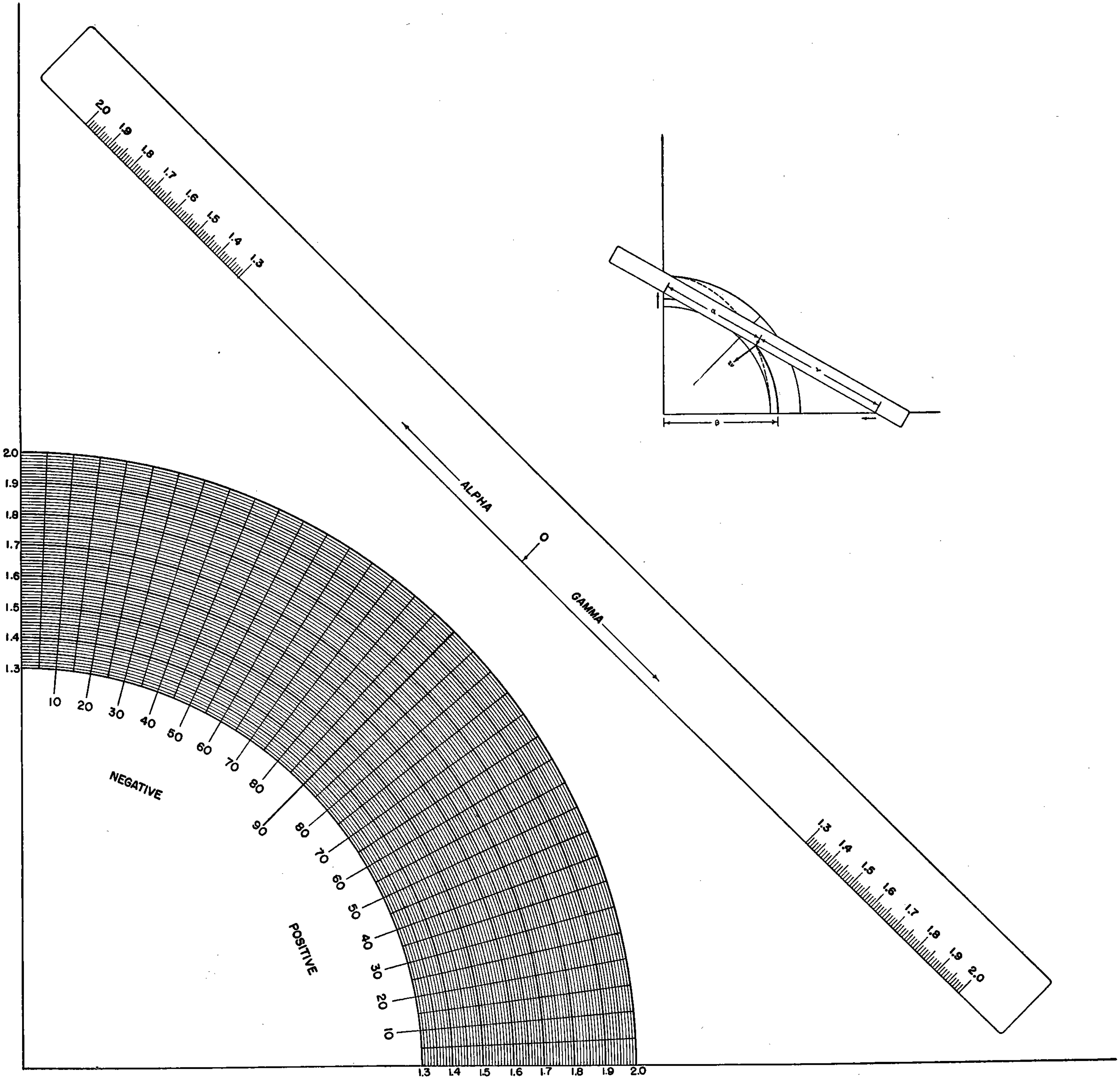

2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
ALPHA
0
GAMMA
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
10
20
30
40
50
60
70
80
90
80
70
60
50
40
30
20
10
NEGATIVE
POSITIVE
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0

FIG. 1

GRAPHIC METHOD OF DETERMINING OPTIC SIGN AND TRUE AXIAL ANGLE FROM REFRACTIVE INDICES OF BIAXIAL MINERALS

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INTRODUCTION

The axial angle and optic sign of any biaxial mineral are functions of the refractive indices, as may be expressed by the equation[1]:

$$\text{Tan}^2\, V_\alpha = \frac{\alpha^2(\gamma^2 - \beta^2)}{\gamma^2(\beta^2 - \alpha^2)}.$$

The same relation may be expressed approximately by the simpler equation[2]:

$$\text{Tan}^2\, V_\alpha = \frac{\gamma - \beta}{\beta - \alpha}.$$

In general, the second equation is more convenient to use, and its application has been facilitated by Wright's graphic solution,[3] later simplified by Smith.[4] The approximate equation, however, whether solved graphically or by computation, involves a certain element of error, which varies directly with axial angle and with total birefringence, and reaches serious proportions in minerals of relatively high birefringence.[5] To avoid these errors and yet retain speed and simplicity of operation, a graphic solution for true axial angle is here introduced.

The graphic solution described below was developed by Lane as a student in Smith's class in optical mineralogy. The manuscript was prepared in collaboration.

DESCRIPTION

The graphic method here presented (Fig. 1) employs two separate parts: a graphic plot, and a graduated scale which is adjusted to the plot for each determination.

[1] Johannsen, Albert, *Manual of petrographic methods,* McGraw-Hill, New York, p. 104; Rosenbusch, H., and Wülfing, E. A., *Mikroskopische Physiographie der Petrographischen Wichtigen Mineralien,* Band 1, Erste Hälfte, E. Schweizerbart'sche Verlagsbuchhandlung, Stuttgart, **1924,** p. 120.

[2] Rosenbusch and Wülfing, *op. cit.,* p. 121.

[3] Wright, F. E., Graphical methods in microscopical petrography: *Am. Jour. Sci.,* vol. **36,** pp. 509–39, 1913.

[4] Smith, H. T. U., Simplified graphic method of determining approximate axial angle from refractive indices of biaxial minerals: *Am. Mineral.,* vol. **22,** pp. 675–81, 1937.

[5] Larsen, E. S., The microscopic determination of the nonopaque minerals: *U. S. Geol. Surv., Bull.* **679,** pp. 10–11, 1921.

The graphic plot consists simply of a series of concentric 90 degree arcs whose radii are proportional to refractive indices within the range 1.3 to 2.0. A radial line bisecting the arcs marks the boundary between the positive and negative field. Each set of 45 degree arcs is in turn subdivided by radial lines into 18 equal sectors, representing axial angles from 0 to 90 degrees in 5 degree intervals, the ratio being 1 degree of arc equals 2 degrees of axial angle.

The graduated scale consists of a strip of stiff drawing paper on which values of indices in the range 1.3 to 2.0 are marked off in both directions from a medial zero point, in the same units as used on the graphic plot. The distance from the zero point to the value for any index will then equal the radius of the arc corresponding to that same index on the graphic plot. To mark the values of indices for particular determinations, sliding pointers of heavy drawing paper may be constructed, and equipped with slits through which the scale is inserted. In preparing the scale, it is essential that one edge be a true straightedge.

The procedure in using the graphic method is indicated by the guide sketch accompanying the chart, and is outlined below:

1. On the graduated scale, mark the value of α to the left of the zero point, and the value of γ to the right of the zero point. These values may be marked either with sliding cardboard pointers or with a pencil.
2. Place the scale on the graphic plot so that the value of α lies on the ordinate and value of γ on the abscissa. Keeping these values on the lines indicated, move the scale until its zero point cuts the arc which represents the value of β.
3. From the intersection determined above, follow the radial line to the inner quadrantal scale, on which optic sign and axial angle may be read directly. Values intermediate between the 5 degree lines are interpolated.

Derivation

The graphic solution presented above is based on the geometric properties of the biaxial indicatrix, which is a triaxial ellipsoid. In the *alpha-gamma* cross section, or optic plane of the indicatrix, it may be seen that the angle between the two primary optic axes is the same as the angle between the traces of the two circular sections. It follows that the acute angle between the acute bisectrix and one optic axis equals the acute angle between the obtuse bisectrix and the trace of a circular section, and that twice the latter equals the axial angle. In the accompanying graphic plot, it is actually the angle between the obtuse bisectrix and the trace of the circular section which is measured, and then doubled. The trace of the circular section corresponding to any particular set of indices is located by finding the intersection of a circle of radius equal to β with the ellipse whose semiaxes represent α and γ.[6] On the graphic plot, the circle used for any particular determination is selected from a series of circles, or rather arcs, whose radii represent different values of β.

The sliding scale is simply a device to describe ellipses for any set of values of α and γ. It is based on the proposition that the figure traced by any point on a straight line of fixed length, moving so that its ends remain in the coordinate axes, is an ellipse whose semiaxes equal the distances from the point to the two ends of the line, respectively.[7] In practice, only a single point on the ellipse—the point where it intersects the circle of radius β—is desired, and the sliding scale is moved only far enough to locate that point. This point, once determined, locates the trace of the circular section, from which the desired acute angle with respect to one of the coordinate axes is noted.

Accuracy

The accuracy of the graphic solution described above is qualified by two factors: (1) the accuracy of the laboratory data used, and (2) the degree of precision with which the chart may be read. Factors influencing

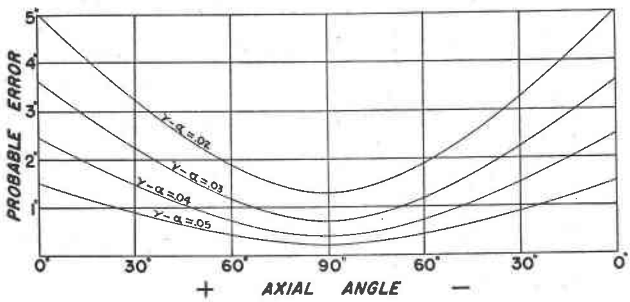

Fig. 2. Curves showing approximate amount of error introduced by inaccuracies in reading the graphic chart, as estimated from about one hundred trial determinations using assumed values.

the accuracy of laboratory determinations of indices have been noted by the senior author in a previous paper.[8] Here it need only be said that the accuracy of the graphic solution varies directly with the ratio between total birefringence of the mineral and the maximum possible error in measurements of indices.

[6] For proof, see Johannsen, *op. cit.*, pp. 92–93, 102.

[7] Given as a problem in textbooks on analytic geometry.

[8] Smith, H. T. U., *op. cit.*, p. 678.

The accuracy attainable in reading the graphic chart depends on the sharpness of the intersection between the circle and the imaginary ellipse described by the zero point on the sliding scale. This varies directly with axial angle and with total birefringence, and varies inversely with the relative thickness of the lines with respect to the scale of the graph. The approximate amount of error introduced by these factors is shown in Fig. 2. Only when the total birefringence is less than about .03 do the errors incident to this factor reach serious proportions.

In summary, the accuracy of the graphic solution here presented is maximum where that of Smith's approximate graphic method is minimum, and vice versa. The two methods thus conveniently supplement one another.

Applications

The applications of the graphic method here introduced are similar to those of the approximate graphic method, previously discussed,[9] except that it is best suited to minerals whose birefringence is relatively high. In short, it may be used (1) to determine 2V when this quantity is not directly measurable; (2) to determine a third index when two indices and 2V are known; and (3) to check the mutual consistency of optical constants when all are directly determined by standard methods. For the teacher, a further application lies in its value as a means of elucidating the geometry of the biaxial indicatrix.

[9] Smith, H. T. U., *op. cit.*, p. 679.