ALIGNMENT CHART FOR CALCULATION OF REFRACTIVE INDEX FROM THE DEVIATION OF LIGHT BY A PRISM

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ABSTRACT

An alignment chart is presented by means of which the refractive index of a prism is obtained from the angle of deviation of light passing through it, and the prism angle.

New interest in the prism methods for measurement of refractive indices has resulted from the development of new designs for goniometers (Wolfe and Franklin, 1949; Wolfe, 1948). Such interest suggests that the prism methods and computations therefor might profitably receive additional study. This note describes an improved nomogram for the computation. Smith (1906) described a chart for use with the minimum deviation method, accurate to the nearest .005. Other reductions of equations (1) or (2) to chart form are not known to the writer.

Equations (1) and (2) are used for computing the refractive index, n, from measurements of the prism angle, a, and the deviation, d, of a light beam passing through it.

$$n = \frac{\sin (a + d)}{\sin a} \tag{1}$$

$$n = \frac{\sin \dfrac{(a + d)}{2}}{\sin \dfrac{a}{2}} \tag{2}$$

It is obvious that any nomogram for one will also give solutions of the other, provided only that the scales for a and d have two sets of numerical calibrations differing by a factor of two. Equation (1) is the form used when the light enters (or leaves) one side of the prism at normal incidence (Ford, 1932, p. 240); equation (2) is the form used when the deviation of light is minimum (Ford, 1932, p. 242).

These equations may be solved by means of the alignment chart reproduced in Fig. 1. A straight line passing through the scales a and d at points corresponding respectively to the prism angle and to the angle of deviation will pass through the n-scale at a point corresponding to the refractive index of the prism. It is to be noted that the inner calibrations of the a-, and d-scales apply to the case of normal incidence of the light rays upon one prism face, and the outer (*slant-lettered*) calibrations, to the case of minimum deviation.

The coordinates of the various points along each scale have been obtained for approximately twice as many points as are reproduced here, and when plotted on a chart measuring 100 cm. by 80 cm., readings of refractive index can easily be obtained that are accurate to the nearest 0.003, or better, from angular data accurate to the nearest minute of arc.

Derivation of the Alignment Chart

Equation (1) can be rewritten as follows:

$$n \sin a - \sin a \cos d - \cos a \sin d = 0 \tag{3}$$

In determinant form, this is:

$$\begin{vmatrix} n & 0 & 1 \\ \cos a & \sin a & 0 \\ \cos d & -\sin d & 1 \end{vmatrix} = 0$$

Rearrangement of the determinant gives the following standard form (Mackey, 1944, p. 85):

$$\begin{vmatrix} 0 & \tan a & 1 \\ 1/n & 0 & 1 \\ \sec d & -\tan d & 1 \end{vmatrix} = 0 \tag{4}$$

Equation (4) is recognized as the equation of a straight line through the three points, $x_1=0$, $y_1=\tan a$; $x_2=1/n$, $y_2=0$; and $x_3=\sec d$, $y_3=-\tan d$. It will be noted that the first point is determined solely by the prism angle a, the second by the refractive index n, and the third by the angle of deviation d. The first lies along the positive y-axis, the second, along the positive x-axis; the third, along a curve in the quadrant of positive x, negative y (Fig. 2A).

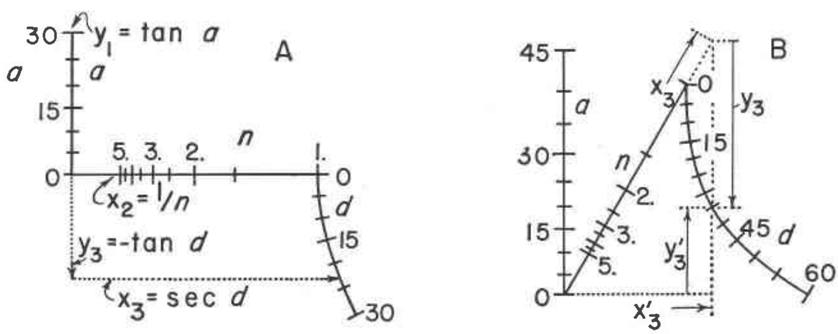
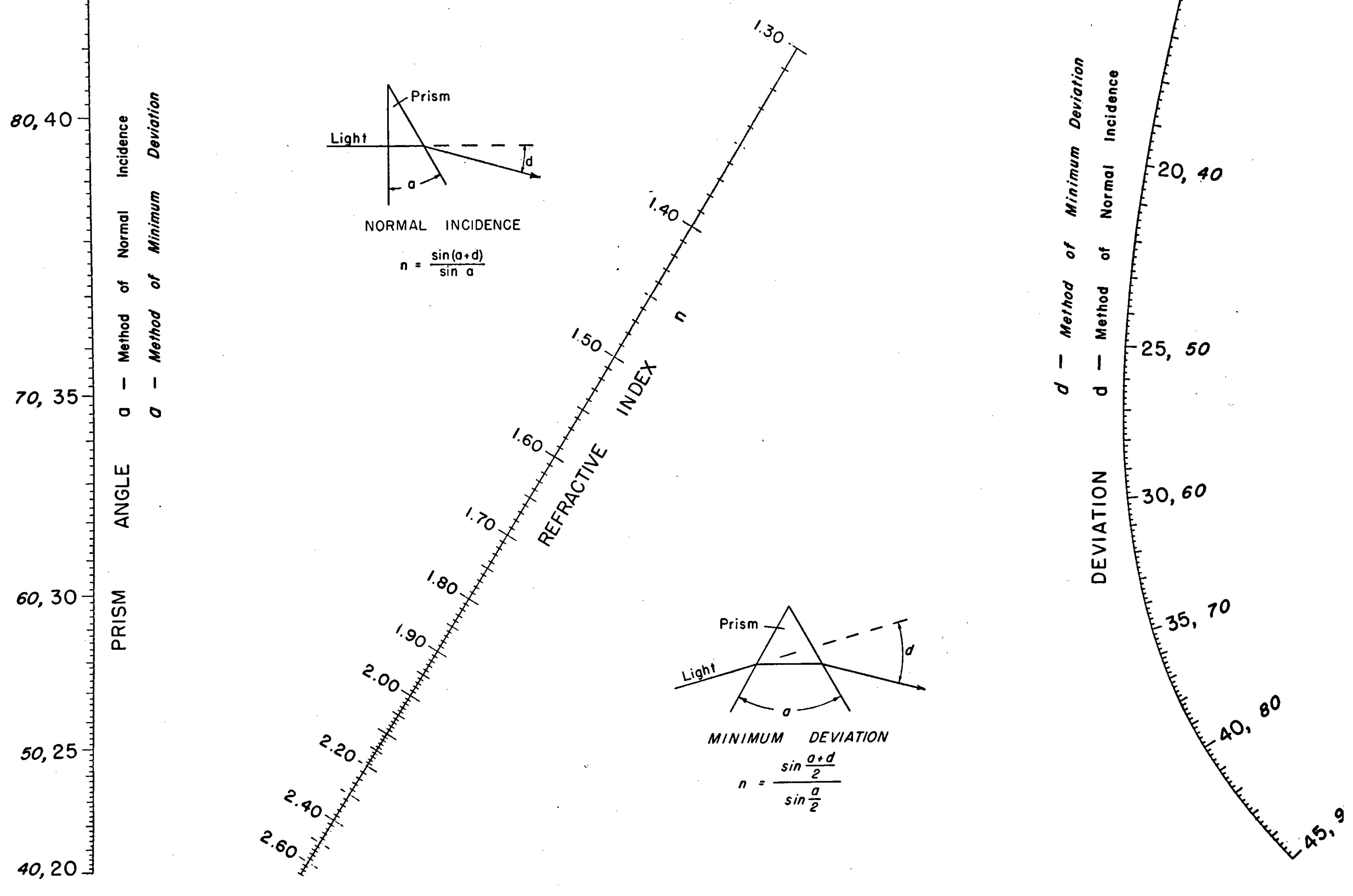


Fig. 2

The angle between the x- and the y-axis does not have to be 90°. The value 30° would give a satisfactory chart which is sketched in Fig. 2B. To facilitate plotting the d-scale on ordinary graph paper, the rectangular coordinates (x_3', y_3') of the scale points may be found after the above choice of 30° for the angle between the x-, and the y-axis is settled (equation

CHART FOR DETERMINING REFRACTIVE INDEX
From Deviation of Light by a Prism



Normal incidence:

$$n = \frac{\sin(a+d)}{\sin a}$$

Minimum deviation:

$$n = \frac{\sin\frac{a+d}{2}}{\sin\frac{a}{2}}$$

5), or the original coordinates (x_3, y_3), measured *parallel to the axes at the new angle of 30°* may be used as suggested in the figure.

$$\left.\begin{aligned} x_3' &= \sin 30^\circ \sec d \\ y_3' &= \cos 30^\circ \sec d - \tan d \end{aligned}\right\} \tag{5}$$

No computing whatever is necessary for the scales of a and n; the x_3 and y_3 values for the scale of d may likewise be found in any table of tangents and secants, but the more convenient rectangular coordinates x_3' and y_3' must be computed.

A projective transformation of equation (4) (Mackey, 1944, p. 97) produces a more useful chart, and at the same time gives it a better shape with selected points of the a-, and d-scales at the corners of a square or rectangle. This transformation is accomplished by multiplying both sides of equation (4) by a determinant (6) in which the a's, b's, and c's are constants to be evaluated.

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0 \tag{6}$$

The product is given as follows:

$$\begin{vmatrix} \dfrac{b_1 \tan a + c_1}{b_3 \tan a + c_3} & \dfrac{b_2 \tan a + c_2}{b_3 \tan a + c_3} & 1 \\ \dfrac{a_1 + nc_1}{a_3 + nc_3} & \dfrac{a_2 + nc_2}{a_3 + nc_3} & 1 \\ \dfrac{a_1 \sec d - b_1 \tan d + c_1}{a_3 \sec d - b_3 \tan d + c_3} & \dfrac{a_2 \sec d - b_2 \sec d + c_2}{a_3 \sec d - b_3 \sec d + c_3} & 1 \end{vmatrix} = 0 \tag{7}$$

We may choose to make the alignment chart 1 meter square, with the a-scale along the left edge ranging from 20° to 45°, and the d-scale curving inward from the right edge and ranging from 10° to 45°. Since calculations are simplified by taking the range of the a-scale to 45°, this square shape was selected, though the upper fifth of the square is omitted in Fig. 1. Placing the origin of coordinates at the lower left corner, we obtain the following eight equations:

Origin: $(x_1=0,\ y_1=0)$

$$x_1 = \frac{b_1 \tan 20 + c_1}{b_3 \tan 20 + c_3} = 0 \qquad y_1 = \frac{b_2 \tan 20 + c_2}{b_3 \tan 20 + c_3} = 0$$

Upper left corner $(x_1=0,\ y_1=1.00$ meter):

$$x_1 = \frac{b_1 \tan 45 + c_1}{b_3 \tan 45 + c_3} = 0 \qquad y_1 = \frac{b_2 \tan 45 + c_2}{b_3 \tan 45 + c_3} = 1$$

Upper right corner $x_3=1,\ y_3=1)$

$$x_3 = \frac{a_1 \sec 10 - b_1 \tan 10 + c_1}{a_3 \sec 10 - b_3 \tan 10 + c_3} = 1 \qquad y_3 = \frac{a_2 \sec 10 - b_2 \tan 10 + c_2}{a_3 \sec 10 - b_3 \tan 10 + c_3} = 1$$

Lower right corner $(x_3=1,\ y_3=0)$

$$x_3 = \frac{a_1 \sec 45 - b_1 \tan 45 + c_1}{a_3 \sec 45 - b_3 \tan 45 + c_3} = 1 \qquad y_3 = \frac{a_2 \sec 45 - b_2 \tan 45 + c_2}{a_3 \sec 45 - b_3 \tan 45 + c_3} = 0 \tag{8}$$

These eight equations contain nine unknown quantities of which one

(c_3) is arbitrarily put equal to unity, reducing the system to eight equations in eight unknowns. Inspection shows that $b_1 = c_1 = 0$, simplifying the system at once. The final solution gives the elements of the multiplying determinant (6) as follows:

$$\left.\begin{array}{lll} a_1 = 0.325911 & b_1 = 0 & c_1 = \\ a_2 = 0.726998 & b_2 = 0.753776 & c_2 = -0.274352 \\ a_3 = -0.749293 & b_3 = -0.520575 & c_3 = 1 \end{array}\right\} \qquad (9)$$

These values are substituted into equation (7) for the construction of the final alignment chart, which is reproduced in Fig. 1. The two equations for the n-scale are known to lead to a straight scale, and can therefore be combined into a single equation of distance x_2' (Fig. 3) along the line from its point of intersection with the y-axis at $y = -0.27435$. As shown by the dashed line in Fig. 3, the upper fifth of the square is omitted. The finished chart is presented as Fig. 1.

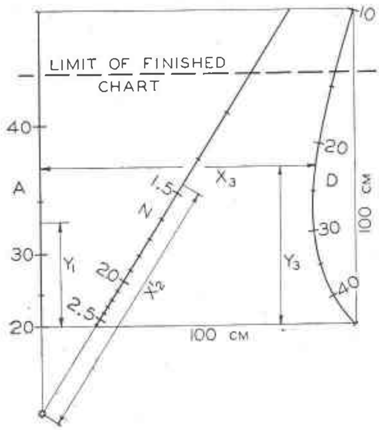


FIG. 3

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Manuscript received Nov. 6, 1950.