*c*) of the dominant zone [001] being the crystallographic direction to constitute a possible lineation, its location was derived from the extinction angle $Z \wedge c$ in 010. Use of the location of *c* in petrofabric analysis, excluded the possibility of selecting favourable sections which, moreover, may be rare in slides of a certain orientation.

Apart from these considerations calling for special precautions against errors and for careful selection of slides with regard to the major structural features, the exact vertical adjustment of crystallographic reference planes was severely hampered by their often imperfect development, their sometimes low inclination to the section and, above all, by the clouded to almost opaque appearance of many sections subparallel to *c* due to strong absorption and alteration (cf. Fig. 4).

However, nearly all sections did show well developed crystal faces whose identification was aided by fairly regular cleavage traces. Although, at first, it seemed possible to use these faces for vertical adjustment, their corroded condition—further impaired by granular alteration products—precluded accurate adjustment as already experienced by Nieland (1932). On the other hand, the traces of faces and cleavages in the plane of section could usually be located within ½ degree (cf. Donnay and O'Brien, 1945, p. 597). The angles between these traces in the plane of section, the apparent dihedral angles or intertracial angles as they will be referred to in this paper, give an indication of the inclination—if any—of the crystal with respect to the section. Knowing from the orientation of the indicatrix, the position in space of one crystallographic plane, 010, the possible locations of *c* are restricted to this plane, generally resulting in only one such location to produce the specific intertracial angles, although it should be realized that an equal but opposite inclination of *c* in 010 produces corresponding supplementary angles (cf. Fig. 3, *A*3 and *B*5). Special conditions apply when 010 is perpendicular to the section giving rise to the same intertracial angles for equal but opposite inclinations of *c* (cf Fig 3, *A*2 and *B*4), unless the section is further specified as either perpendicular to *c* showing true dihedral angles between all traces of *hk*0 planes (cf. Fig. 3, *A*1) or parallel to *c*, when all traces of *hk*0 planes are parallel (cf. Fig. 3, *B*1–3). The other orientations perpendicular to 010 give the only cross-sections ambiguous with respect to the location of *c* in 010 owing to the fact that the twofold axis of symmetry *b* lies in the plane of section. The result of this is that dihedral angles of 90°, such as (010)$\wedge$(100) and (010)$\wedge$(001) are preserved throughout rotation and that others such as (010)$\wedge$(110) and (010)$\wedge$($\bar{1}$10) are changed in the same sense retaining the symmetry of outline of a section perpendicular to *c*. These sections require a special treatment as will be shown below.

Geometry

It may be useful to visualize the geometrical relations of the problem before offering a stereographical solution.

The orientation of a crystal in space may be described by two intersecting faces; each of which may, in turn, be fixed by two non-parallel lines in it. The latter description can be used to determine the principal sections of the indicatrix, of which ZX coincides with 010 in the case of aegirine-augite. Other crystallographic planes, however, are merely limited in orientation by containing one line, i.e. their trace in the section. Nevertheless, in the monoclinic system *h*0*l* planes such as (100) and (001), which are parallel to [010], are completely fixed in orientation when their trace in any section not parallel to *b* and the direction of *b* have been determined. Moreover, for any monoclinic pyroxene or amphibole the angle between the (110) planes and *b* is constant, the value for aegirine-augite being approximately 46°30′. This means that they are

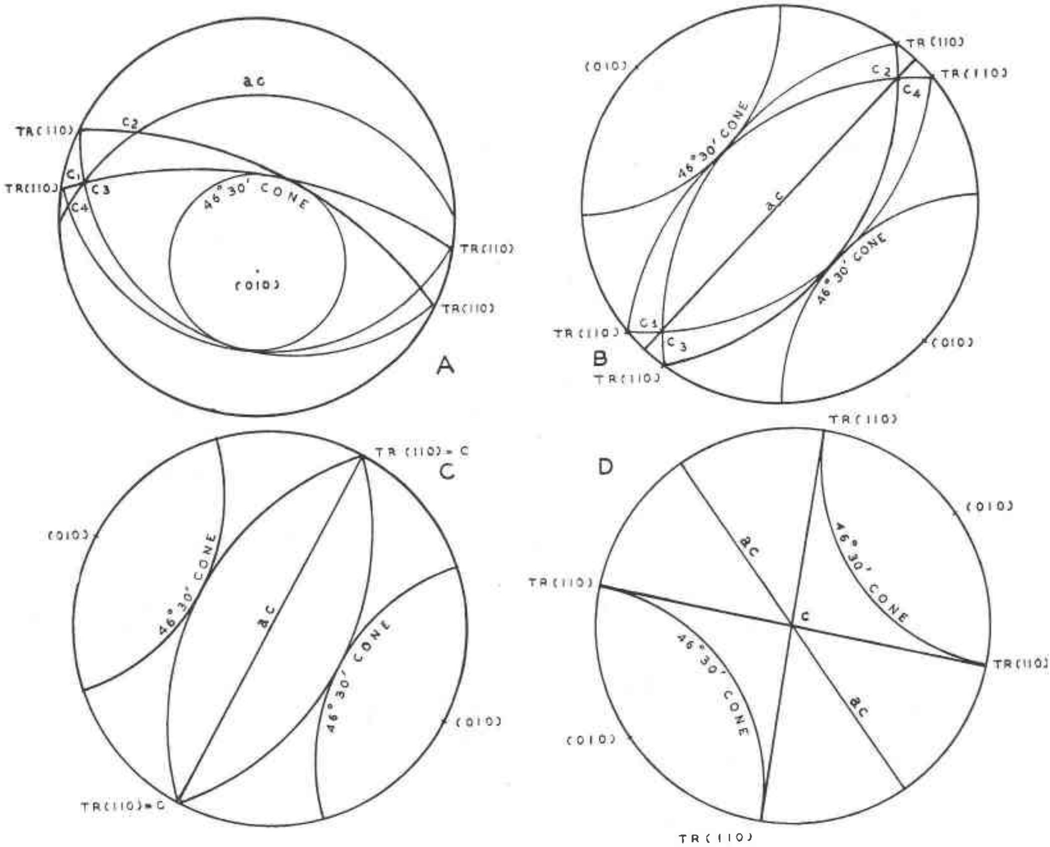

FIG. 1. Stereographic projections of the geometrical relations in an aegirine-augite crystal between the *b* axis and the (110) crystallographic planes as described by their traces in the plane of section. In the general case (*A*) there are four possible planes tangent to the 46°30′ cone about *b*, two of which have a common intersection in the *ac*-plane. In sections ⊥ 010 generally two common intersections in the *ac*-plane are offered by the four planes (*B*). Sections ||*c* (*C*) and sections ⊥*c* (*D*) have only two possible (110) planes and only one common intersection in the *ac*-plane.

tangential planes to a cone with apical angle of 46°30′ about *b* intersecting the sectional trace in the apex. The stereographic projections of Fig. 1 are in the plane of the section and designed to show the geometrical relations between (110) traces in the section and the cone about *b* with its apex in the centre of the projection, where all directions in the crystal intersect. From the general case, figured in 1*A*, it is evident that there is a maximum of two tangential planes that will simultaneously pass through a particular (110) trace in the section. As there are generally two (110) traces present, this results in a maximum of four possible orientations of the (110) planes of which commonly only one pair, belonging to different traces, will constitute a [001] zone by common intersection in 010, thus satisfying the symmetry requirements.

However, two common intersections of different (110) planes in 010 may be available resulting in two possible locations of *c*, when the cone axis *b* bisects the angle between the (110) traces. This happens only in sections perpendicular to 010 where the twofold axis of symmetry lies in the plane of section so that both pairs of (110) planes contact the cone along opposite lines (cf. Fig. 1*B*).*

Two sections perpendicular to 010 have only one possible pair of (110) planes tangent to the cone about *b*, caused by either mutual coincidence of (110) traces at right angles to the cone axis (cf. Fig. 1*C*) or by coincidence of different (110) traces with the cone (cf. Fig. 1*D*). The former condition is common to all sections of the *c* zone where parallelism of *hk*0 traces obviates the necessity of constructing *c*, the latter applies to sections at right angles to the (110) form and hence perpendicular to *c*.

Stereographic Construction

These principles may now be applied to the actual location of *c* in the stereographic projection.

First of all, plotting of the indicatrix on the Wulff net is performed in the conventional manner using the plane of section as the surface of projection. 010 and the orientation of Z within it are established from measurements on the universal stage. The inner stage is then set horizontal and the traces of relevant faces and cleavages (with their directions averaged if identical) are successively set parallel to the N-S. cross-hair, the angular positions being read from the arc of the inner stage. The traces, representing intersections of crystallographic planes with the section, are now plotted. They all lie on the primitive circle (cf. Fig. 2*A*). All *h*0*l* planes can be constructed as great circles through *b* and their trace in the section, if these do not happen to coincide. To facilitate the construction of other structural elements, such as planes of the (110) form, a second projection of the crystal may then be plotted from the first by a rotation designed to bring $b = Y$ at the centre (cf. Fig. 2*B*). The trace directions of the crystallographic planes are marked on the upper hemi-

* Actually sections parallel to *b* through monoclinic crystals have a planar point group symmetry (*C*2*l*) containing a centre and 2 lines of symmetry which are the *h*0*l* and 0*k*0 traces. As *c* is confined to 010, the crystal symmetry is satisfied in positions symmetrical to a vertical plane through the *h*0*l* trace. For similar reasons of symmetry all planes of the (110) form are interchangeable as far as they serve for the location of *c* in 010.

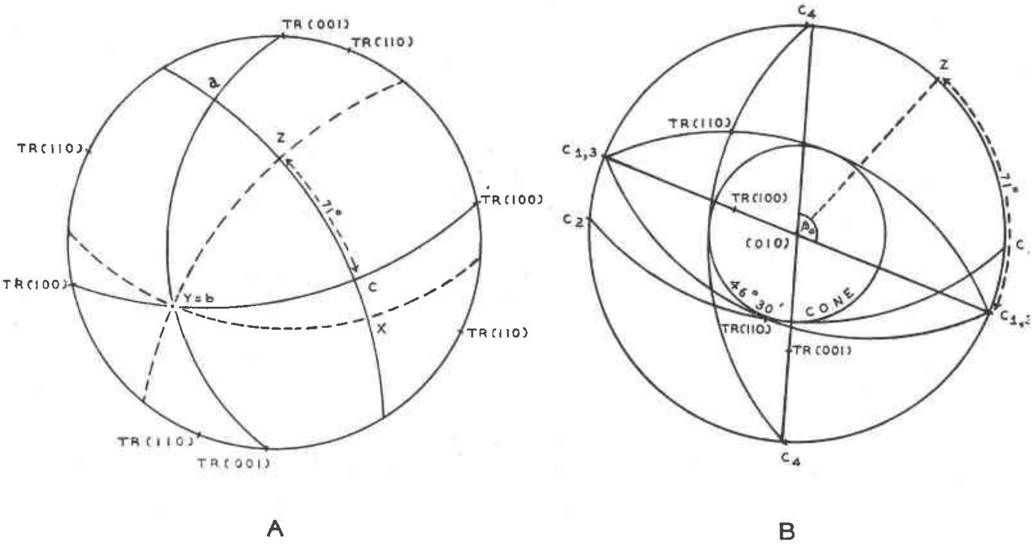

FIG. 2. Stereographic constructions of the extinction angle $Z\wedge c$ from the orientation of the indicatrix and from the traces of crystallographic planes in a section through an aegirine-augite crystal. In A the c axis is found by construction of $h0l$ planes and their intersection in ac, using the initial projection on the plane of section. B is a projection of the same crystal on the ac plane, plotted from A, in which the construction of (110) planes tangent to a 46°30′ cone about b is greatly facilitated.

sphere with their appropriate Miller indices and the prefix *TR*.

Either trace of the (110) form, by rotations about b 180° apart, falls on opposite 46°30′ great circles from the centre, representing a tangential plane to the corresponding cone about b, reproduced by the symmetry in 010. Also, if more than 46°30′ radially from b, either (110) trace has two possible locations on each 46°30′ great circle resulting in two different tangential planes and therefore generally in two possible locations of c in 010. The same applies to the other (110) trace, but from the four possible locations of c thus produced at least one of the latter should be shared with the former to satisfy the symmetry of the form. Failure to do so implies lack of accuracy in orientation and construction and may be overcome by adhering to the mean location between the 2 most nearly coinciding orientations resulting from different traces. A further check may be available in the trace of any $h0l$ plane in 010. In Figs. 1*A* and 2*B* the correct location of c is shown by $c_1=c_3$. However, c_2 may also coincide with c_4 (cf. Fig. 1*B*) when a series of steps can be taken to decide which is the correct location by:

(i) Reading the alternative extinction angles $Z\wedge c_1=c_3$ and $Z\wedge c_2=c_4$ from the primitive circle graduation and comparing the values so obtained with the range of extinction

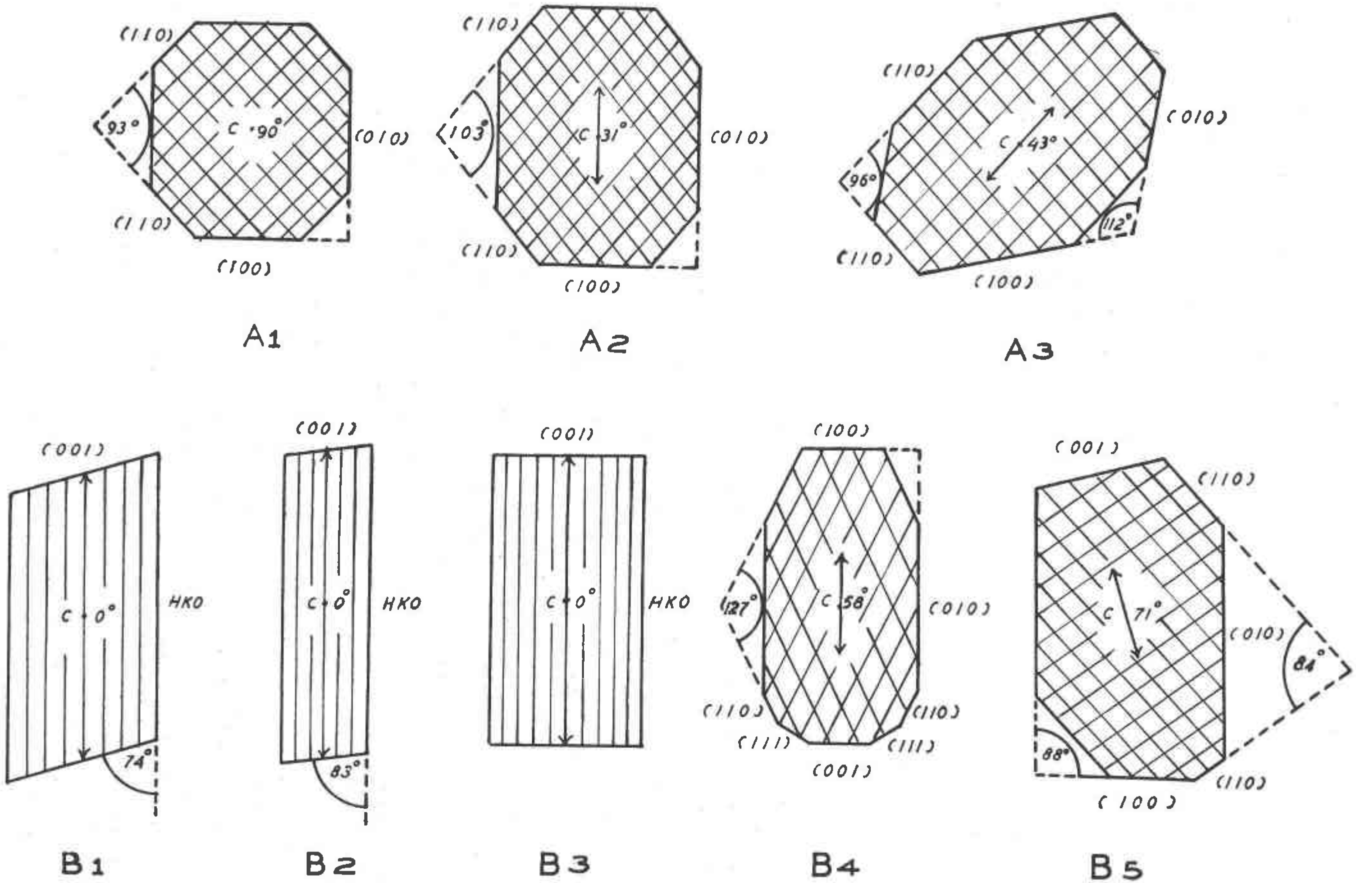

FIG. 3. Ideal sections of an aegirine-augite crystal typical of the treatment required for recognition of their orientation. Possible directions and inclinations of c to the section are indicated as well as some of the relevant intertracial angles. Symbols refer to the schedule of typical sections given in the text. Ambiguity of orientation in A_3 and B_5 will cease to exist once the sense of inclination of (010) is established. Axial elements: $(110)\wedge(\bar{1}10)=87°$; $\beta=74°$.

angles given by any of the better text books for the species under consideration. Failure of elimination necessitates:

(ii) Gauging the sense of inclination to the section of at least one of the (110) planes. This is done in the conventional universal stage manner and is a much safer operation than the exact vertical adjustment as pointed out by Schmidt (1928, p. 206). The sense of inclination of c in 010, as derived from these data, is indicated by an arrow from the centre of the initial projection along the ac radial and is subsequently rotated to a corresponding position along the primitive circle of the second projection.

Finally there may be either no alternative great circles to suit each trace (cf. Fig. 1*D*) or no difference between traces (cf. Fig. 1*C*) both resulting in a unique location of c, perpendicular or parallel to the section respectively.

Schedule of Typical Sections

The general procedure of stereographic construction may now be supplemented by a practical subdivision of monoclinic pyroxene-amphibole sections according to the treatment required for recognition of their orientation.

c being the dominant zone axis sought, the sections may be roughly divided into:

(*A*) those $\perp$ or sub $\perp$ to c, being approximately octagonal in outline;

(*B*) those $\|$ or sub $\|$ to c, being approximately rectangular or parallelogram shaped in outline.

In each case the forms of faces are identified as far as possible by their relations to the characteristic (110) cleavages and to (010) as determined by means of the indicatrix. Preference is given to $hk0$ planes as direct indicators of c in 010.

(*A*) In this division three types of sections may be recognized as to require a different treatment. These are:

(1) *Those $\perp$ to c*, which are easily recognized by true dihedral angles between all traces of $hk0$ planes. For petrofabric purposes all adjustment and plotting may be dispensed with. Plotting of the indicatrix is necessary for the determination of $Z \wedge c$, whereby the principal section of the indicatrix coinciding with 010 should be vertical and c in the centre of the initial projection.

(2) *Those $\perp$ to 010, sub $\perp$ to c* may be distinguished from those $\perp$ to c by true dihedral angles of 90° between 100 and 010 but apparent dihedral angles between the traces of other planes such as 110 and $\bar{1}10$, whilst the principal section of the indicatrix coinciding with 010 appears to be vertical in the initial projection. Location of c depends on the traces of (110) planes.

(3) *Other sections sub $\perp$ to c* do not reveal any true dihedral angles between $hk0$ traces unless $\perp$ to 100. Location of c is obtained by construction of 100 in the initial projection through b and its trace in the section. An alternative is available in the construction of (110) planes, which may be used as an unequivocal check on the location of c for steep inclinations of 010 to the section (cf. Fig. 4*A*).

(B) Sections parallel or subparallel to c may be subdivided into five types according to the treatment required:

(1) *Those* ‖ *to 010* are recognized by perfect parallelism of $hk0$ traces combined with a cardinal orientation of the indicatrix which has the principal section coinciding with 010 horizontal. True dihedral angles are enclosed between the traces of all $h0l$ planes. The trace of any $hk0$ plane in the section gives the location of c. $Z \wedge c$ is read from the arc of the microscope stage.

(2) *Those* ‖ *to c, sub* ‖ *to 010* are identified by parallelism of $hk0$ traces with the principal section of the indicatrix coinciding with 010 inclined at less than 40° to the section. They are treated in essentially the same way as B_1 provided 010 is set horizontal for reading $Z \wedge c$.

(3) *Those* ‖ *to c,* ⊥ *or sub* ⊥ *to 010* constitute a case similar to B_2 with the exception that 010 cannot be set parallel to the microscope stage for direct measurement of $Z \wedge c$. It may be read, however, from the great circle connecting Z and the $hk0$ trace in the initial projection.

(4) *Those* ⊥ *to 010, sub* ‖ *to c* are not essentially different from those mentioned under A_2 but subparallelism of $hk0$ traces may prove to be confusing, in which case hkl traces, if identifiable, can be used to advantage.

(5) *Other sections sub* ‖ *to c* are similar to those sub ⊥ to c the treatment and recognition of which are given under A_3. However, the 001 trace is now more likely to appear alongside that of 100 or even as the only $h0l$ trace available. As both may be used for the location of c in this type of section, care should be taken to correctly identify the faces they represent. If both are present they will be seen to make different angles with 010 whereby the 001 trace is characterized by absence or shorter development of adjoining (110) traces compared with those near the 100 trace (cf. Fig. 4B). If only (001) or (100) traces are present, they will be parallel and the relevant face giving rise to a subparallel location of c is represented (cf. Fig. 4C).

Applicability, Advantages and Limitations

The proposed method is applicable to random sections of crystals with known axial elements, identifiable faces or cleavages, and known crystallographic orientation of one principal section of the indicatrix, commonly the optic axial plane, as provided by the majority of monoclinic pyroxenes and hornblendes. It is virtually independent of the degree of perfection and preservation of faces and cleavage planes, because the crystallographic reference planes are described by their trace directions in the horizontal plane of section and their relation to a principal section of the indicatrix only. Once the orientations of the two measured principal sections of the indicatrix have been corrected on the Fedorov or Hallimond (1950) nomograms for errors due to tilt of the section on vertical adjustment, no more errors of this type are introduced in the construction of crystallographic reference planes. Especially for low inclinations of the latter to the section the accuracy of construction is thus improved while time consuming corrections are reduced to a minimum.

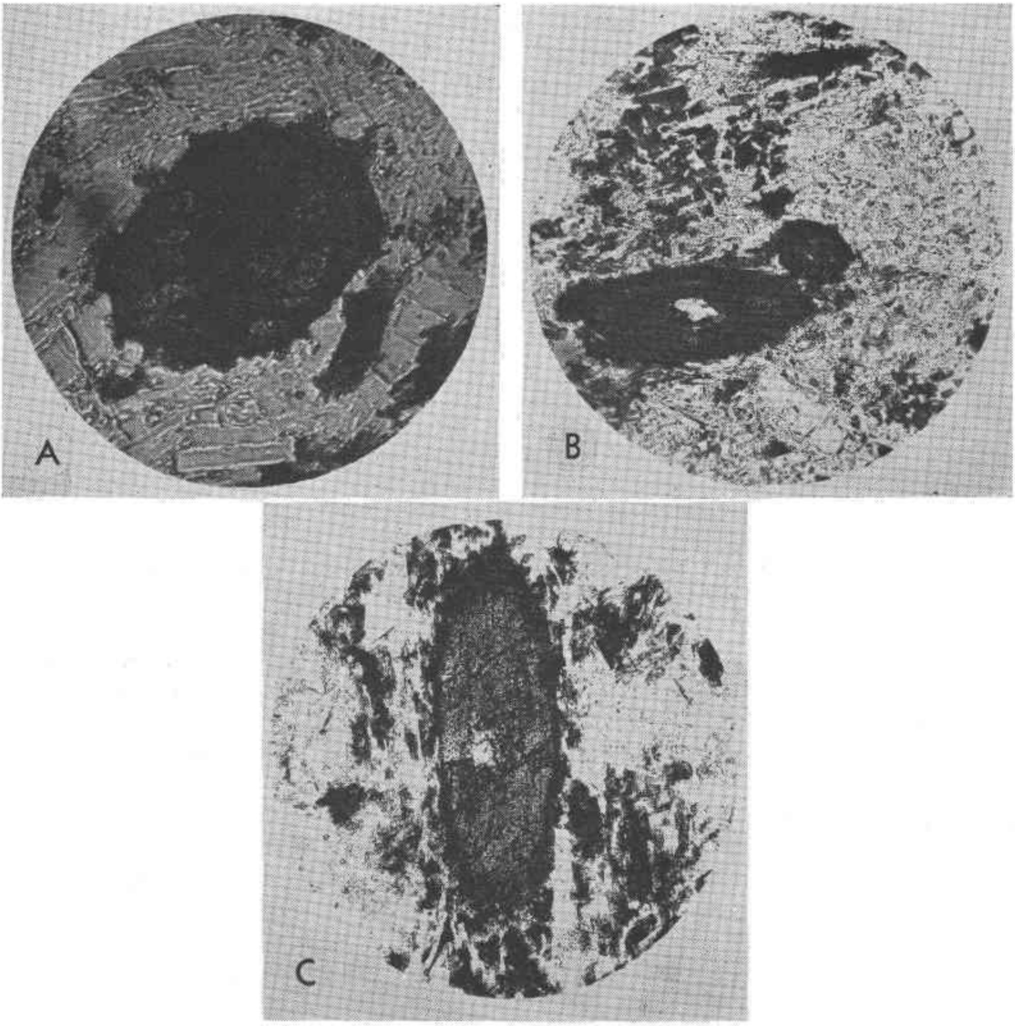

FIG. 4. Three photomicrographs of general orientations as shown by the outline of aegirine-augite phenocrysts in selected sections of alkaline trachytes from the Macedon district, Victoria. Identification of planes is aided by the presence of traces of one (110) cleavage in *A*, of 001, 010 and 100 faces in the larger phenocryst in *B*, and of (001) and (100) partings in *C*. *A* conforms to A_3, *B* and *C* to B_5 of the schedule. *C* is nearly parallel to 010. Lin. mags: *A* and *B*, 350; *C*, 100.

Generally c may be located within 2° in 010 if the relevant traces can be adjusted with an accuracy of $\frac{1}{2}$°. This may not be possible for low inclinations of the relevant planes to the section as is often the case with *hk*0 faces and cleavages in sections of monoclinic pyroxenes and hornblendes subparallel to c. However, this effect is countered by the only slight change in location of c brought about by a specific variation of trace direction in these sections as compared with that due to the same cause in sections subperpendicular to c, where the *hk*0 traces are usually adjustable within less than $\frac{1}{2}$°. Also, in the latter type of section, there is a greater variety of identifiable *hk*0 traces offering a close check on the location of c (cf. Fig. 1*C*, *D;* 3*A*, *B;* 4). This constancy of error within 2° for all orientations makes the method especially suitable for statistical treatment of dimensional orientation as used in the petrofabric analysis of rocks for possible lineations provided by monoclinic pyroxenes and hornblendes. For individual crystals the extinction angle or optical orientation, indicating the chemical composition, should be determined by means of the most accurate pertinent method.

The principal limitation of the proposed method is set by the requirement of identifiable faces or cleavages. Thus the mineral under consideration should be of idiomorphic development throughout the slide unless at least two characteristic and identifiable cleavages are available as in the monoclinic pyroxenes and hornblendes. In the latter minerals the identification of faces is greatly facilitated by the presence of either or both (110) cleavages. Idiomorphic development may be rare, but in certain extrusive and hypabyssal rocks a lineation can be restricted to the phenocrysts. Idioblastic development of amphiboles is, on the other hand, a common feature.

Schmidt (1928) applied predetermined extinction angles from selected sections to the optical orientation of unfavourable sections once the sense of inclination of (110) and c were established. For quick, though rough results in microscopic dimensional lineation, universal stage work is made superfluous by adapting Lowe's method (1946) of correlating average apparent lineations to use in oriented sections. However, $Z \wedge c$ may vary for individual crystals, whilst apparent lineations may be too faint to be recognized by visual averaging of elongations.

It is indeed very likely that lineations suggestive of the mode of emplacement are to be found in apparently massive igneous rocks by petrofabric analysis of selected specimens, whereby individual location of dominant zone axes will add to accuracy with very little extra effort as shown above. Results of this nature will be discussed in a separate paper.

Use of the 5 axes universal stage is not essential, but it may be helpful in identifying sections subparallel to ZX with the aid of the Berek-Dodge procedure as advocated by Emmons (1943), especially when the optic axial angle is suspected to be rather small and Bx_a subhorizontal.

ACKNOWLEDGMENT

The author is greatly indebted to Dr. C. M. Tattam, Associate Professor of Geology in the University of Melbourne, and to Dr. J. McAndrew, Officer of C.S.I.R.O. Mineragraphic Section, Melbourne, for critical reading of the manuscript and valuable suggestions.

REFERENCES

SCHMIDT, W. (1928), Zur Regelung Zweiachsiger Mineralien in Kristallinen Schiefern: *N. Jahrb. Min., A*, **57**, 203–222.

BURRI, C. (1931), Zur Bestimmung der Auslöschungsschiefe Monokliner Hornblenden und Augiten auf (010) mittels beliebiger Schnitte: *Schweiz. Min. Petr. Mitt.*, **11**, 285–289.

NIELAND, H. (1932), Zur Bestimmung von Auslöschungsschiefe und Kristallformen mittels des Drehtisches nach Fedorow: *Centralbl. Min., A*, 215–218.

WENK, E. (1936), Zur Genese der Bändergneise von Ornö Huvud: *Bull. Geol. Inst. Univ. Upsala*, **26**, 59–60.

NEMOTO, T. (1938), A new method of determining extinction angles of monoclinic minerals, especially pyroxenes and amphiboles by means of random sections: *Journ. Fac. Sc. Hokkaido Imp. Univ.*, IV, **4**, 108–112.

TURNER, F. J. (1940), Note on determination of optic axial angle and extinction angle in pigeonites: *Am. Mineral.*, **25**, 821–825.

TURNER, F. J. (1942). Determination of extinction angles in monoclinic pyroxenes and amphiboles: *Am. Jour. Sc.*, **240**, 571–583.

EMMONS, R. C. (1943), The Universal Stage with 5 axes of Rotation: *Geol. Soc. Am. Memoir*, **8**, 28–39.

DONNAY, J. H. F. and O'Brien, W. A. (1945), Microscope goniometry: *Jour. Ind. Eng. Chem., An. Ed.*, **17**, 593–597.

LOWE, K. E. (1946), A graphic solution for certain problems of linear structure: *Am. Mineral.*, **31**, 425–434.

GILBERT, Ch. M., AND TURNER, F. J. (1949), Use of the Universal stage in sedimentary petrology: *Am. Jour. Sc.*, **247**, 3–7.

BURRI, C. (1950), Das Polarisationsmikroskop: Basel, Birkhäuser Verl., 151–159, 272–279, 286–293.

HALLIMOND, A. F. (1950), Universal stage methods: *Mineral. Mag.*, London, **83**, 12–22, 77–80.

SABINE, P. A. (1950), Optical properties and composition of acmitic pyroxenes: *Mineral. Mag.*, **29**, 120–122.

Manuscript received Oct. 10, 1952.