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A SIMPLE POINT COUNTER FOR THIN-SECTION ANALYSIS

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ABSTRACT

A manually operated point counter for thin section analysis is described. The machine is sturdy, inexpensive, and easily operated. Its precision has been tested by analyzing 47 thin sections in duplicate and computing the analytical error, or standard deviation of a single analysis, from the observed variance of the differences. The error distribution is effectively binomial and the precision of the instrument is somewhat better than that of the Wentworth-Hunt and Hurlbut integrators. Average operating speed is about four times that of the Wentworth-Hunt and twice that of the Hurlbut machine.

INTRODUCTION

Estimates of modal composition have always been of central importance in petrography, and the various instruments and techniques designed to provide them are too well known to require review here. It seems, however, to have escaped notice that the theoretical underpinning of modal analysis has changed at least four times; thin section analyses are still called "Rosiwal analyses," though it is almost safe to say that no microscopist has made a Rosiwal analysis since the introduction of the Shand micrometer in 1916.

In the method of Delesse as modified by Sollas the individual sample was an entire microscopic field, and the items in this sample were the areas of grains or portions of grains contained in the field. In the Rosiwal method the sample consisted of a number of parallel lines so spaced that no two cut the same grain; the items in the sample were the distances marked out by the intercepts of grain boundaries on these lines. In Shand's procedure the condition that no grain be cut by more than one line is abandoned,* and the lines, or traverses, are distributed evenly over the surface of the specimen regardless of grain size. The importance of this change is two-fold: it permits much greater precision in the estimate of the composition of a given specimen, and it both permits and requires a

* On rereading the original description of the Shand stage I find it specifies that no grain be cut by more than one traverse. Formal abandonment of this condition is apparently due to Wentworth (*Jour. Geol.*, 1923).

sharp distinction between the reproducibility of a single analysis on the one hand, and differences between thin sections on the other.

The basic sampling design of the Shand stage is carried over intact in the Wentworth-Hunt, Dollar, Leitz, and Hurlbut instruments, all of which have the advantage that several constituents may be estimated simultaneously. This sampling design is identical with the well known "trapezoid rule" for graphical integration (1, p. 272).

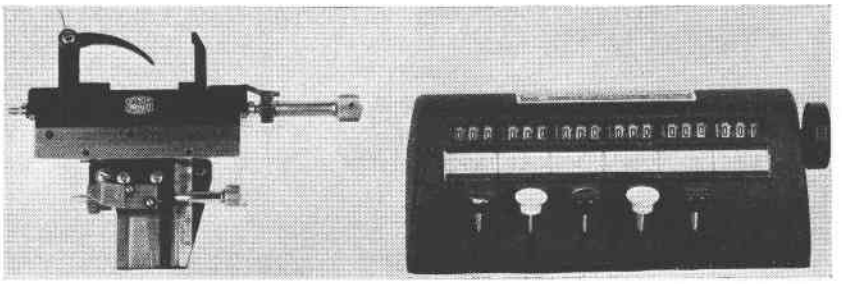
A new basis for thin section analysis was introduced by Glagolev (2, 3) in 1933. In Glagolev's procedure the regular spacing between traverses is retained, but the traverse itself is broken into a series of equally spaced points. The operator identifies the material under the cross-hair intersection and depresses the appropriate key of a tabulating device; the depression of any key on the tabulator triggers a mechanism which translates the stage a fixed distance along the line of traverse. This procedure is repeated until the proper distance has been traversed, the stage is reset and a new line run, and so forth. The sample is thus a bilaterally symmetrical (but not necessarily isotropic) grid of points.

In the course of the last few months I have made nearly 300 thin section analyses with a hand operated version of Glagolev's device. The instrument has proved superior in several respects to any machine now on the market. It is sturdy, inexpensive, and easily assembled from stock parts. It has about the same advantage in time over the Hurlbut instrument as the latter has over the Wentworth; an hour's work on the Wentworth will usually take about a half-hour on the Hurlbut and fifteen minutes on the point counter. The tabulator dials are set at zero at the start of each analysis and are read only once, at the end of the analysis; with a slide rule the number frequencies are easily converted to percentages in two or three minutes. Whereas other instruments can handle only a fixed number of constituents simultaneously, additional counters may be added to the tabulating unit of the point counter at will. The instrument may be used on much finer grained material than can be handled with the Hurlbut integrator and its precision is somewhat superior to that of the Hurlbut and Wentworth instruments.

The Point Counter

A standard Spencer or Bausch and Lomb mechanical stage is easily adapted for point counting. The knurled nut at one end of the horizontal thread is replaced by a click wheel of the type shown at *A* in Fig. 1; in my instrument the notches are spaced on the wheel so that the distance between centers is equivalent to a horizontal traverse of 0.3 mm. A spring stop is mounted on the frame of the stage and adjusted so that each translation of 0.3 mm. is signalled to the operator by an easily audible click. A similar arrangement on the rack and pinion which controls the

"vertical" motion of the stage insures equal spacing of the traverse lines;† this wheel is calibrated so that each translation is 0.5 mm. and I have so far systematically spaced the traverses at 1 mm.—or two clicks—apart. Using a standard area of 1 inch by $\frac{3}{4}$ inch this means that each analysis consists of about 1400 points spaced at 0.3 mm. in one direction and 1 mm. in the other. Holes in the slide are ignored, but if there are many of these there is no point in running the analysis. The outline of the area to be analyzed is traced onto the thin section from a template, in India ink; some irregularity in the number of points is introduced by inaccurate tracing.



A B

FIG. 1. Point Counter (A) and Tabulating Unit (B)

The tabulator at present consists of a five-unit Clay-Adams blood cell counter (*B* in Fig. 1). This is an extremely convenient instrument, but will record only five constituents. A similar and somewhat cheaper six-unit tabulator is made by the Denominator Corporation, but the keys on this are rather widely spaced. Where more than five constituents are to be recorded, a complete block of the same type or separate Veeder counters may be added.

Greater precision may be obtained by reducing the vertical traversing interval as well as by inserting additional horizontal cross hairs in the ocular. With two of these and a vertical interval of 0.5 mm. 8400 counts could be obtained from the standard $\frac{3}{4}$ inch $\times$ 1 inch area used in the tests described below. This would more than double the time required for analysis and it would require four- instead of three-digit counters. These can be purchased separately but are not available in blocks.

PRECISION OF THE POINT COUNTER

Under proper circumstances precision is best determined by making replicate analyses on the same sample. Where there is doubt about the

† Mr. F. A. Rowe, who installed the click wheels on the stage shown in Fig. 1, informs me that his work required about 6 hours.

independence of the replications, however, this procedure may give rise to unwarranted optimism about the precision of the measurement.

For some time I have felt that my study of the precision of the Hurlbut integrator (1) was open to question on this score. In that study traverses spaced at 0.1 mm. were made over the surface of a thin section and recorded separately on numbered cards. The cards were then assembled in ten sets, the first consisting of traverse numbers 0, 10, 20, . . . , the second of 1, 11, 21, . . . , and so forth, each set being regarded as a separate analysis made at a traverse interval of 1 mm. This gave a group of ten analyses from which standard deviations were computed for each of five constituents. Now the 0.1 mm. spacing of the original traverses was very small in relation to the grain of the rock, so it is reasonable to suppose that in general adjacent traverses would be more similar to each other than distant ones. Thus the initial traverses of each synthetic analysis, numbers 0, 1, 2, . . . , would be more similar than if they had been chosen randomly from the entire array; this would be true also of the second traverses, 10, 11, 12, . . . , and so forth for the entire set.* The effect of positive correlation between adjacent runs would be to reduce dispersion of the synthetic analyses. Even by this rather elaborate *tour de force* only ten replications were obtained and these were combined with similar computations made from the same data for synthetic analyses with traverse intervals of 0.9 and 1.1 mm.; for this reason too it would be likely that the observed value would somewhat underestimate the true dispersion.

For the present test a different procedure was adopted. In the machine as described above there are two possible settings of the thin section. Each of 47 thin sections (of the Milford, N. H., Westerly, R. I., and Barre Vt., granites) was analyzed twice. The entire suite was first run with the label of each slide at the left side and then with the slide rotated 180°; the test was spread out over three weeks and the replications were made under varying conditions of illumination, operator fatigue, and pressure for time. Only virtually perfect thin sections were used, so that variations introduced by ignoring or "identifying" holes in the thin sections are not included in the result. Except for this, however, the conditions of the test are very like those encountered in daily routine.†

The testing procedure yields a series of 47 paired differences for each

* Unfortunately the record of the original traverses has gone the way of all scrap paper, so it is no longer possible to determine whether this was in fact the case.

† In my own work I have found it best to refrain from analyzing slides of inferior quality. There is always a strong temptation to use results obtained from such slides if they are not discordant and to discard them if they are. Under such circumstances there is little point in making the analysis.

constituent. The variance of any set of differences is the mean of the squared deviations, or

$$s_d^2 = \frac{1}{n} \sum_{i=1}^{n} (X_i - \mu) \tag{1}$$

and since μ is by definition zero (each pair of analyses being run on the same slide) equation (1) may be written

$$s_d^2 = \frac{1}{n} \sum_{i=1}^{n} (X_i)^2. \tag{2}$$

From the design of the experiment the variance of a difference is the sum of two independent variances which are equal to each other and to the square of the precision error—e.g., the "precision variance"—of a single analysis, or

$$s_d^2 = 2s_a^2 \tag{3}$$

so that, finally, the precision error is given by

$$s_a = \sqrt{s_d^2/2} \tag{4}$$

where s_d^2 is computed from the data according to equation (2). Results of the test are shown in Table 1.

TABLE 1. DUPLICATE ANALYSES OF 47 THIN SECTIONS OF GRANITE

Mineral	Mean $\bar{x}$	Standard Deviation s_x	Variance of Differences s_d^2	Precision Error of a Single Analysis s_a
Feldspar+Musc.	65.84	3.77	2.98	1.22
Quartz	26.45	2.75	1.46	0.85
Biotite	6.65	2.82	0.63	0.56
Others	1.06	0.42	0.22	0.33

The precision error given in column 4 of Table 1, and as used throughout this paper, is a standard deviation. On the assumption that the error distribution is normal there is a two to one chance that a single analysis will not differ from the "true value" by more than this standard deviation, the "true value" being defined simply as the mean of a great many analyses of the same slide. Similarly, the probability of a difference between "true" and observed values as large as $2s_d$ is no more than 5%. By the method used to reach Table 1, however, no slide is analyzed more than twice, so that "true" values are not available for comparison.

The reason for defining precision error in terms of dispersion about the "true value" is just that in every day work we usually regard the observed as an estimate of the "true"; it is essential therefore to be able

to specify, with known probability, the range in which the "true value" may lie.

The tests to which the instrument is likely to be subjected will probably be on a pretty small scale, and where pairs of values are compared, whether obtained by one operator or two, the proper index of dispersion to be applied to the differences is the square root of the appropriate entry in column 3 of Table 1. It is important to realize that theoretically *no* difference is too large to occur, and that in a large enough sample about a third of the differences should be larger than s_d. For example, of the 47 biotite differences obtained in this test, 31 are less than s_d for biotite, which is 0.795, 13 lie in the range $s_d \leq d \leq 2s_d$, and 3 are larger than $2s_d$; the largest is 2.3, or nearly three times s_d. Distributions of differences for the other constituents are comparable.

Although the selection of points in a single analysis is not random in the usual sense, for the entire array may be regarded as fixed once the initial point is chosen, it may be shown that the results in Table 1 conform rather nicely with what would be expected if the sampling were truly random. In a perfectly adjusted machine of the type pictured in Fig. 1, a thin section area $\frac{3}{4}$ inch $\times 1$ inch offers a total of 2800 points of which 1400 are used for any single analysis. If the points forming each analysis were randomly chosen the case would conform exactly to hypergeometric sampling, in which a sample of fixed size is drawn, without replacement, from a lot containing a finite number of items. The standard deviation for sampling of this type is given by

$$s_h = \sqrt{n \cdot \frac{M}{N} \cdot \frac{N-M}{N} \cdot \frac{N-n}{N-1}} \tag{5}$$

where n = number of items in the sample.
N = number of items in the lot.
M = number of items of one particular kind in the lot.

If knowledge about the lot is lacking, as will usually be true, the fractions M/N and $(N-M)/N$ are estimated from the sample. As N increases, the term $N-n/N-1$ approaches unity and in the limit as $N \to \infty$ equation (5) reverts to the common binomial form

$$s_b = \sqrt{npq}. \tag{6}$$

If $N = 2n$ the variance will be approximately half and the standard deviation a little over 70% of the binomial parameters. If $N = 4n$ the hypergeometric standard deviation is 86%, and if $N = 10n$ it rises to 95%, of the binomial parameter. For both N and n large the advantage of hypergeometric sampling is rapidly lost with increase in the ratio N/n.

In a perfectly adjusted machine the sampling would be truly hyper-

geometric, with $N=2n$. As the machine wore, end play would increase in the thread, the notches on the clicking wheels would enlarge and the tension of the springs would decrease. All of these changes tend to increase the number of possible points (the lot size, N), and an increase in the speed of operation would have the same effect.

We should expect then that if the sampling were effectively random, estimates of analytical error would be conformable with parent standard deviations intermediate in size between the simple binomial and the hypergeometric with $N=2n$. This in fact proves to be the case. The average number of points per thin section was 1391. Using the mean values of Table 2 as estimates of p, and M/N, and taking $n=1391$, $N=2800$, equations 5 and 6 may be solved to yield theoretical estimates of precision error comparable to the observed values shown in Table 1. Equations 5 and 6 give the error as a number of points; in Table 2 the results are shown on a percentage basis (100s/1391).

TABLE 2. COMPARISON OF THEORETICAL AND OBSERVED ANALYTICAL ERROR

Mineral	Mean	Error of a Single Analysis as % of the whole		
		Hypergeometric	Observed	Binomial
Feldspar+Musc.	65.84	0.90	1.22	1.27
Quartz	26.45	0.84	0.85	1.19
Biotite	6.65	0.47	0.56	0.67
Others	1.06	0.20	0.33	0.27

The chief use of the error estimates of Table 2 is in judging single analyses. With the procedure described above, the least count of the instrument is 1/1400 so that an analysis should not be reported beyond tenths of a per cent. To this degree of approximation $s_h \leq s_{obs} \leq s_b$, as may be seen by rounding off the error entries of Table 2 to the first decimal.

For practical purposes the analytical error may be regarded as binomial. Over a short period of time results obtained from a single instrument may benefit by the hypergeometric character of the sampling. But if the machine is constantly used and the replications are made at long intervals the error distribution will probably be effectively binomial. This will almost certainly be so if the same slide is analyzed on a number of different instruments, and in any case the difference is small. For the curious fact that a clearly non-random sampling procedure behaves as if it were random I have no satisfactory explanation. In these thin sections the average grain diameter is several times the distance between points, so that adjacent points are certainly not independent. But there

is no connection between the locations of grain boundaries and the selection of points. It may be that the randomness operates in terms of clusters or runs of points.

Concluding Remarks

Enthusiasm at finding a simple and relatively rapid method for analyzing thin sections has led me to make rather generous claims for the instrument described in this note. By way of conclusion I should like to offer evidence substantiating these claims, to anticipate a few criticisms, and to give an example or two illustrating the utility of the instrument.

It has already been suggested that in the only similar precision test of the Hurlbut integrator (1) the testing procedure is not entirely comparable with that used here. Both tests, however, were attempts to estimate the same error; both being run on granite, each offers estimates of precision error at about the same composition levels. The results are compared in Table 3 below.

Table 3. Comparison of Precision Tests for Hurlbut Integrator and Point Counter

Composition Level	Hurlbut Integrator*			Point Counter†		
	$\bar{x}$	Prec. Error	Prec. Error $\frac{}{(2n)^{1/2}}$	$\bar{x}$	Prec. Error	Prec. Error $\frac{}{(2n)^{1/2}}$
Principal						
Feldspar	61.0	1.93	0.35	65.8	1.22	0.18
Quartz	29.6	1.08	0.20	26.4	0.85	0.12
Minor	7.8	0.94	0.17	6.6	0.56	0.08
Accessory	1.6	0.47	0.09	1.1	0.33	0.05

* Data from Table 6 of reference 1.

† Data from Table 1, this paper.

For normal distributions σ itself is normally distributed with standard deviation $\sigma_\sigma = \sigma/\sqrt{2n}$; each entry in columns 3 and 6 of Table 3 is an estimate of the error contained in the value immediately to the left. In each case the estimated precision error of the point counter is less than that of the Hurlbut integrator and in three of the four comparisons the difference is greater than the sum of the estimated σ_σ values.

A more refined test of Table 3 would be to use the observed difference at any level divided by either σ_σ estimate at that level as a normal deviate, on the assumption that the true difference is zero. Still another would be to compute the standard deviation of each difference on the assumption that the samples were *not* drawn from the same population.

Both these tests, as well as the simple comparison suggested in the preceding paragraph assume that the error distribution is "sufficiently normal," but there seems little reason to doubt this except possibly at the "accessory" composition level. All three procedures lead to about the same result: the point counter is certainly as precise as the Hurlbut integrator and probably a little more so.

No comparably detailed direct test of the Wentworth-Hunt instrument seems to have been made. My own work (1) suggests that if the traverses are evenly spaced—something which could not be accomplished without slightly modifying pre-war models—it has about the same reproducibility as the Hurlbut, except at the "accessory" level, where it seems a little better. (Strictly the test showed only that there was no necessity to assume any difference between the precision of the two instruments.)

The sampling design of all instruments which attempt to measure a continuous line is theoretically superior to that of the point counter. Their difficulty is in practice not theory. Manually operated instruments of this type require a reading (and recording) of each dial at the end of each traverse; each reading contains an error and though these errors *tend* to balance out, in general they will not exactly compensate in the course of a single analysis. Errors made by the eye in using the manually operated machines are made by the hand in operating the Hurlbut stage; almost invariably one over- or under-runs grain contacts. A Hurlbut stage which would operate at high speed without over- or under-running would no doubt be considerably superior to the point counter described in this note; in effect it would be a point counter counting a very large number of points.

While the possibility of errors in identification can never be entirely excluded, it should always be held to a minimum in any work of this type. To get results of the necessary quality and quantity it is desirable to work at high speed; minerals which cannot be instantaneously, or almost instantaneously, distinguished from each other should not be separately recorded. Identification errors will be erratic, highly subjective, and, in practical terms, virtually unmeasurable. Only an operator whose bravery exceeds his wisdom will attempt analyses when he has reason to suspect that errors of identification will be more than a trifling component of the total precision error.

Finally, there is the question of the speed at which analyses can and should be run. This will no doubt vary with the ability and experience of the operator, but I believe that the relative rates at which the point counter, the Hurlbut integrator, and the Wentworth-Hunt stage are conveniently operated will be about as stated in the abstract. My experience with the point counter suggests that a rate of between 75 and 100

counts per minute is not at all excessive after a little practice; this is an over-all figure and includes delays for identification, change of focus, movement of the vertical traverse knob, etc. It can be maintained without difficulty for the 15 to 20 minutes required for an analysis. These figures are based largely on work with fine grained granites, 7 constituents being recorded in each analysis. Although the tabulation of additional constituents is at first very troublesome, I believe anyone who has even a passing acquaintance with touch-typing will find that after a little practice—and a few worthless analyses—the number of constituents separately recorded has no appreciable effect on the rate of operation. Grain size is a critical factor in determining the rate of operation, since if the grains are small the runs of identical points will all be short. Very fine grained rocks cannot be done at all, but it is my impression that the point counter allows more leeway in this direction than any of the continuous-line recorders.

Prolonged use of any of the instruments mentioned in this note leads to fatigue and unreliable results. It is always best to take a five or ten minute break between analyses, and I have so far never spent a full day analyzing thin sections. A half-dozen analyses represents a pretty fair morning's work.

It is virtually certain that a long analysis done slowly will be better than a short one done quickly, but it is not at all sure that the increased precision will be either very large or very useful. Quantitative modal analysis has fallen so far behind interpretive petrology that rather severe measures are in order; in the present status of our subject many good analyses are worth more than a few excellent ones. For practical purposes we need require only that the precision error of a single analysis be small in relation to the expected differences between rocks. In many experimental designs the precision error, if it is known, can be extracted from the total variance. In many others the hypothesis being tested is so general that the operation of extracting the precision error from the total variance is scarcely worthwhile.

This particular point is much more readily made by illustration than by argument. A deep lavender fluorite, occurring in very small grains, is the most conspicuous accessory constituent of the Westerly, R. I., granite. The mineral is present in every one of 24 specimens, each specimen coming from a separate quarry and the quarries being anywhere from a hundred yards to over ten miles distant from each other. Yet in 24 analyses, of about 1400 counts each, not a single grain of fluorite happened to fall at the cross-hair intersection. Proper analysis for such a minor constituent would require a very extended, time-consuming count. For a first approximation it seems to me far more useful to know that

each of 24 samples yielded less than 0.1% fluorite than it would be to know that each of 6 samples yielded fluorite in the range, say, 0.005 to 0.05%; in terms of time expended this is about the choice we face in this case.

Again, in over 60 specimens from in and near a transition between "granodiorite" and "gabbrodiorite" in Norfolk County, Massachusetts, the replacement habit of quartz is always prominent. These rocks are extensively altered and I believe that from an examination of the thin sections most geologists—myself included—would be willing to regard the metasomatic or hydrothermal introduction of quartz as a factor of first importance in determining their composition. Yet nearly all of my specimens contain less than 6 or more than 28 per cent of quartz; the introduction of quartz, apparently so extensive, has not even masked, much less eliminated, one of the most prominent original differences between the dominant members of the complex. All of the rocks are now a little richer in quartz than they may once have been, but some, and possibly a good deal, of this excess quartz may have formed by decomposition of minerals already present. Hydrothermally introduced quartz can hardly amount to more than a few per cent of the total rock.

These results will be described in detail in a later report; they are mentioned here only as an illustration of the utility of moderately precise quantitative estimates. The range of quartz content is here very wide, so that a precision error of 1% in each analysis is not of much consequence. Forty or fifty moderately precise analyses of well chosen samples provide grounds for an extremely useful, if rather rough, estimate of the general importance of hydrothermal quartz; 10 or 20 analyses would give no more than an indication, regardless of their precision. For the same investment in time there is a much greater return from numerous moderately precise analyses—providing their precision is known—than there would be from a few excellent analyses.

The point counter described in this note may be used either for very detailed work or for the rapid, moderately precise estimates which seem to me of so much greater present importance. In terms of both time and money it brings quantitative modal analysis within reach of every petrographic laboratory.

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