

MODELS TO AID IN VISUALIZING THE OPTICAL PROPERTIES OF CRYSTALS

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INTRODUCTION

The study of crystal optics is largely a matter of solid geometry, and demands clear visualization of optical properties in their three-dimensional relations. To aid students in visualizing these relations, numerous models of wood, wire, glass, and plaster have been devised,¹ and have proven to be of definite value. The models described in this paper were designed, in part to supplement, and in part to substitute for standard types already in use. Following the precedent of Wright² and Rogers,³ emphasis is placed on the indicatrix, or index ellipsoid, and its relations to crystallographic directions and to common optical phenomena. Relations heretofore shown only by two-dimensional drawings, by solid opaque models, or by elusive wire "bird cages" are clearly represented in three dimensions by transparent models made of sheet celluloid.

The models described below were constructed for the Petrographic Laboratory of the University of Kansas by student workers as a project under the National Youth Administration. Particular credit is due to Mr. Hugh Magruder and to Mr. Raymond Leonard for skilled workmanship and for ingenuity in working out details of construction. Photographs of the models were made by Mr. Oren Bingham, director of the University Photographic Bureau.

MATERIALS, EQUIPMENT, AND METHODS

Sheet celluloid constitutes the basic material for the models. This material has the advantages of being easily tooled and cemented, and of being practically unbreakable. It may be purchased in thicknesses of .08 in. and .125 in., both sizes being used in the various models. Originals

¹ Wright, F. E., A device to aid in the explanation of interference phenomena: *Am. Jour. Sci.*, vol. 26, p. 536, 1908.

Johannsen, Albert, Manual of Petrographic Methods, McGraw-Hill, New York, 1914, Fig. 162.

Rosenbusch, H., and Wülfing, E. A., Mikroskopische Physiographie der Mineralien und Gesteine, Band I, Erste Hälfte, Stuttgart, 1924, Figs. 84, 90, and 94.

Rogers, A. F., A model for biaxial crystals: *Am. Mineral.*, vol. 19, pp. 206-208, 1934.

Catalogues of Dr. F. Krantz, Bonn, Germany.

² Wright, F. E., The index ellipsoid (optical indicatrix) in petrographic microscope work: *Am. Jour. Sci.*, vol. 35, pp. 133-138, 1913.

³ Rogers, A. F., Introduction to the Study of Minerals, McGraw-Hill, New York, 1937, pp. 176-181.

of the models described below were constructed of the highly inflammable nitrate celluloid, purchased from the Frederick Post Co., Box 803, Chicago, Illinois. Later it was learned that the relatively non-inflammable acetate celluloid might be purchased from the Plastics Department, E. I. Du Pont de Nemours & Co., Arlington, New Jersey, under the trade name "Plastacele." The latter is recommended.

Tools required for construction of the models are few and simple, and are listed below:

Jeweler's hand saw	Scale
Carpenter's square	Protractor
Carpenter's compass	Ellipsograph
Scriber	Woodworking lathe
Clamps	Medium sandpaper
Straightedge	

For the scribe, a heavy sewing needle or a victrola needle held in a pin vice may be used. The ellipsograph is easily made by cutting a narrow slot in a strip of brass or bakelite about 9 inches long and about .6 in. wide (Fig. 1). A slender, flat-headed pin (A) is fixed in position just beyond one end of the slot, and in line with its center. A second similar pin (B) is placed in the slot so as to slide freely to any given point and then be locked in position. Between these two pins, a scribing point (C) is inserted in the slot, so as to be adjustable to any position between A and B. In using the instrument, a steel square serves as a guide, as explained under Fig. 1.

Celluloid is tooled in much the same way as wood. Straight lines are laid out with straightedge, scale, protractor, and carpenter's square, and marked with a scribe. Curved lines are marked off with the carpenter's compass or ellipsograph, as the case may require. Parts are then cut out with a jeweler's hand saw, using a one-inch board with a deep V-shaped notch as a support for the celluloid. Sandpaper is used for smoothing sawed edges, and, if necessary, for reducing parts to correct dimensions. In carrying out these operations, particular care should be exercised to avoid scratching the celluloid, which is relatively soft and easily marred.

In cementing celluloid, the surfaces to be joined are first made to fit closely at all points. Each surface is then coated with a dilute solution of celluloid in acetone, a few moments allowed for partial drying, a second coat applied, and the parts then are pressed firmly together and held in position for a few minutes until initial set is acquired. Permanent set is attained in a few hours. If cementing is properly done, the joint, or "weld," will be fully as strong as other parts of the celluloid. In acquiring the technique of cementing, some experimentation with scrap material may be desirable. Proper viscosity of the cement is important: if too low,

the fluid is too mobile and difficult to control; if too high, the fluid is syrupy and makes an uneven, messy joint.

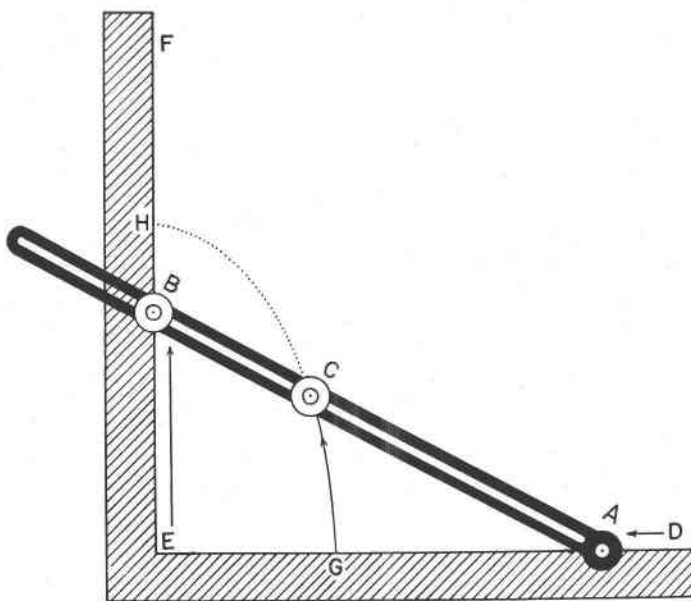


FIG. 1. Ellipsograph. An ellipse of any desired dimensions may be drawn, one quadrant at a time, by setting the distance AC equal to the semimajor axis, and the distance BC equal to the semiminor axis. Next, the two pins, A and B, are placed along one inside edge (DE) of a steel square aligned with the coordinate axes of the ellipse, with pin B at the corner E. Then, keeping the pins in contact with the edges of the square, A is moved from D to E, and B is moved from E to F. Point C inscribes the ellipse GH.

SKELETON INDICATRIX MODELS

The importance of the indicatrix in microscopical petrography has been emphasized by Wright,⁴ Tunell,⁵ and Rogers.⁶ Standard models of the indicatrix are mainly of the wooden or wire type, which leave much to the imagination. The transparent skeleton models (Fig. 2) described below have several advantages over the more elaborate wooden models. The principal circular and elliptical cross sections, representing indices of rays travelling parallel to optical and ellipsoidal axes, are all visible at

⁴ *Op. cit.* (Ref. No. 2).

⁵ Tunell, George, The ray-surface, the optical indicatrix, and their interrelation: an elementary presentation for petrographers. *Jour. Wash. Acad. Sci.*, vol. 23, pp. 325-338, 1933.

⁶ *Op. cit.* (Ref. Nos. 1 and 2).

the same time, without dismounting, and others may readily be visualized by interpolation. In addition, vibration directions and relative velocities of the different rays are shown by colored "arrows" normal to directions of propagation, and spaced at intervals inversely proportional to indices. Finally, the models are easily constructed and inexpensive.

For the uniaxial indicatrix, the circular section, the two principal elliptical sections, and an intermediate elliptical cross-section are shown. Following the practice of most textbook diagrams, this and other models

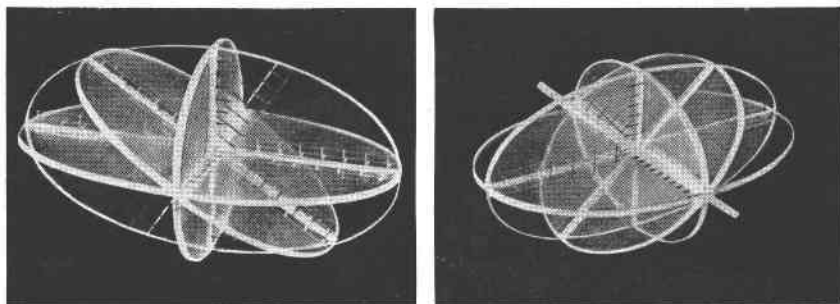


FIG. 2. Skeleton indicatrix models. The uniaxial model is at the left, the biaxial at the right.

are proportioned to represent imaginary minerals of greatly exaggerated birefringence, in order to emphasize the ellipsoidal form of the indicatrix. Suggested specifications are indicated in the diagram, Fig. 3, and are listed below:

- Two dovetailing ellipses,
Major axis—7 in.
Minor axis—4 in.
- Two semicircles, cut to dovetail with one of above ellipses,
Radius—4 in.
- One ellipse, dissected in 4 parts,
Major axis—5.2 in.
Minor axis—4 in.
- Thickness of celluloid throughout—.125 in.
- Spacing of lines representing vibration directions for:
 - O ray—10 mm. (blue)
 - E ray—6 mm. (red)
 - E' ray (travelling normal to intermediate ellipse)—8 mm. (brown)

The above specifications are for a uniaxial positive indicatrix; if desired, a similar model might be constructed for the negative indicatrix.

In marking vibration direction lines on the celluloid, a fairly deep groove is cut with the scribe, and left as rough as possible. This is filled with colored India ink from a fine-pointed pen, allowed to dry, and the

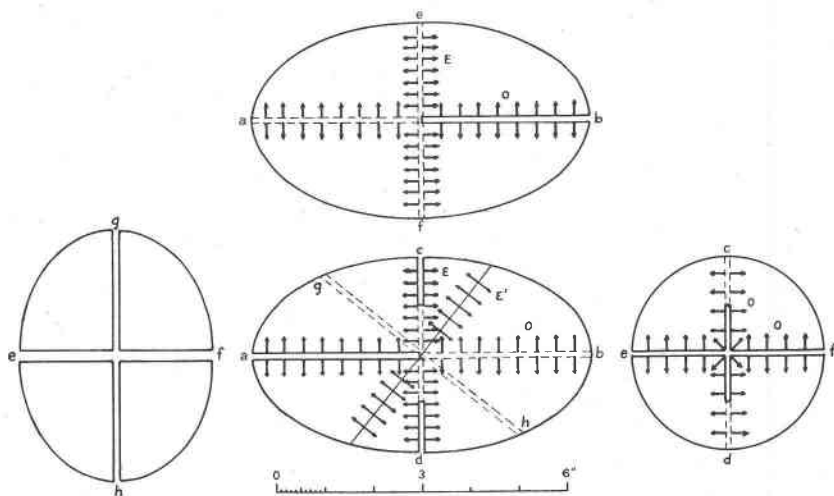


FIG. 3. Uniaxial indicatrix model, unassembled. Dashed lines indicate where parts fit together, and letters indicate how they fit.

inking repeated until the color is sufficiently strong. In assembling the model, dovetailing surfaces are coated with cement and then quickly slid together before the cement has time to dry. The joint is then further strengthened by applying one or more coats of cement externally.

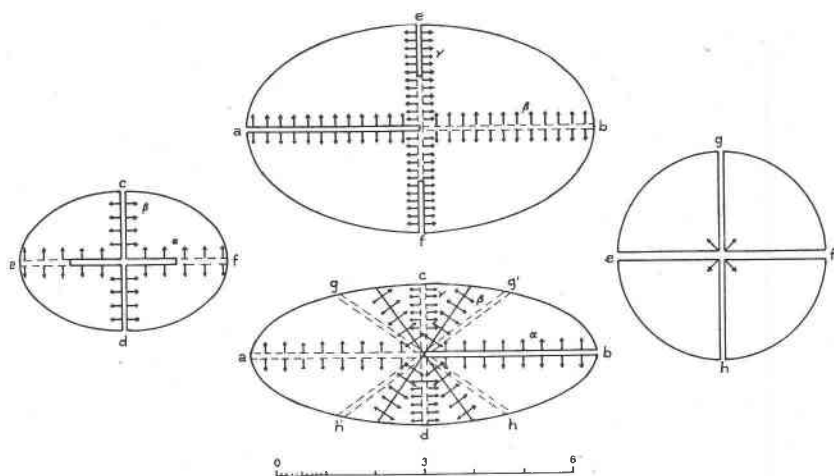


FIG. 4. Biaxial indicatrix model, unassembled.

For the biaxial indicatrix, the three principal elliptical cross-sections and the two circular sections are shown. Vibration directions and relative velocities are marked for rays travelling parallel to the two optical axes and the three ellipsoidal axes. Suggested specifications for the biaxial negative indicatrix are indicated in Fig. 4, and are listed below:

- One ellipse, of .125 in. celluloid,
 - Major axis—7 in.
 - Minor axis—4.2 in.
- One ellipse, of .125 in. celluloid, dovetailing with above,
 - Major axis—7 in.
 - Minor axis—2.8 in.
- One ellipse, of .125 in. celluloid, dovetailing with first ellipse above,
 - Major axis—4.2 in.
 - Minor axis—2.8 in.
- Two circles, of .08 in. celluloid, each dissected into four parts,
 - Radius—4.2 in.
- Spacing of vibration direction lines,
 - α —10 mm. (black)
 - β —7 mm. (red)
 - γ —4 mm. (blue)
- Axial angle—70°.

This model is assembled in the same way as that for the uniaxial indicatrix.

VIBRATION DIRECTIONS MODEL

In introducing students to the study of uniaxial minerals, the concept of vibration directions in relation to double refraction is readily demonstrated with calcite crystals and polaroid plates. For biaxial crystals, however, no correspondingly simple demonstration is readily available. In its stead, the writer has found it helpful to use a transparent celluloid model of an orthorhombic crystal (Fig. 5) which shows simply the vibration directions and relative velocities for light propagated through the crystal in different directions. These properties are represented by lines of different color and spacing on celluloid surfaces mounted within the crystal model, and corresponding to its three planes of symmetry. Fig. 6 shows the layout for the interior part of the model, and specifications are given below:

- Symmetry planes mounted inside crystal model:
 - 1 piece 4×5 in., to dovetail with
 - 1 piece 3×5 in.;
 - 2 pieces 3×1.96 in., to dovetail with above.

Spacing of vibration direction lines:

α —10 mm. (black)

β —7 mm. (blue)

γ —4 mm. (red)

Axial angle— 82° (negative)

Crystal model proper:

6 pieces cut to form an orthorhombic crystal $3 \times 4 \times 5$ in., inside measurement.

Thickness of celluloid—.08 in. throughout.

The core of the model, made up of the inscribed symmetry planes, is assembled first, and the crystal model built around it.

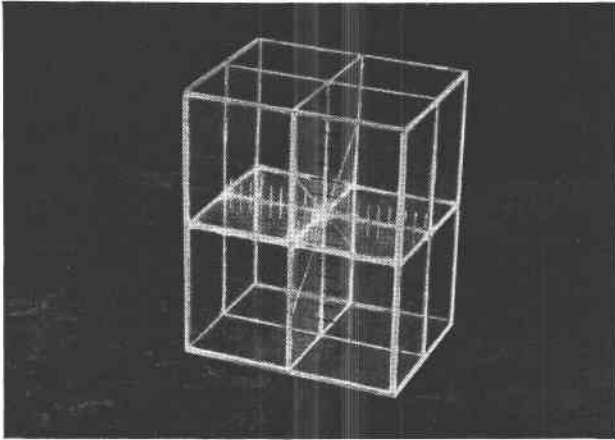


FIG. 5. Vibration directions model.

This model serves to illustrate the introductory discussions on biaxial minerals as presented by Winchell,⁷ and by Hartshorne and Stuart.⁸ By rotating the model around any one of the crystallographic axes, it may be seen that the index for the ray vibrating in the plane of that axis remains constant, while that for the ray vibrating in the plane normal to the axis varies between fixed values. Thus it is seen, as stated by Winchell,⁹ "not only that the indices are different for two rays of light traveling in the same direction, but vibrating in different directions, but

⁷ Winchell, A. N., *Elements of Optical Mineralogy, Part I—Principles and Methods*, 5th Ed., Wiley, *New York*, 1937, pp. 158-159.

⁸ Hartshorne, N. H., and Stuart, A., *Crystals and the Polarizing Microscope*, Arnold, *London*, 1934, pp. 74-75.

⁹ *Ob. cit.*, p. 159.

also that the indices are the same for rays traveling in different directions, but vibrating in the same direction. The direction of vibration, and not the direction of transmission, determines the index of refraction."

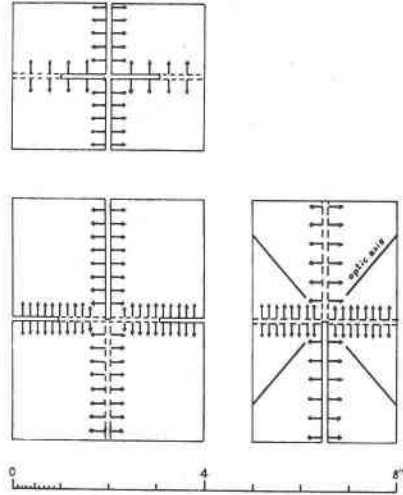


FIG. 6. Core of the vibration directions model, unassembled.

MODELS SHOWING RAY-SURFACES IN RELATION TO INDICATRIX

In discussions of optical theory, ray- or ray-velocity surfaces are commonly introduced, thus giving rise to a need for some means of showing the relation of these surfaces to the indicatrix. Following the usage of Rosenbusch and Wülfing,¹⁰ this may be done by showing both

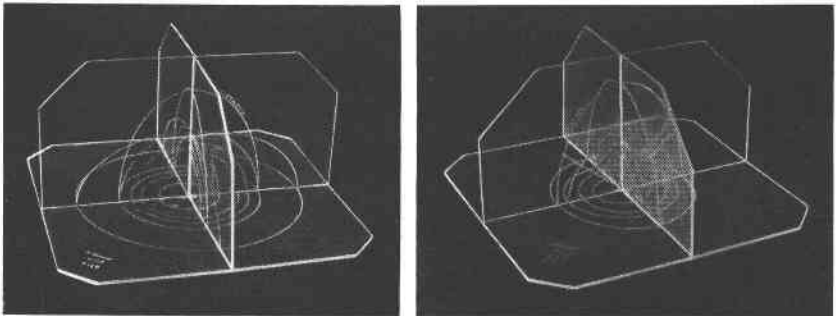


FIG. 7. Models showing relations of ray-surfaces to indicatrix. The uniaxial model is at the left, the biaxial at the right.

¹⁰ *Op. cit.* (Ref. 1), Figs. 82, 83, and 93.

with respect to a unit circle. Taking the radius of the circle as equal to one unit, the indicatrix is drawn, in the three mutually perpendicular cross-sections, by plotting numerical values of indices in the direction of *vibration*. The ray-surfaces are then drawn by plotting reciprocals of

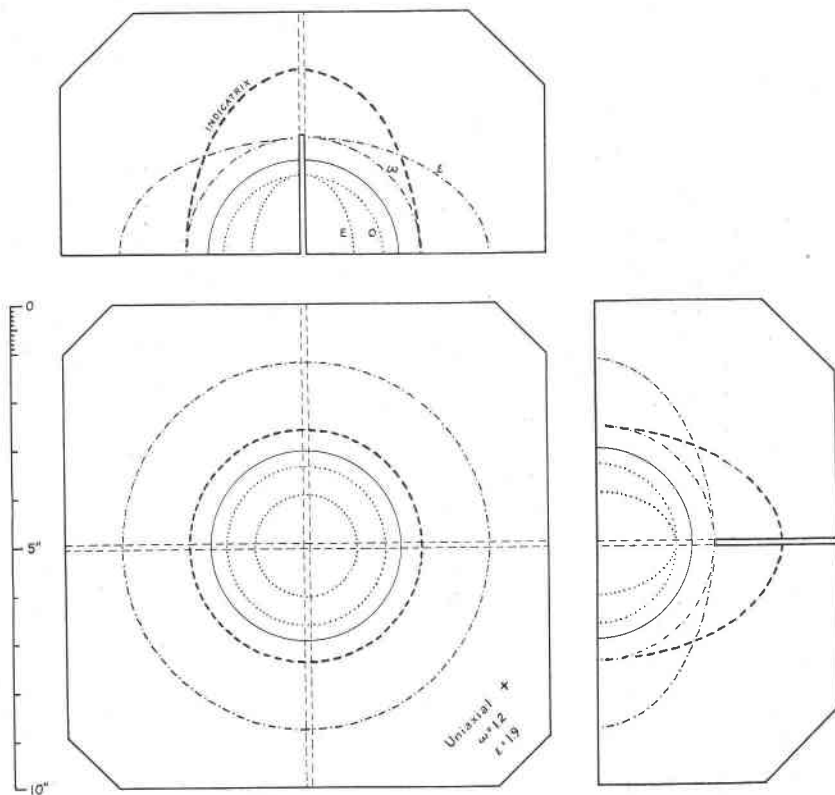


FIG. 8. Uniaxial ray-surface-indicatrix model, unassembled. The unit circle is shown by a light solid line. Dotted lines indicate ray-surfaces. The heavy dashed line represents the indicatrix. Dot and dash lines represent index surfaces. Light dashed lines show where parts fit together.

indices in the direction of *propagation*. If desired, index surfaces¹¹ may be shown by plotting values of indices in the direction of propagation; these are shown in the uniaxial model, but are omitted from the biaxial model in the interest of simplicity.

¹¹ Rosenbusch and Wülfing, *op. cit.*, pp. 91-92, 95-96, 108-109.

The uniaxial model is shown assembled in Fig. 7, and unassembled in Fig. 8. It is designed to represent an optically positive crystal of indices: $\omega = 1.2$, $\epsilon = 1.9$. Specifications are given below:

Base of .125 in. celluloid 10 in. square, inscribed with circles of the following radii:

- 2 in. (unit circle, blue)
- 1.63 in. (ray surface for O ray, black)
- 1 in. (ray surface for E ray, black)
- 2.4 in. (indicatrix and index surface for ω , red)
- 3.8 in. (index surface for ϵ , brown).

Two dovetailing upright pieces of .08 in. celluloid, each 5×10 in., and each inscribed with:

Semicircles of radii:

- 2 in. (unit circle, blue)
- 1.67 in. (ray-surface for O ray, black)
- 2.4 in. (index surface for ω , brown)

One semiellipse (ray-surface for E ray, black),

- Semimajor axis—1.67 in.,
- Semiminor axis—1 in.

Two semiellipses, one with major axis vertical (indicatrix, red), and one with major axis horizontal (index surface for ϵ , brown),

- Semimajor axis—3.8 in.
- Semiminor axis—2.4 in.

The two upright pieces are most conveniently laid out as a single sheet 10×10 in., which is sawed in two after the circles and ellipses are drawn. In cementing the upright pieces to the base, it is helpful to hold them in position by means of needle points hammered through the base so as to project upward about 1 mm., and then broken off below flush with the lower surface. These will prevent slipping while the cement is drying.

The biaxial model is shown unassembled in Fig. 9, and assembled in Fig. 7. Specifications below are for a model representing an optically positive crystal of indices 1.2, 1.4, and 2.0.

Base (XY section) of .125 in. celluloid 10 in. square, inscribed with:

Circles of radii:

- 2 in. (unit circle, blue)
- 1 in. (ray surface for slow ray, black)
- One ellipse (ray surface for fast ray, black),
- Semimajor axis—1.67 in.
- Semiminor axis—1.43 in.

One ellipse (indicatrix, red),

- Semimajor axis—2.8 in.
- Semiminor axis—2.4 in.

Upright (XZ section) of .08 in. celluloid, 5×10 in., inscribed with:

Semicircles of radii:

- 2 in. (unit circle, blue)
- 1.43 in. } ray-surfaces, black
- Semiellipse, }

Semimajor axis—1.67 in.

Semiminor axis—1 in.

Semiellipse (indicatrix, red),

Semimajor axis—4 in.

Semiminor axis—2.4 in.

Optic axes (primary, red) with axial angle of 79° .

Upright (YZ section) of .08 in. celluloid 5×10 in., dovetailing with above, and inscribed with:

Semicircles of radii:

2 in. (unit circle, blue

1.67 in. }

Semiellipse, } ray-surfaces, black

Semimajor axis—1.43 in.

Semiminor axis—1 in.

Semiellipse (indicatrix, red),

Semimajor axis—4 in.

Semiminor axis—2.8 in.

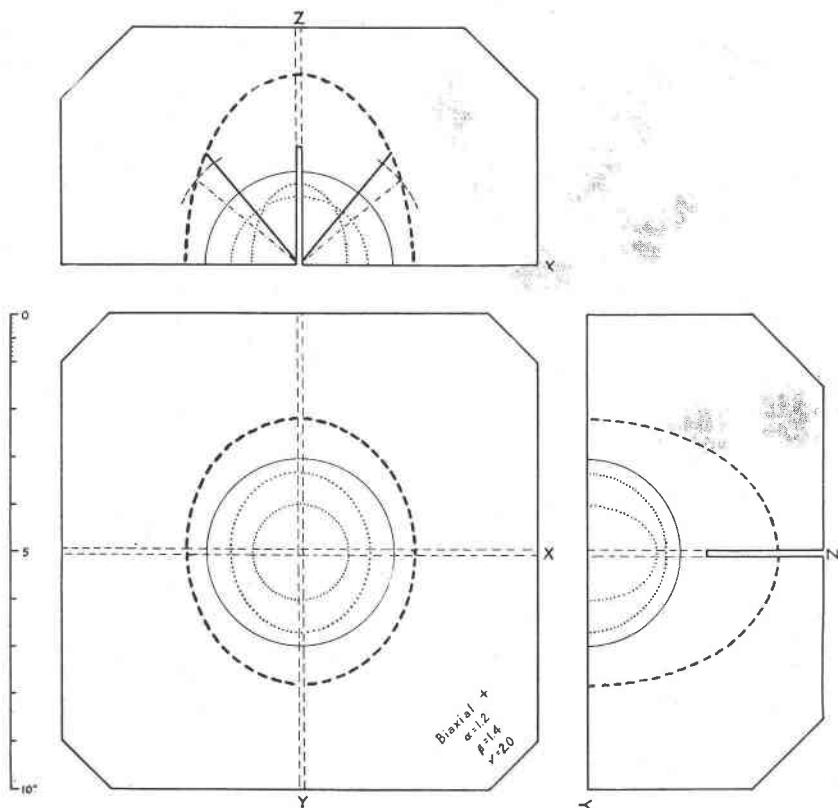


FIG. 9. Biaxial ray-surface-indicatrix model, unassembled. The unit circle is shown by a light solid line, the ray-surfaces by dotted lines, the indicatrix by a heavy dashed line, the primary optic axes by heavy solid lines, and the traces of the circular section on the XZ plane by dot and dash lines.

On the XZ upright section, traces of the circular sections are located by finding the intersections of a circle of radius 2.8 in. (β) with the indicatrix. The optic axes are drawn perpendicular to the traces of the circular sections. The model is assembled in the same way as that for the uniaxial crystal: circles and ellipses are inscribed on the upright pieces before they are sawed apart, then these two pieces are dovetailed together, and finally they are cemented to the base.

MODELS SHOWING ORIENTATION OF THE INDICATRIX IN THE CRYSTAL

In studying mineral grains in oil immersion or in thin section, it is generally desirable to note the orientation of the indicatrix in each grain

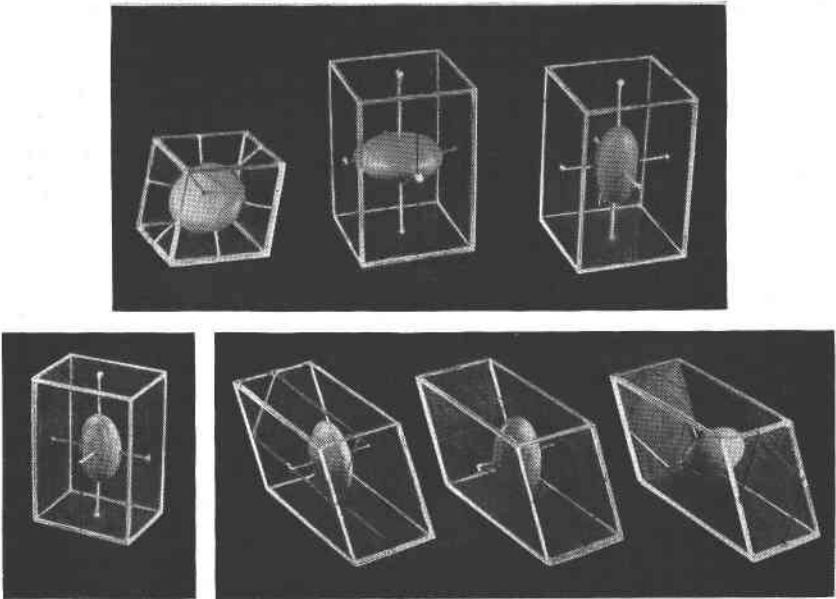


FIG. 10. Models showing orientation of the indicatrix in different crystals. Top row, left to right: rhombohedral negative; tetragonal negative; tetragonal positive. Bottom row, left to right: orthorhombic, $X=a$; $Z=c$; monoclinic, $X=b$; monoclinic, $Y=b$; monoclinic, $Z=b$.

examined. To aid in visualizing the different possible orientations and their relations to cleavages and to crystal faces, wooden indicatrix models are mounted inside celluloid crystal models (Fig. 10).

For uniaxial crystals, positive and negative tetragonal, hexagonal, and rhombohedral models may be constructed. Specifications are given below:

Crystal models:

Tetragonal— $2.5 \times 2.5 \times 4$ in., inside measurementHexagonal— $1.4 \times 1.4 \times 1.4 \times 4$ in., inside measurementRhombohedral— $2 \times 2 \times 2$ in., inside measurement, with acute angle of 79° between edges of sides

Thickness of celluloid—.08 in.

Indicatrices:

Prolate ellipsoids of revolution,

Axes 2 in. and 1 in.

Oblate ellipsoids of revolution,

Axes 1.8 in. and 1 in.

The wooden ellipsoids are made of gum wood or some other fine-grained, moderately hard wood, and are cut to shape on a woodworking lathe. Holes corresponding to the crystallographic axes are drilled through the wood *before* turning to shape, and are aligned so as to *just miss* one another at the center. These holes should be about $1/16$ in. in diameter. After receiving a coat of varnish or shellac, the ellipsoids are mounted inside the crystal models, which are completely assembled except for the top. String of one color is used to represent the horizontal crystallographic axes, and string of a different color to represent the vertical axis. The string is doubled, threaded through the ellipsoid and the crystal model, knotted at both ends, and then tightened by twisting and fixed in position by coating the knots with celluloid cement.

For biaxial crystals, orthorhombic and monoclinic models showing different ellipsoidal orientations were constructed. For orthorhombic crystals, six different models are necessary to show all possible orientations of the indicatrix. One of these is shown in Fig. 10. Crystal models $2 \times 2.8 \times 4$ in., inside measurement, are used. The ellipsoids are triaxial, with major, intermediate, and minor axes of 1.8, 1.2, and .8 in., respectively. These are made from ellipsoids of revolution by carefully sanding them to triaxial form by hand. String of three different colors is used in mounting the ellipsoids inside the crystal models.

For monoclinic crystals, three models suffice to show all possible orientations of the indicatrix. In these three models, the three axes of the ellipsoid, respectively, are mounted on thin metal rods, which, in turn, are mounted in the crystal models to correspond to the *b* crystallographic axis, with projecting ends bent over to provide a means of turning the wooden model inside the crystal. Thus the two remaining ellipsoidal axes may be rotated into any desired angular relation with the *a* and *c* crystallographic axes. The monoclinic crystal models are $2 \times 2.8 \times 4.2$ in., inside measurements and show the clino axis making an angle of 60° with the plane of the other two axes.

In any of the biaxial models described above, prism faces in addition

to pinacoids may be represented by means of colored lines inscribed on the inside of the model.

UNIVERSAL STAGE MODEL

In demonstrating the manipulation of the universal stage, a large scale model is helpful. Such a model (Fig. 11) is easily constructed from a disc of .125 in. celluloid. Fig. 12 gives dimensions and details. All working parts of the model are cut as concentric rings from the single

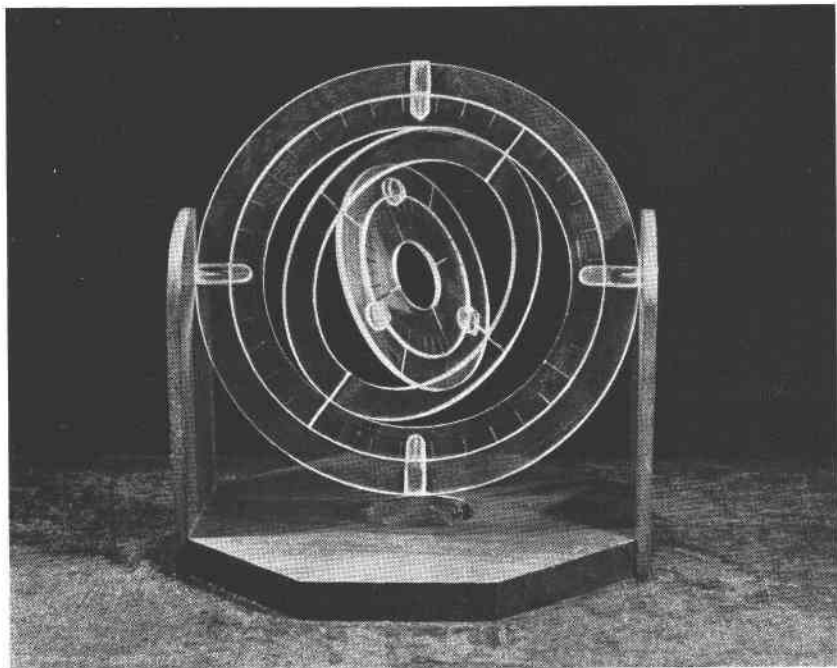


FIG. 11. Universal stage model.

disc. Rings are hinged together by means of heavy darning needles forced into drill holes barely large enough to receive them. Rings designed to rotate concentrically within others are held in place by four paired sets of projecting arms. Angular distances around the rings may be marked off in units of 10° or less, if desired. The completed assembly is mounted over a wooden base to which two upright arms are affixed. In using this model, wooden indicatrix models, like those used in the crystal models, may be attached to the inner ring with modeling clay or adhesive.

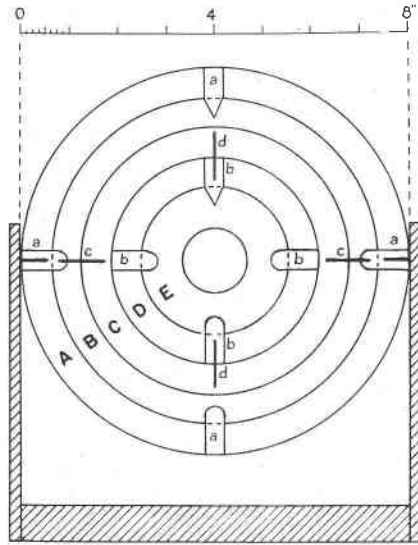


FIG. 12. Diagram showing assembly of universal stage model. Ring *A* is hinged to the sides of the frame. Ring *B* rotates concentrically within *A*, being held in place by the arms *a*. Ring *C* is hinged to *B* at *c*. Ring *D* is hinged to *C* at *d*. Ring *E* rotates concentrically within *D*, being held in place by the arms *b*. The center of ring *E* is cut out, in order to accommodate a small wooden indicatrix model.

CONCLUDING STATEMENT

The foregoing descriptions of models are intended to be suggestive rather than definitive. Numerous modifications in detail may suggest themselves to the reader, either to improve the models or to adapt them for specific objectives. Furthermore, a wide field remains for the designing of additional models to illustrate further particulars of optical theory and of petrographic practice. Some of these might well be suggested by perspective drawings in textbooks. Certainly the value of visual models in saving time and increasing the efficiency of teaching more than justifies the time and effort expended in designing them.