THE MORPHOLOGY OF COLUMBITE CRYSTALS

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Abstract

The space-group of columbite, obtained from the morphology, by the Donnay method (1938) is *Pman*, in a new setting ($a:b:c=0.4023:1:0.3580$) chosen so as to comply with the convention $c<a<b$. In the same setting, the space-group found by *x*-ray methods (Sturdivant, 1930) is *Pcan*. The difference lies in the interpretation of the zone [100].

The classical law of Bravais does not agree with the observed facts as well as the generalized law (Donnay-Harker, 1937), which, although not perfect, is decidedly better.

Introduction

Columbite, $(Fe,Mn)(Nb,Ta)_2O_6$, crystallizes in the orthorhombic system. It is of special interest on account of the anomalous zone of the (0*kl*) faces, which makes it impossible to reconcile the space-group determined from the morphology, with the space-group indicated by *x*-rays. In the first part of the paper the space-group is derived by the morphological method (Donnay, 1938, *a b c*) and special reference is made to the anomalies. The second part is devoted to a comparison of the classical law of Bravais with the generalized law (Donnay-Harker, 1937) in order to determine which of the two conforms more closely to the observed facts.

Observation Data

V. Goldschmidt's axial elements and setting are adopted provisionally. His letters are also used throughout to denote the forms (*Atlas der Krystallformen*, 1913). Of the ninety figures given in Goldschmidt's *Atlas*, eighty-two were studied with a view to estimate the relative importance of the forms occurring on columbite crystals (eight figures lacking clarity were disregarded).

The cleavage forms, *b*{010}, rather distinct, and *a*{100}, less so, coincide with the dominant growth forms, a fact which, however, cannot be taken into account in estimating form importance (Donnay, 1938, *c*). The importance of the forms is based primarily on the *frequency* of occurrence; their *size* is also taken into consideration, but is given less weight.

Several types of data are sought: (1) the relative importance of the forms for each crystal figured in the *Atlas;* (2) the relative importance of the forms for each locality; (3) the relative importance of the forms for the species as a whole. Tables 1 to 12 present the data on the first two points. In these tables the figure numbers in Goldschmidt's *Atlas* are indicated in the left hand column and are grouped according to locality.

TABLE 1

Greenland

Fig. no. in Gdt.	Observer / Form	010	111	011	110	130	021	100	001	012	221	231	211	121	131	241	201	161	991	101	133	160	163	411	150	032
		b	*u*	*k*	*g*	*m*	*h*	*a*	*c*	*l*	*s*	π	*n*	β	*o*	*t*	*e*	*x*	*r*	*i*	α	*y*	σ	ϕ	*z*	*f*
23	Des Cloizeaux	1	2	3	6	5		7	4	8	11	12	13	15	14	10				16						9
25	Breithaupt	1	3	8	2	4	10	6	7											9	11				4	
34	Schrauf	1	2	4	6	8		3		5	7															
35	Schrauf	1	3	3	2	8	6			3	10	6			9											
36	Schrauf	1	2	6	9	3	7	4	8	5	10	12		11												
37	Schrauf	1	4	7	3	5	2	6	8		10	9	11													
38	Schrauf	1	2	4	8	2	4		7	4		9	10	11	11			11								
39	Schrauf	1	2	3	4	5	6		7		8		9													
40	Schrauf	1	3	5	2	4	5			5	9	8	10		12		11									
41	Schrauf	1	3	5	7	6	9	4	2	8	10	13	15	12	16	11	14		17							
42	Schrauf	1	5	3	6	4	9	7	2	8	15	14	16	12	13	10	18	19	17	21		11	22	20		
43	Schrauf	1	3	2	6	4		5			8		10	7		9										
46	Schrauf	1	2	5	4	7	3	12		6	8	10	9	11												
47	Schrauf	1	3	2	7	4	6			5	8	10		11	14			13			9		12			
49	Schrauf	1	2	3	7	5		6	4																	
50	Schrauf	1	2	3	8	9		10	4	7					6					5						

EXPLANATION OF TABLES

In these tables each form is designated by Goldschmidt's letter and Miller indices ($a:b:c=0.4023:1:0.3580$). The figure number in Goldschmidt's *Atlas* is given in the first column, the name of the observer in the second column. The number assigned to each form in the remaining columns is its rank in the sequence of decreasing importance for the forms of the figure considered. In each table the forms appear as column headings in the order of decreasing importance for the locality considered.

TABLE 2

Bodenmais, Bavaria

Fig. no. in Gdt.	Form / Observer	010 *b*	001 *c*	100 *a*	130 *m*	111 *u*	201 *e*	110 *g*	160 *y*	211 *n*	101 *i*	011 *k*	131 *o*	012 *l*
9	Dana	1	3	2	7	4	5	8	6	9				
11	Lévy	1	2	3	4	5	6							
12	Lévy	1	3	2		6		5			4			
13	Lévy	1	2	6	4	7	3	5						
14	Lévy	2	1	7	5			2			6	4		
16	Rose	1	2	3	3	6	5							
17	Rose	1	2	3		5	4	6	6					
28	Schrauf	1	2	3	4	6							5	7
30	Schrauf	1	2	3	7	4	5	8		6				
48	Schrauf	1	2	3	4		5							
56	Rath	1	2	3	6	7	5	4		8				

TABLE 3

Ilmen Mountains

Fig. no. in Gdt.	Form / Observer	010 *b*	100 *a*	111 *u*	201 *e*	130 *m*	110 *g*	150 *z*	121 *β*	131 *o*	001 *c*	160 *y*
20	Auerbach	1	2	3								
21	Auerbach	1	5	4	2	6	3					
22	Auerbach	1	5	4	2	6	3					
26	Nordenskjöld	1	5	3	2	6	4					
27	Nordenskjöld	1	2	3								
33	Schrauf	1	4	3	2	5	6					7
52	Maskelyne	1	3	6		5	7	4			2	
64	Kokscharow	1	3	5		2	4		6	7		
65	Kokscharow	1	3	4	6	2	5		7			
66	Kokscharow	1	4		3	2	5					

TABLE 4
Haddam, Connecticut

Fig. no. in Gdt.	Observer	Form 010 *b*	100 *a*	001 *c*	130 *m*	110 *g*	111 *u*	160 *y*	201 *e*	211 *n*	101 *i*
3	Torry	1	2		4	5		3			
4	Torry	1	3	2	4	4		4			
8	Dana	1	2		5	6	3	4			
29	Schrauf	1	3	2	6	7	5		4		8
31	Schrauf	1	6	2	3	4	4			7	
54	Des Cloizeaux	1	3	2		5	4				
55	Des Cloizeaux	1	2		4		3				

TABLE 5
Middleton, Connecticut

Fig. no. in Gdt.	Observer	Form 010 *b*	100 *a*	001 *c*	130 *m*	111 *u*	201 *e*	160 *y*	131 *o*	110 *g*	211 *n*	011 *k*	221 *s*
7	Johnston	2	3		6	4	5	1		7	7		
10	Dana	1	4	3	2	6	10	7	9	8	11	5	
18	Rose	1	3	2	8	5	4	9	6	10	7		
32	Schrauf	1	3	5	2	6	9		7	8	10	4	

TABLE 6
Anneröd, Norway

Fig. no. in Gdt.	Observer	Form 010 *b*	100 *a*	201 *e*	111 *u*	211 *n*	001 *c*	110 *g*	130 *m*	085 *μ*	131 *o*	150 *z*	121 *β*	101 *i*	221 *s*
57	Brögger	1	3	2	7	5	4	6	8			9			
58	Brögger	1	2	3	5	4	6	8	7						
59	Brögger	1	2	10	5	3	6	9	4	7	8		11	12	13
60	Brögger	1	5	3	2	8	9	4	7	6					

TABLE 7
Norway (Miscellaneous)

Fig. no. in Gdt.	Form / Observer	010	100	111	201	130	001	110	150	101	211
		b	*a*	*u*	*e*	*m*	*c*	*g*	*z*	*i*	*n*
76	Milch	1		3	2	5		4			
78	Brögger	1	2	5	4	6	3	7			
79	Brögger	1	3	4	6	5	2	8	9		7
80	Brögger	1	3	4	6	2		5			
83	Brögger	1	4	3	2	5		6	7		
86	Brögger	1		2	4	5		3			
87	Brögger	1	2	5	4		6			3	

TABLE 8
Black Hills, South Dakota

Fig. no. in Gdt.	Form / Observer	010	130	100	170	011	131	150	201	133	111	032	211
		b	*m*	*a*	*d*	*k*	*o*	*z*	*e*	*α*	*u*	*f*	*n*
67	Blake	1	2	4	3	5	5			7			
68	Blake	1	2	5	3	5	5	3		8		9	10
69	Blake	1	4	2		3	8		5		6		7

TABLE 9
Standish, Maine

Fig. no. in Gdt.	Form / Observer	010	001	100	130	110	111	011	131	201	150	121	211
		b	*c*	*a*	*m*	*g*	*u*	*k*	*o*	*e*	*z*	*β*	*n*
63	Dana	2	1	3	4	5	7	9	8	12	6	10	11
70	Dana	1	2	2	4	5	6			7			
71	Dana	1	2	3	4	7	8	4	6	9			

TABLE 10
Rumford, Maine

Fig. no. in Gdt.	Form / Observer	032	010	011	130	100	110	201	170	111	231	131	211
		f	*b*	*k*	*m*	*a*	*g*	*e*	*d*	*u*	*π*	*o*	*n*
72	Foote	2	1		4	3	5					6	
73	Foote	1	3	2	4		6	5	7	8	9	11	10

TABLE 11

Paris, Maine

Fig. no. in Gdt.	Form / Observer	170 *d*	001 *c*	100 *a*	010 *b*	110 *g*	130 *m*	131 *o*	211 *n*
74	Warren	1	2	4	3	5	6	7	
75	Warren	1	2	3				5	4

TABLE 12

Rabenstein Nr. Zwiesel, Bavaria

Fig. no. in Gdt.	Form / Observer	010 *b*	001 *c*	100 *a*	130 *m*	201 *e*	160 *y*	111 *u*
1	Leonhard	1	2	2	6	5	4	7
19	Rose	1	2	3	4	5		

The forms in the column headings are arranged from left to right in estimated order of decreasing importance for the locality considered. The number assigned to a form indicates its rank in the sequence of decreasing importance. This sequence for all the localities is in turn averaged and is presented in Table 13, in which the estimated sequence of decreasing importance for the species as a whole is shown in the column headings.

Table 13

Relative importance of forms according to localities

Table no.	Locality	Observer	010	100	001	111	130	110	201	011	211	121	131	150	221	160	021	012	101	231	170	032	133	161	241	991	085	163	411	141
		Form	*b*	*a*	*c*	*u*	*m*	*g*	*e*	*k*	*n*	*β*	*o*	*z*	*s*	*y*	*h*	*l*	*i*	*π*	*d*	*f*	*α*	*x*	*t*	*r*	*μ*	*σ*	*φ*	*R*
1	Greenland		1	7	8	2	5	4	16	3	12	13	13	24	10	21	6	9	19	11		25	20	17	15	18		22	23	
2	Bodenmais		1	3	2	5	4	7	5	11	9		12			8		13	10											
3	Ilmen Mts.		1	2	10	3	5	6	4			8	9	7		11														
4	Haddam		1	2	3	6	4	5	8		9					7			10											
5	Middleton		1	2	3	5	4	9	6	11	10		8		12	7														
6	Anneröd		1	2	6	4	8	7	3		5	12	10	11	14				13								9			
7	Norway		1	2	6	3	5	7	3					8					9							10				
8	Black Hills		1	3		10	2		8	5	12		6	7							4	11	9							
9	Standish		1	3	2	6	4	5	9	7	12	11	8	10																
10	Rumford		2	5		9	4	6	7	3	12		11							10	8	1								
11	Paris		4	3	2		6	5			8		7								1									
12	Rabenstein		1	3	2	7	4		5							6														
Fig. no. in Gdt.		Observer																												
2	?	Phillips	1	2	2	4	8	5	5		8					7														
6	Chesterfield, Massachusetts	Shepard	1	2	3	4	5	5																						
15	?	Mohs Zippe	1			3	4						4	4			2													
44	?	Schrauf	1		8	2	2	7		6	14	10	12		9		4	5		13				11						
45	?	Schrauf	1	3	7	2	10	8		4		12	13		11		5	6		9										
51	Montevideo	Maskelyne	1	3	2	6	5	8			4				7															
61	Craveggia, Ossola	Strüver	1	2	4	3	8	6		9	5			7																
62	Sawarka, Urals	Arzruni	1	2	4	6				3	7							5												
77	Yamano, Japan	Wada	1	2	4	5	3	6	7		8																			
88	Ramona, California	Eakle	1	4		3	5	2			8	10	6		9		6													11
90	Ankaratra, Madagascar	Lacroix	1			5	2	3	3		6																			

N.B.—The localities from which only one crystal is figured in the *Atlas* are included in Table 13. The final sequence, obtained for the species as a whole, is shown in the column headings. Note that (m, g) are about equally important, likewise (β, o) and (α, x). The final order of forms can thus be written as follows:

$$b\ a\ c\ u\ (m, g)\ e\ k\ n\ (\beta, o)\ z\ s\ y\ h\ l\ i\ \pi\ d\ f\ (\alpha, x)\ t\ r\ \mu\ \sigma\ \phi\ R$$

Discussion of Zonal Characters

The relative importance of the forms in each zone is determined from the above tables:

A. *"Central" zones.*

(1) In the zone of the (hkk) faces, the observed order is: $u\ n\ \alpha\ \phi$. The face u is undoubtedly dominant in this zone, occurring always before n for every drawing in the *Atlas*, and found in twenty-three localities compared with seventeen for n.

(2) In the zone of the (lkl) faces, the order is: $u\ (o\ \beta)\ x\ R$. The face u is dominant, occurring twenty-six times, whereas o occurs only fourteen and is considerably smaller, in the observed figures. It is difficult to judge between o and β; they are very nearly equal, with o perhaps a trifle larger. They are assigned equal importance.

(3) In the zone of the (hhl) faces, there are only two faces, u and s, shown on the projection (Fig. 1); $r(991)$ is insignificant and has been omitted throughout in the calculations. The face u clearly dominates s, the occurrence ratio being 26:7.

These three central zones have a common dominant u which is therefore the unit face (111), confirming Goldschmidt's choice. The zone of the (hkk) faces is simple; the anomalies in the section considered are (311) and (122), both missing although $\alpha(133)$, $n(211)$ and $\phi(411)$ are present. The zone of the (lkl) faces is also simple, in spite of the absence of (151) and the anomalously equal importance of $o(131)$ and $\beta(121)$; the faces present are $u(111)$, $\beta(121)$, $o(131)$, $R(141)$ and $x(161)$. Likewise the zone of the (hhl) faces is simple; (112) and (113) are missing but this anomaly does not obscure the character of the zone. If this conclusion were not evident at sight, it would follow of necessity from the two previous considerations, namely, the zones of the (hkk) and (lkl) faces are simple. Indeed if two central zones are simple, the third must be also, as it results from the theory of space groups.

B. *"Axial" zones.*

(1) In the zone of the $(0kl)$ faces the observed order of importance is: $k(011)$, $h(021)$, $l(012)$, $f(032)$. This order holds for nearly every crystal drawing examined. Although these forms are rarely observed and are in no case observed together on the same crystal, yet where only one form occurs it is practically always k, which must therefore be the dominant. The forms h and l are about equal as regards frequency of occurrence; neither is definitely larger than the other. This is the zone which does not conform to the space-group found by x-ray work: it therefore deserves

closer analysis. There are five localities where (0*kl*) faces have never been observed, *viz*: the Ilmen Mts.; Haddam, Conn.; Norway; Paris, Me., and Rabenstein. Of the remaining localities Greenland gives the most complete data. On the sixteen crystals figured, *k* occurs in every case, *h* and *l* occur jointly on eight crystals with varying relative importance; *h* is found alone on four crystals and *l* also alone on four crystals. With two exceptions these observations indicate that *k* is the dominant, the other two indicate *h* as the most important face. The order may safely be written *k*(*h*, *l*) *f*. Data on the localities, not previously mentioned, are as follows: at Bodenmais, from eleven figures, Lévy reports *k* once and Schrauf reports *l* once: *k* is more important than *l*. At Middleton, Conn., *k* alone is observed on two of the four figures. Three crystals from the Black Hills all show *k*, and *f* is observed on one; here *k* is more important than *f*. Three crystals from Standish, Me., and two from Rumford, Me., show *k* twice and *f* once, respectively. For other localities, each represented by one figure, *k* occurs four times; *h*, four times; *l*, three times. In none of the crystals drawn is *l* the dominant of the zone. It is regrettable that there is not more information regarding these forms. However, from the available morphological data, we must conclude that this is a simple zone with *k*(011) dominant.

(2) In the zone of the (*h*0*l*) faces there are only two forms: *e*(201) and *i*(101), of which *e* is undoubtedly the dominant, as it occurs forty-three times whereas *i* occurs only nine. This is a simple zone with the dominant "shifted" from unit position toward *a*.

(3) In the zone of the (*hk*0) faces the following forms occur: *g*(110), *m*(130), *z*(150), *y*(160), and *d*(170). The forms *g* and *m* show equal observed importance in the final order; it is impossible to give more weight to one than to the other. However, despite this and the anomaly of *y* being more important than *d*, the zone can only be interpreted as double with *g* dominant.

C. *Pinacoids.*

In relative importance the three pinacoids rank as follows: *b*, *a*, *c*. With very few exceptions, *b* is the largest face on the figures studied and in most cases *a* is observed to be more important than *c*.

Space-group Determination from Morphological Data

A gnomonic projection (Fig. 1) is drawn to present the observed data. All forms are plotted except σ r μ α which either are doubtful or have such high indices that they cannot be considered in the discussion of zonal character. The approximate importance of the various forms is shown by the size of the gnomonic poles. For the less important forms this method of notation is of necessity less accurate. The scale of poles on the left hand side of the projection represents the zone of the $(hk0)$ faces and is a gnomonic projection on the side pinacoid b. Another projection, on the front pinacoid a, is shown at the bottom of the figure.

The conclusions derived from the above study are summarized in a stereographic projection (shown in the lower right hand corner, Fig. 1). In this inset the zonal character is interpreted in each case; a double line representing a double zone and a single line representing a simple zone. Faces with coprime indices are represented by black dots; faces with doubled indices, by open circles. The dominant face only is shown in each zone.

If we examine the results in the previous section, we see that the three central zones are simple and have a common dominant $u(111)$. Therefore the lattice is a primitive lattice (P), with (111) dominant. This is in agreement with Goldschmidt's setting. Next we consider the axial zones. The zone of the $(0kl)$ faces is a simple zone with the unit face $k(011)$ dominant, hence the (100) plane, if a plane of symmetry, is not a glide plane. The zone of the $(h0l)$ faces is a simple zone with $e(201)$ dominant, *i.e.* the dominant is "shifted" toward a and consequently the (010) plane is an a glide plane of symmetry. The zone of the $(hk0)$ faces is a double zone with the unit face $g(110)$ dominant, hence the (001) plane is an n glide plane of symmetry (Donnay, 1938). From these considerations the symbol of the *morphological aspect* (Donnay-Harker, 1937) is P^*an. It also follows that the pinacoids must be written $c\{001\}$, $a\{200\}$, $b\{020\}$, in terms of the *multiple indices* (Donnay-Harker, 1937). P^*an corresponds to two space-groups: $Pman$ in the holohedry $(2/m\ 2/m\ 2/m)$, and $P2an$ in the antihemihedry $(2\ m\ m)$. The symmetry class $2\ m\ m$ is ruled out because, in the present setting, the $z[001]$ axis is an axis of two-fold symmetry.

If we consider the multiple indices of the pinacoids $a\{200\}$, $b\{020\}$, $c\{001\}$; the effective interplanar spacings will be in the ratios $d_{200}:d_{020}:d_{001}=0.2012:0.5:0.3580$. Thus the theoretical order of importance of these forms would be $b\ c\ a$, as against the observed rank $b\ a\ c$. This is an anomaly to the generalized law of Bravais.

Despite the anomalies encountered in our interpretation of the zonal characters and the relative importance of the pinacoids, it is plain that the morphological data lead to a unique determination of the space-group $Pman$.

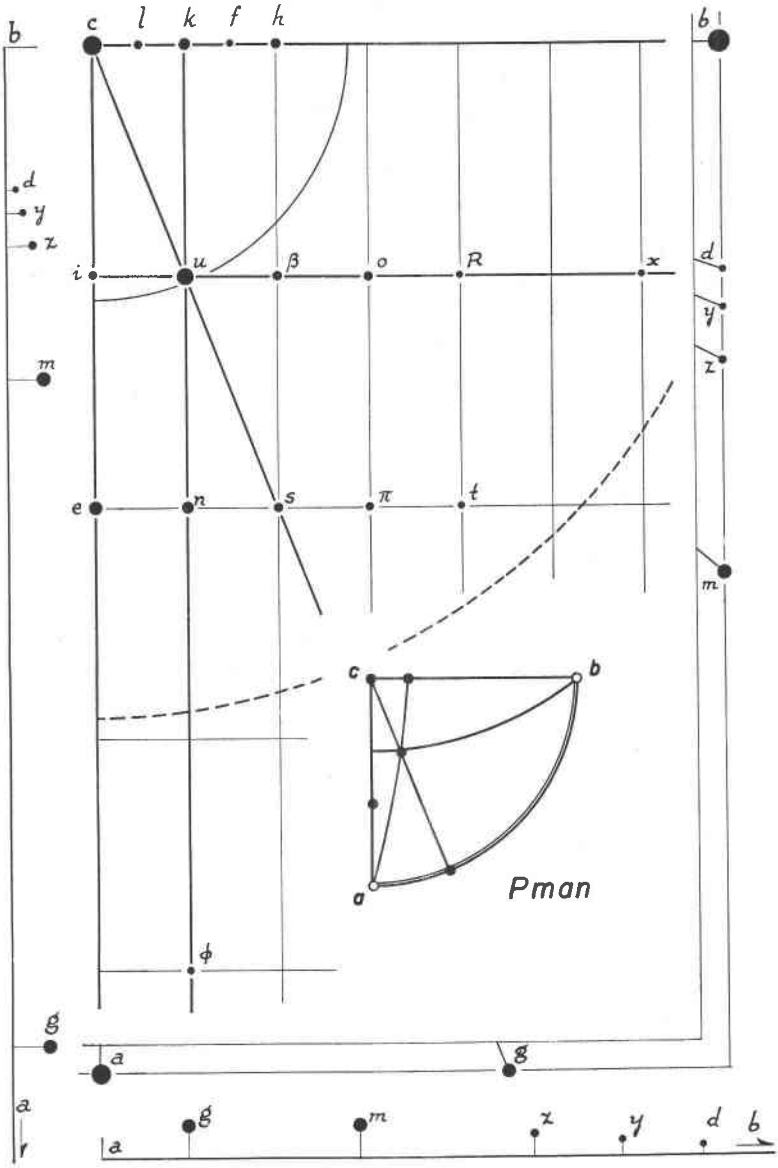

FIG. 1

Gnomonic projection of columbite:

$a:b:c=0.4023:1:0.3580 \quad p_0:q_0:r_0=0.8899:0.3580:1$

The radius of the solid arc is that of the sphere of projection ($r_0=1$). The radius of the dotted arc is that of the sphere of Bravais probability. The inset shows a stereographic projection of the main zones and derivation of the space group.

Comparison Between the Classical Law of Bravais and the Generalized Law

The theoretical order of decreasing importance of the forms in columbite crystals is determined, for the two laws, by a graphical method (Peacock, 1938). A *sphere of Bravais probability* (Peacock, 1938) is taken with a radius large enough to include most of the important forms and small enough to avoid consideration of too many unknown forms. It is indicated by the dotted arc on the projection, Fig. 1. In Table 14, the two laws are compared; the first column, headed P^{***}, and the second column, headed *Pman*, show the forms, in decreasing order of importance, according to the classical law and the generalized law, respectively. In the last three columns a direct comparison is made; the first column shows the forms peculiar to the classical law, the center column has the forms common to both laws, and the last column, the forms which only occur with the generalized law. It will be seen that there are many more anomalies in the classical law than in the generalized law. Therefore, from the point of view of the forms observed, the second law, although not perfect, is more satisfactory.

Comparison with X-ray Results

J. H. Sturdivant (1930), using the oscillation and the Laue methods, has found a set of x-ray extinctions leading to a space-group, which in our setting is *Pcan*. It will be seen that the morphological development of the space-group is confirmed insofar as it concerns all the central zones of (hkl) faces and the axial zone of ($h0l$) and ($hk0$) faces. However, the morphology of the zone of the ($0kl$) faces indicates that the (100) plane is a mirror plane of symmetry whereas Sturdivant's results show it to be a c glide plane. As for the pinacoids, the Sturdivant space-group demands the halving of all three, hence they must be written $a\{200\}$, $b\{020\}$, $c\{002\}$, which leads to the theoretical order $b\ a\ c$, identical with that of the classical law of Bravais and in agreement with the observed fact.

Professor Martin A. Peacock has kindly checked and confirmed the Sturdivant space-group by means of Weissenberg pictures taken on Topsham, Me., material. The case of columbite, therefore, appears to be one of bona fide conflict between the x-ray results and the morphological data for the zone of the ($0kl$) faces. Does the structural arrangement of atoms in columbite simulate a mirror plane of symmetry where the c glide plane actually exists? Is it similar, in this respect, to NaCl, which, although face-centered, acts like a primitive cubic lattice in the morphological development? These questions cannot be answered positively with the available structural data.

TABLE 14. COMPARISON OF THE CLASSICAL LAW OF BRAVAIS (P^{***}) WITH THE GENERALIZED LAW (*Pman*)

Theoretical sequences of forms in order of decreasing importance

P^{***}	*Pman*	Comparison		
		P^{***}	Common	*Pman*
b 010	*b* 020	*b* 010	—	
a 100	*g* 110		—	*b* 020
g 110	*c* 001	*a* 100	—	
c 001	*k* 011		*g* 110	
k 011	*h* 021		*c* 001	
120	*u* 111		*k* 011	
h 021	*m* 130	120	—	
i 101	031		*h* 021	
u 111	β 121	*i* 101	—	
m 130	*o* 131		*u* 111	
031	041		*m* 130	
β 121	*a* 200		031	
140	*R* 141		β 121	
o 131	*z* 150	140	—	
041	*l* 012		*o* 131	
210	*e* 201		041	
R 141	051		—	*a* 200
z 150	*n* 211	210	—	
l 012	*s* 221		*R* 141	
e 201	151		*z* 150	
051	240		*l* 012	
n 211	*f* 032		*e* 201	
230	π 231		051	
s 221	061		*n* 211	
151	*t* 241	230	—	
f 032	*x* 161		*s* 221	
π 231	*i* 202		151	
y 160			*f* 032	240
061			π 231	
t 241		*y* 160	—	
x 161			061	
			t 241	
			x 161	
			—	*i* 202

N.B.—Known forms are indicated by the Goldschmidt letters.

Transformations

In the Schrauf, 1877 V. Goldschmidt, 1913- Taylor, 1939 setting, the structural unit cell has the following identity periods (Sturdivant's values):

$$a_0 = 5.730 \text{ Å}, \; b_0 = 14.238 \text{ Å}, \; c_0 = 5.082 \text{ Å}.$$

The various cells used by other authors, and their respective settings, are shown in Fig. 2 and explained in Table 15.

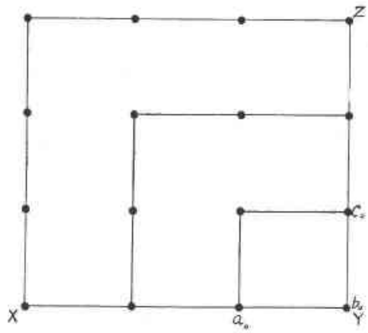

Fig. 2. The lattice of columbite.

Table 15

Transformation of settings

	Dana 1892	V. Gold-Schmidt 1886	Schrauf, 1877 V. Gold-schmidt, 1913 Taylor, 1939	Groth {1882, 1889	Breithaupt, 1858 Sturdivant, 1930	Schrauf 1864
X	$3a_0=\bar{b}$	$a_0=b$	$a_0=a$	$2a_0=a$	$a_0=\bar{c}$	$a_0=\bar{b}$
Y	$b_0=\bar{a}$	$b_0=c$	$b_0=b$	$b_0=b$	$b_0=\bar{b}$	$b_0=\bar{a}$
Z	$3c_0=\bar{c}$	$c_0=a$	$c_0=c$	$2c_0=c$	$c_0=\bar{a}$	$c_0=\bar{c}$

N.B.—In each column, a, b, c, indicate the a, b, c, of the author considered.

It is seen that only three cells have been used; all of which have the same length in the direction of Taylor's b-axis. In the directions of Taylor's a and c, the lengths are in some cases the same and in other instances doubled or tripled. The other differences lie in the setting adopted. Among those who have correctly chosen the unit cell, the settings used are as follows:

V. Goldschmidt, 1886	$a<b<c$
Schrauf, 1877; V. Goldschmidt, 1913; Taylor, 1939	$c<a<b$
Breithaupt, 1858; Sturdivant, 1930	$a<c<b$
Schrauf, 1864	$c<b<a$

The convention $c<a<b$ is chosen for this paper in accordance with recent proposals (Donnay and Mélon, 1933; Donnay, Tunell and Barth, 1934; Peacock, 1937).

Table 16 summarizes the transformations with correct fractional values.

TABLE 16

Transformation Matrices

From ↘ To ↗	Dana, 1892	V. Goldschmidt, 1886	Schrauf, 1877 V. Goldschmidt, 1913 Taylor, 1939	Groth, {1882 1889}	Breithaupt, 1858 Sturdivant, 1930	Schrauf, 1864
Dana, 1892	$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$	$\begin{matrix} 0 & 0 & \bar{\frac{1}{3}} \\ 0 & \bar{\frac{1}{3}} & 0 \\ \bar{1} & 0 & 0 \end{matrix}$	$\begin{matrix} 0 & \bar{\frac{1}{3}} & 0 \\ \bar{1} & 0 & 0 \\ 0 & 0 & \bar{\frac{1}{3}} \end{matrix}$	$\begin{matrix} 0 & \bar{\frac{2}{3}} & 0 \\ \bar{1} & 0 & 0 \\ 0 & 0 & \bar{\frac{2}{3}} \end{matrix}$	$\begin{matrix} 0 & 0 & \frac{1}{3} \\ 1 & 0 & 0 \\ 0 & \frac{1}{3} & 0 \end{matrix}$	$\begin{matrix} 1 & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{matrix}$
V. Goldschmidt, 1886	$\begin{matrix} 0 & 0 & \bar{1} \\ 0 & \bar{3} & 0 \\ \bar{3} & 0 & 0 \end{matrix}$	$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$	$\begin{matrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{matrix}$	$\begin{matrix} 0 & 2 & 0 \\ 0 & 0 & 1 \\ 2 & 0 & 0 \end{matrix}$	$\begin{matrix} \bar{1} & 0 & 0 \\ 0 & 0 & \bar{1} \\ 0 & \bar{1} & 0 \end{matrix}$	$\begin{matrix} 0 & 0 & \bar{1} \\ 0 & \bar{1} & 0 \\ \bar{1} & 0 & 0 \end{matrix}$
Schrauf, 1877 V. Goldschmidt, 1913 Taylor, 1939	$\begin{matrix} 0 & \bar{1} & 0 \\ \bar{3} & 0 & 0 \\ 0 & 0 & \bar{3} \end{matrix}$	$\begin{matrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{matrix}$	$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$	$\begin{matrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{matrix}$	$\begin{matrix} 0 & 0 & \bar{1} \\ 0 & \bar{1} & 0 \\ \bar{1} & 0 & 0 \end{matrix}$	$\begin{matrix} 0 & \bar{1} & 0 \\ \bar{1} & 0 & 0 \\ 0 & 0 & \bar{1} \end{matrix}$
Groth, {1882 1889}	$\begin{matrix} 0 & \bar{1} & 0 \\ \bar{\frac{3}{2}} & 0 & 0 \\ 0 & 0 & \bar{\frac{3}{2}} \end{matrix}$	$\begin{matrix} 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \end{matrix}$	$\begin{matrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{matrix}$	$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$	$\begin{matrix} 0 & 0 & \bar{\frac{1}{2}} \\ 0 & \bar{1} & 0 \\ \bar{\frac{1}{2}} & 0 & 0 \end{matrix}$	$\begin{matrix} 0 & \bar{1} & 0 \\ \bar{\frac{1}{2}} & 0 & 0 \\ 0 & 0 & \bar{\frac{1}{2}} \end{matrix}$
Breithaupt, 1858 Sturdivant, 1930	$\begin{matrix} 0 & 1 & 0 \\ 0 & 0 & 3 \\ 3 & 0 & 0 \end{matrix}$	$\begin{matrix} \bar{1} & 0 & 0 \\ 0 & 0 & \bar{1} \\ 0 & \bar{1} & 0 \end{matrix}$	$\begin{matrix} 0 & 0 & \bar{1} \\ 0 & \bar{1} & 0 \\ \bar{1} & 0 & 0 \end{matrix}$	$\begin{matrix} 0 & 0 & \bar{2} \\ 0 & \bar{1} & 0 \\ \bar{2} & 0 & 0 \end{matrix}$	$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$	$\begin{matrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{matrix}$
Schrauf, 1864	$\begin{matrix} \frac{1}{9} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{matrix}$	$\begin{matrix} 0 & 0 & \bar{1} \\ 0 & \bar{1} & 0 \\ \bar{1} & 0 & 0 \end{matrix}$	$\begin{matrix} 0 & \bar{1} & 0 \\ \bar{1} & 0 & 0 \\ 0 & 0 & \bar{1} \end{matrix}$	$\begin{matrix} 0 & \bar{\frac{1}{4}} & 0 \\ \bar{\frac{1}{2}} & 0 & 0 \\ 0 & 0 & \bar{\frac{1}{2}} \end{matrix}$	$\begin{matrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{matrix}$	$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$

Dana (1892) = Rose (1845), Hausmann (1847), Miller (1852), Des Cloizeaux (1855).

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