A. C. GILL'S DEVELOPMENT OF THE CONCEPT OF UNIQUE DIAMETERS IN CRYSTALLOGRAPHY

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ABSTRACT

A unique diameter is a line, or diameter, unlike any other in the crystal. All parallel lines are the same line crystallographically. Some properties, physical, chemical, and/or crystallographic, along a unique diameter are different from those along any other line in the crystal.

Several criteria based on physical and chemical properties, the groupings of faces and angles between faces, and the dimensions of crystals may be used to recognize unique diameters. Likewise, laws controlling their distribution in crystals and their relations to the various elements of symmetry may be formulated. Since their arrangement in each crystal system is different from that in any other, except that the tetragonal and hexagonal are alike, unique diameters, in conjunction with like diameters, may be used to define the six crystal systems independent of symmetry and axes of reference, and the selection and orientation of axes of reference may be based on them and certain like diameters.

Some of the applications and relationships of this concept are: (1) the assignment of crystals to systems without the use of symmetry or hypothetical axes of reference; (2) the easy selection of axes of reference; (3) the determination of the crystal system to which a mineral belongs from its cleavage fragment; (4) a clarification of some of the relationships and conditions observed in optical mineralogy; and (5) the simplification of the teaching of crystallography, especially in short courses where it is desirable to present only the commoner forms and not to discuss classes.

This concept presents a basis for the definition and treatment of systems founded on inherent, independent properties and not on hypothetical axes of reference nor on groupings of classes.

INTRODUCTION

The origin of the concept of unique diameters is somewhat obscure. However, the material here presented is substantially that used for many years by the late Professor A. C. Gill in his successful teaching of crystallography at Cornell University. According to statements made by Professor Gill,[1] the idea originated with some German crystallographer, who is reported to have published a note on the subject during the latter part of the last century. Professor Gill obtained the basic germ of the concept from the above-mentioned German crystallographer either by personal communication or from his published note and developed it into a logical sequence of ideas. The material in the form presented in this paper is substantially that developed by Professor Gill and used by him in his lectures on crystallography. In all his considerations of crystals, Professor Gill emphasized the significance of this method, and it

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[1] Dr. Evans B. Mayo, personal communication, the writer, and others. However, Drs. Hermann F. Vieweg and James H. C. Martens, who were also closely associated with Dr. Gill, and a few others, have no recollection of his crediting the conception of the idea to another person.

was under his instruction that the writer obtained his knowledge of the concept.

As no trace of the original notice, if any existed, has been located and since this material in its current form has never been published, it seems desirable to present it at this time.

DEFINITIONS

In any crystal structure, either any line, or diameter, must be unique or there must be others like it in all respects. *A unique diameter, then, is a line different from any other in the crystal.* Examples of unique diameters are the "*c*" axes in the tetragonal and in the hexagonal systems and the three axes of reference in the orthorhombic system.

Like lines, or diameters, are lines which are identical in all respects. The two lateral axes of crystals in the tetragonal system are like lines, as are the three lateral axes of those in the hexagonal system. Every line in crystals of the isometric system has two or more lines like it.

Along every line, whether unique or not, there are two directions, which are opposed to each other in sense. These directions may or may not be alike. A sphere of quartz partially dissolved by hydrofluoric acid[2] is a case in point. The two directions along the "*c*" axis of reference are alike; therefore, the two ends of this axis are attacked in the same manner. On the other hand, the opposing directions along each of the lateral axes of reference are unlike; therefore, the positive and the negative ends of these axes are differently affected in such a manner that the negative end of any axis is appreciably dissolved, whereas the positive end is only slightly (much less) affected.

All parallel lines in a crystal are considered to be the same (one) line, all parallel planes to be the same (one) plane, and all parallel directions to be the same (one) direction if taken in the same sense. These parallel lines, planes, or directions are alike in every respect.[3] Therefore, the use

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[2] Penfield, S. L., and Meyer, Otto, Results obtained by etching a sphere and crystals of quartz with hydrofluoric acid: *Conn. Ac., Tr.* **8,** 158–165 (1890); *Yale Bicen. Pub., Contr. Min.*, 160–167 (1901).

Gill, A. Capen, Beiträge zur Kenntniss des Quarzes: *Zeits. Kryst.*, **22,** Bd. 2, 3–34 (1893).

Dana, E. S., *Textbook of Mineralogy,* 4th ed. by W. E. Ford, John Wiley and Sons, New York, **1932,** p. 213.

[3] The writer is aware of the fact that, in the atomic structure of minerals, the planes formed by sheets of atoms are different from the planes in space lying between these sheets of atoms. Likewise, the lines formed by rows of atoms are not like the lines in space between these rows of atoms. Furthermore, in the atomic structure of many minerals, adjacent rows or sheets of atoms themselves are different. However, when the largest unit which does not contain a repetition is taken, it is repeated indefinitely throughout the structure and accordingly similar properties result, such as repeated potential planes of cleavage be-

of the term "diameter" as synonymous with "line" is justified, since, in any set of parallel lines (which is considered as a single line), there is always one which passes through the center of the crystal.

Some properties, physical or chemical, along or at the ends of a unique diameter must be different from those along or at the ends of any other diameter in the crystal. This shows that the atomic structure along a unique diameter is different from that along any other line in the crystal.

All like planes, lines, or directions in a crystal must be alike in every respect. Therefore, all properties, physical and chemical, manifested by one plane, line, or direction must be shown in like degree by all like planes, lines, or directions. The atomic structure of like planes, lines, or directions is identical.

Physical properties of unique and like lines (and the planes perpendicular to these lines) are well illustrated in gypsum. Crystals of this mineral have a plane in which all diameters are unique, the plane parallel to *b*{010}, and one, and only one, other unique diameter, the normal to this plane. On *b*{010}, which is perpendicular to the single unique diameter, there is perfect cleavage, pearly luster, and a hardness of 2; whereas, on *a*{100}, which is normal to a unique diameter of the plane of unique diameters, there is imperfect cleavage yielding a conchoidal surface, vitreous luster, and a higher hardness. These assemblages of physical properties are peculiar to the faces mentioned. However, the $\{\bar{1}11\}$ cleavage is perpendicular to two diameters which are alike, and, while having properties different from the two cleavages mentioned above, has two directions of cleavage, which are alike in their properties. Other excellent examples of like properties exhibited by faces perpendicular to like directions (like faces) are seen in galena, halite, sphalerite, hornblende, etc. Hornblende also illustrates different properties of different diameters.

Chemical properties of unique and like lines are well illustrated by the corroded quartz sphere previously mentioned.[4] Such a sphere is attacked rapidly in the direction of the unique diameter, the "*c*" axis of reference, to a degree different from that along any other line in the crystal. At the positive extremities of the lateral axes of reference, the crystal is attacked very slowly, but in a similar manner at each positive extremity (like directions). Likewise, the negative extremities of the lateral axes of reference are attacked in a manner similar to one another but different from and greater than the positive ends, yet less than the attack at the ends of the "*c*" axis.

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tween adjacent planar units (one plane) and similar behavior of light along parallel linear units (one line). The former is commonly spoken of as *a single* direction of cleavage.

[4] Penfield, S. L., and Meyer, Otto, *op. cit.* Gill, A. C., *op. cit.* Dana, E. S., 4th ed. by W. E. Ford, *op. cit.*

CRITERIA OF UNIQUE DIAMETERS

The following criteria may be used in recognizing unique diameters:

1. The dimensions of the crystal not equal,—some longer or shorter than others.

An example of this is the greater or less length of the "*c*" axes of tetragonal and hexagonal crystals in comparison with the lateral axes.

This criterion must, of course, be used with extreme care in dealing with actual crystals, because of malformation, but it is suggestive in use with wooden models and to a much less extent with actual crystals themselves. It should, however, always be verified with one of the following criteria.

2. Groupings of faces and angles between faces around the ends of or in relation to one line different from all others.

Examples of this may be seen in tetragonal and hexagonal combinations of prisms and bipyramids. In these systems the "*c*" axes are the unique diameters.

This criterion is the most serviceable for quick determination in all ordinary work.

3. Chemical and physical properties along or at the ends of one line different from those along or at the ends of any other line.

Again the quartz sphere referred to above[4] may be used as an example. Solution along the "*c*" axis, or unique diameter, was considerably more pronounced than along any other line in the crystal.

The behavior of light along the "*c*" axes, or unique diameters, of tetragonal and hexagonal crystals is an example of physical properties along unique diameters being different from those along any other lines in the crystal.

DISTRIBUTIONS OF UNIQUE DIAMETERS IN CRYSTALS

The distributions of unique diameters in crystals are subject to the following restrictions:

1. If two diameters are unique and at right angles to each other, then in their plane either all diameters are unique or these are the only two; but, in either case, the line perpendicular to their plane must be unique.

Crystals in the orthorhombic system, in which the three axes of reference are the unique diameters, serve as examples in the latter case (in which the two unique diameters are the only two in their plane). In the former (in which all diameters in their plane are unique), crystals in the monoclinic system may be used, since the plane of the axes "*a*" and "*c*" is a plane of unique diameters and the axis "*b*" is unique. In the plane of unique diameters, any two such lines at right angles to each other may be selected to fulfill the conditions of this case.

2. If two diameters are unique and oblique to each other, all diameters in their plane are unique and also the line normal to their plane.

Crystals in the monoclinic system may again be used as an example, since the plane of the axes of reference "*a*" and "*c*" is a plane of unique diameters as is the line normal to this plane. Here, however, two unique diameters oblique to each other, such as the axes "*a*" and "*c*," may be selected to fulfill the conditions of the case.

3. If there is a plane of unique diameters, and, in addition to the unique line normal to this plane, which must necessarily be present, there exists still another unique line, then all diameters in the crystals must be unique.

The triclinic system serves as an example of this case. In this system all diameters are unique. However, we may select a plane, which will be a plane of unique diameters, and a normal to this plane, which will also be unique. Then any other line selected in the crystal will be unique, and all lines in the crystal are unique.

The monoclinic system offers the reverse of this. There is a plane of unique diameters, the plane of the axes of reference "*a*" and "*c*," and there is a unique line normal to this plane, the axis "*b*." However, any line other than these lines has another line like it in the crystal, and these diameters are, therefore, the only unique ones present.

4. If there is a plane of symmetry, all unique diameters must lie in this plane or be perpendicular to it. No unique direction can be normal to a plane of symmetry. Unique directions may, however, lie in the plane of symmetry.

The plane of symmetry of the monoclinic system illustrates this case, since it is coincident with the plane of unique diameters and there is a unique diameter normal to it but nowhere else.

The tetragonal and hexagonal systems also illustrate this case, since all planes of symmetry either include or are normal to the "*c*" axis, or unique diameter.

No unique direction can be normal to a plane of symmetry, because a direction on one side of the plane will be repeated on the other side in opposite sense and will, therefore, not be unique. However, those directions lying in the plane of symmetry are not repeated by it and may, therefore, be unique. In the mineral hemimorphite, the two orthorhombic axes "*a*" and "*b*" are unique lines and are normal to planes of symmetry. The opposing directions along each of these axes are alike. However, along the vertical axis "*c*," which is also unique but which is not perpendicular to a plane of symmetry, the opposing directions are unique.

5. If there is a 180° axis of symmetry, all unique diameters must be coincident with or normal to this axis. No unique direction can be normal

to a 180° axis of symmetry. Two such directions may, however, be coincident with such an axis.

Orthorhombic and monoclinic crystals illustrate this case—as do tetragonal and hexagonal ones also. In orthorhombic crystals, the axes of reference being the unique diameters and, in one case or another, axes of symmetry, these axes are always coincident with or normal to a 180° axis of symmetry.

In monoclinic crystals, the "*b*" axis may or may not be an axis of symmetry, but no other line can be in this system. However, when it is, since it and the plane normal to it are unique, all unique diameters are either coincident with or normal to this axis.

The mineral hemimorphite illustrates the latter part of this case. This mineral belongs to the hemimorphic class, which is characterized by two planes of symmetry with a 180° axis of symmetry at their intersection, the latter being the "*c*" axis of reference. In crystals of this mineral, the "*a*" and "*b*" axes are unique lines, and the two opposing directions along them and normal to the 180° axis of symmetry are alike. On the other hand, the "*c*" axis, or axis of symmetry, is a unique line, but the two directions on this line are unique also and, therefore, coincident with a 180° axis of symmetry.

6. If there is a 120°, a 90°, or a 60° axis of symmetry, it may or may not be unique, but no other diameter in the crystal can be. (Regarding a 60° axis, this statement concerns only the restrictions imposed by the axis itself.)

In the isometric system 120° and 90° axes of symmetry can be found which are not unique, and no other line in the crystal is unique. No 60° axis of symmetry which is not unique occurs in crystals.

In the tetragonal and hexagonal systems, 90° and 60° (or 120°) axes of symmetry, respectively, may be found. These lines are unique, but no other is in these crystals.

7. A center of symmetry imposes no limitations on unique diameters. However, if there is a center of symmetry, there can be no unique directions in the crystal, for every direction is like its opposite.

The triclinic system, in which all diameters are unique, is an illustration. In the normal, or pinacoidal class, which has only a center of symmetry, all lines are unique, but no directions are, hence the forms are all two-faced forms, or pinacoids. On the other hand, in the hemihedral, or pedial class, which has no symmetry at all, all lines, as well as all directions, are unique, hence the forms are one-faced forms, pedions.

Possible Arrangements of Unique Diameters in Crystals

Unique diameters have the following arrangements in crystals and no others:

1. No diameters unique.
2. One unique diameter.
3. Three, and only three, unique diameters at right angles to one another.
4. All diameters in one plane unique and only one other unique diameter, which is normal to this plane.
5. All diameters unique.

DEFINITIONS OF THE SIX CRYSTAL SYSTEMS IN TERMS OF UNIQUE AND LIKE DIAMETERS

1. The isometric system has no unique diameters. It has, among others, three like diameters at right angles to one another.
2. The tetragonal system has one, and only one, unique diameter; and, in the plane normal to it, any two diameters at right angles to each other are alike.
3. The hexagonal system has one, and only one, unique diameter; and, in the plane normal to it, any three diameters at 120° with one another are alike.
4. The orthorhombic system has three, and only three, unique diameters, which are at right angles to one another.
5. The monoclinic system has one plane in which all diameters are unique, and only one other unique diameter; namely, the normal to this plane.
6. The triclinic system has all diameters unique.

SELECTION OF AXES OF REFERENCE AND ORIENTATION OF CRYSTALS FOR DESCRIPTION BY MEANS OF UNIQUE AND LIKE DIAMETERS

1. *Isometric system.* The three like lines at right angles to one another are chosen as the axes of reference. There is only one such possibility.
2. *Tetragonal system.* The unique diameter is set as the vertical axis, the "*c*" axis. In the plane at right angles to this unique diameter, two important like lines perpendicular to each other (axes of symmetry, if possible) are chosen as the lateral, or "*a*," axes.
3. *Hexagonal system.* The unique diameter is set as the vertical axis, the "*c*" axis. In the plane normal to this unique diameter, three prominent like lines at 120° to one another (axes of symmetry, if feasible) are selected as the lateral, or "*a*," axes.
4. *Orthorhombic system.* The three unique diameters are chosen as the axes of reference. One of these (the most convenient) is set vertically and designated by the letter "*c*." The longer of the other two is set right and left, designated by the letter "*b*," and called the macro-axis. The shorter of the lateral axes is then front and back, is designated by the letter "*a*," and is called the brachy-axis.

5. *Monoclinic system.* The plane of unique diameters is set vertically and front and back. The unique line perpendicular to this plane is, therefore, right and left, is designated by the letter "*b*," and is called the ortho-axis. In the plane of unique diameters, a prominent line (the most convenient) is selected, set vertically, and designated by the letter "*c*." Another such line, in general at oblique angles to "*c*," is set sloping downward toward the observer, designated by the letter "*a*," and called the clino-axis.

6. *Triclinic system.* Three prominent lines are chosen as the axes of reference. Lines which give the simplest relations in description are chosen. These are commonly axes of prominent zones. One of these axes (the most convenient) is set in a vertical position and designated by the letter "*c*." The longer of the other two is usually set right and left and sloping downward toward the right, if feasible, designated by the letter "*b*," and called the macro-axis. The third, then falls in a general front-back position, is designated by the letter "*a*," and is called the brachy-axis.

APPLICATIONS OF THE CONCEPT

1. By using this concept, crystals can be assigned to systems without reference to symmetry or to hypothetical axes of reference.

2. The crystal system to which a mineral belongs can often be determined from cleavage fragments of the mineral. In many cases, where the specific crystal system cannot be determined, the mineral can be narrowed down to one of several systems without the use of symmetry.

For example, in studying a cleavage fragment of selenite, it can be seen that there are two unique diameters oblique to each other, one normal to the {100} cleavage and one bisecting the angle of the $\{\bar{1}11\}$ cleavage and lying in {010}. These lines are unique, since each of these cleavages manifests properties different from those of any other plane in the crystal. In the latter cleavage, the line bisecting the angle between the cleavage surfaces and lying in the plane of {010} must be unique, because there are no others like it in the crystal. Since these two unique diameters are oblique to each other, they must lie in a plane of unique lines, in accordance with the second case of distribution of unique diameters. The {010} cleavage lies in this plane and, for reasons similar to those cited above, is perpendicular to a unique diameter. This arrangement, a plane of unique diameters and a unique diameter perpendicular to this plane, fulfills conditions found in the monoclinic and the triclinic systems. However, this crystal must belong to the monoclinic system, because of the $\{\bar{1}11\}$ cleavage, which is parallel to a four-faced form and, therefore, cannot be triclinic and because every line other than those in the plane of unique diameters and the one normal to this plane is, and can be seen to be, like some other line in the crystal, a condition found only in monoclinic crystals.

One other example might be cited. In a cleavage fragment of hornblende, the line of intersection of the cleavage surfaces is unique, as are the two lines bisecting the angles between the cleavage surfaces and lying in the plane perpendicular to them. There can be determined, then, from such a fragment three unique lines at right angles to one another. Such a fragment must be, from these data, in either the orthorhombic or the monoclinic systems; for the two like directions of cleavage exclude the triclinic and the absence of a face cutting the "*c*" axis makes it impossible to distinguish between these two systems.

3. If unique diameters are lines each of which has some properties different from those of any other line in a crystal, it follows that optical properties are likely to be greatly influenced by them. This is found to be the case. In the isometric system, which has no unique diameters, the optical indicatrix is a sphere. In tetragonal and hexagonal crystals, the "ϵ" axis of the optical indicatrix coincides with the single unique diameter. This optical indicatrix is an ellipsoid of revolution, and the "ϵ" axis is a unique axis. In orthorhombic crystals, the principal axes of the optical indicatrix coincide with the three unique diameters. The principal axes of the indicatrix for biaxial crystals are themselves unique. In the monoclinic system, two such principal axes lie in the plane of unique diameters and the third coincides with the unique diameter which is normal to this plane. In the triclinic system since all diameters are unique, there is no set relationship between crystallographic and optical directions; the principal axes of the optical indicatrix can coincide with any three unique diameters at right angles to one another.

4. Similarly, in a consideration of the atomic structure of crystals, certain rows of atoms can be seen to be unique and certain rows have others like them. In fact, the term "unique rows of atoms" has been used by some *x*-ray specialists in discussing the atomic structure of minerals.

5. This concept can be used to advantage in a short course in mineralogy—a one semester course, for example—where the amount of time available does not warrant a discussion of classes. Systems can then be defined in terms of unique diameters, and the commonest forms in the several systems described without reference to classes other than a discussion of symmetry and a statement of the basis on which the division of crystals into classes is made. Crystallography then is a complete, integral unit to the student, and this method avoids the feeling on his part of having covered only certain specific sections of the subject.

This method of treatment can also be used in longer courses in which all or many of the classes are discussed; and, in this application, it presents a simple, direct, real approach to systems.

SUMMARY

This concept presents a basis for the definition and treatment of systems founded on inherent, independent properties and not on hypothetical axes of reference, nor on groupings of classes.

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