SIMPLIFIED GRAPHIC METHOD OF DETERMINING APPROXIMATE AXIAL ANGLE FROM REFRACTIVE INDICES OF BIAXIAL MINERALS

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INTRODUCTION

The mathematical interrelation between axial angle, optic sign, and refractive indices of biaxial minerals gives a method of determining axial angle and sign, and provides a check on the consistency of index determinations by the immersion method. This relation is expressed exactly by the accurate equation and approximately by Mallard's simplified form of that equation, both of these being given in standard textbooks on the subject (Johannsen, 1914, pp. 103–104; Rosenbusch and Wülfing, 1924, pp. 119–121; Winchell, 1931, pp. 159–160). In determinative mineralogy, the approximate equation is more convenient to use, and generally is sufficiently accurate for practical purposes. Its application is facilitated by the graphic solution introduced by Wright (1913, pl. 6; 1911, pl. 9).

Wright's graphic solution, helpful though it is, retains certain of the disadvantages, or inconveniences, of the equation on which it is based. It requires that partial birefringences be computed first, and that optic sign be noted indirectly, by inspection. It gives V rather than $2V$, and, unless the terms are reversed, it always gives V with respect to the same fixed axis of the indicatrix, regardless of optic sign. To the beginner this is a source of considerable confusion, and it is a real inconvenience even to the experienced worker, leading frequently to a neglect of the method when it might well be used as a check on laboratory determinations.

In the graphic method here presented these disadvantages are eliminated. Partial birefringences need not be computed nor their ratios noted. A sliding scale carrying a range of values for refractive indices is simply aligned with a chart, and reference lines are traced through the chart to an upper scale on which the optic sign and axial angle are read directly.

THE SIMPLIFIED GRAPHIC METHOD

The graphic method here described employs two separate parts, a graphic plot, or chart, and a sliding scale which is adjusted to the chart for each determination.

The Chart. The chart (figure 1) is an adaptation of Wright's graphic solution (1913, pl. 6) for the equation:

$$\tan^2 V_\alpha = \frac{\gamma-\beta}{\beta-\alpha}.$$

Wright's original plot forms the central square, or grid, of the present chart. Lines representing values along the abscissa and ordinate of this square are projected to a straight line, or base line, which makes angles of 45° with the sides of the square. The radial lines of Wright's plot, and an additional complementary set of similar lines, are projected to an upper straight line, or scale, parallel to the base line. On this upper scale the values of $2V$ for positive minerals are read on the right side of the central point, and for negative minerals on the left side. Angles used in plotting the radial lines were calculated from the equation:

$$A = \arctan \tan^2 V$$

where A is the angle between the nearest side of the grid and the radial line leading to a particular value for $2V$. For convenience of any who may wish to redraft the chart, the calculated angles are listed in Table 1.

The Sliding Scale. The sliding scale is simply a linear scale on which the values for refractive indices within a given range are plotted in proportionate units. The unit value, 0.001, may range from one millimeter to one centimeter. Numerical values are plotted from left to right in ascending order.

In using the graphic method, maximum ease and accuracy of reading are attained when, in the reading position, one (or both) of the indices on the sliding scale falls well toward an end of the base line on the chart. Consequently, for minerals of low birefringence, it is desirable to employ a sliding scale based on a larger unit value than for minerals of high birefringence. For the latter a single scale, such as the upper one in figure 1, may suffice to cover the ordinary range. For the former, it is convenient to have a series of overlapping scales based on a larger unit value, or values. The lower scale in figure 1 is a representative example. If prepared scales are not practicable, a simple substitute is easily made by marking off on a strip of cross section paper the values for any particular set of indices, in equal units of any convenient magnitude.

PROCEDURE

Procedure in using the graphic method is suggested by the skeletonized guide sketch accompanying the chart, and is summarized below.

1. A sliding scale is selected which has a range adapted to the indices of the mineral considered. Lacking a suitable scale already drafted, the

values of the indices may be marked off in proportionate units on a strip of cross section paper.

2. The sliding scale is moved along the base line of the chart until the value of β coincides with the medial line of the chart.

3. Converging lines leading from the values of α and γ on the sliding scale are traced to their mutual intersection in the central grid. This step is facilitated by the use of a transparent celluloid square, or even of a cardboard square.

4. The radial line passing through the above-determined intersection is traced to the upper scale, on which optic sign and axial angle are read directly.

TABLE 1. ANGLES (A) PLOTTED ON THE CHART FOR VARIOUS VALUES OF $2V$

$2V$	A	$2V$	A
0	0° 0′	54	14°33′
10	0 26	56	15 47
20	1 47	58	17 05
22	2 10	60	18 26
24	2 35	62	19 51
26	3 03	64	21 20
28	3 33	66	22 52
30	4 06	68	24 28
32	4 42	70	26 07
34	5 20	72	27 50
36	6 02	74	29 35
38	6 46	76	31 24
40	7 33	78	33 15
42	8 23	80	35 09
44	9 16	82	37 05
46	10 13	84	39 02
48	11 13	86	41 01
50	12 16	88	43 00
52	13 23	90	45 00

ACCURACY

The accuracy of this method is dependent on three factors: (1) the accuracy of the approximate equation, (2) the precision attained in laboratory determination of indices, and (3) the accuracy of the chart itself. The effects of each of these is considered below.

Accuracy of the approximate equation. Certain errors inherent in the approximate equation were noted by Wright (1913, pp. 530–532), and have been analyzed in detail by Larsen and Berman (1934, pp. 6, 7). Summarizing briefly, the magnitude of these errors varies directly with birefringence and axial angle, and inversely with refringence. For a mineral whose lowest index is 1.50, the maximum error is less than 1° when total birefringence is 0.05, less than 1°30′ when total birefringence is 0.10, and nearly 4° when birefringence is 0.30. As α increases, the maximum error for any given value of total birefringence decreases slightly. For constant values of refringence and birefringence, the error is maximum when $2V$ is 90°, and becomes negligible as $2V$ approaches zero. Correction factors for compensating these errors are given by Larsen.

Errors in Laboratory Determinations. Obviously, results from the graphic method can be only as accurate as the data used. Index determinations by the immersion method have an accuracy which varies roughly from $\pm.003$ to $\pm.001$, or less, depending on the skill of the worker, the equipment used, and the nature of the material studied. Errors arise from (1) dispersion, (2) incorrect orientation of the mineral for particular readings, (3) failure to match liquid and mineral exactly, and (4) inexact calibration of the liquid. The errors arising from dispersion may be eliminated by using monochromatic light. Correct crystallographic orientation of mineral grains may be assured by obtaining centered interference figures, by using universal stage methods, or, in certain instances, by noting cleavages and extinction angles. Close matching of liquid and mineral may be attained by chance, by decreasing the interval between successive liquids in the series, by mixing liquids, or, better, by using index variation methods. Proper calibration of the liquid depends mainly on care in comparing temperatures at the time of calibration and at the time of mineral determination, and on guarding against deterioration of the liquid during the interval between these two steps. A consideration of these several factors provides a basis for evaluating the accuracy of data in particular instances, Only when the total birefringence of the mineral is much greater than the maximum possible error in index determinations can a very close approximation to the true axial angle be expected from any mathematical method.

Accuracy of the Chart Itself. The accuracy of the chart is determined both by the precision with which it is constructed and by the degree of exactness with which it may be read. The former factor is relatively constant throughout the chart, the lines being drafted with an accuracy of about ± 5 minutes of arc. Errors in $2V$ introduced by possible deviations of this amount increase toward the lower ends of the scale, but, in practice, are negligible.

The ease and exactness with which the chart may be read varies with the spacing of the radial lines, and thus is different for different parts of the chart. Greatest exactness is possible where the radial lines are well separated, and least exactness where they are crowded. Crowding occurs in two gradational zones—toward the two sides of the central grid, affecting only those radial lines representing the lower values of $2V$, and toward the central or converging point of the radial lines, affecting all lines. The consequences of the former are inherent in the formula on which the chart is based, and unavoidable, but concern only a relatively small group of minerals. The effects of crowding at the central zone, however, may be avoided by selecting, for each determination, a sliding scale which throws the intersection point beyond the crowded area. Used thus with discrimination, the chart, throughout the greater part of its range, will give a value of $2V$ matching to the nearest degree that given by calculation with the approximate equation.

Considering all sources of error, it may be stated that the graphic method is most satisfactory in determining axial angle for minerals whose total birefringence is neither very low nor unusually high—ranging, roughly, from a few units in the second decimal place to a few units in the first decimal place, the lower limit varying with the degree of precision attained in index determinations. Correct optic sign is given for all minerals whose axial angle does not fall very near to 90°.

Applications

The graphic method is useful both as a means of determining axial angle and optic sign, and as a check on the general consistency of optical data when all constant are determined directly. For minerals on which interference figures are impossible to obtain, it constitutes the only method of determining $2V$ and sign (excepting, of course, the equivalent calculations or the use of universal stage methods). Similarly, if two indices, axial angle, and optic sign are known, the graphic solution may be used to determine the third index. As suggested by Larsen and Berman (1934, p. 6), however, a more general application lies in using the method as a check on the mutual consistency of optical constants when all are determined directly by standard methods. If any one (or two) of the set falls beyond the position on the chart prescribed for it by the others, an error is indicated, and a consideration of the relative degree of certainty with which the several constants were measured may suggest the source of the error. Both of these applications of graphic methods in connection with the universal stage and the double variation method have been noted by Emmons in Winchell (1931A, pp. 235, 239; 1931B, pp. 132, 140, 142).

To the teacher, the graphic chart provides a means of repeatedly

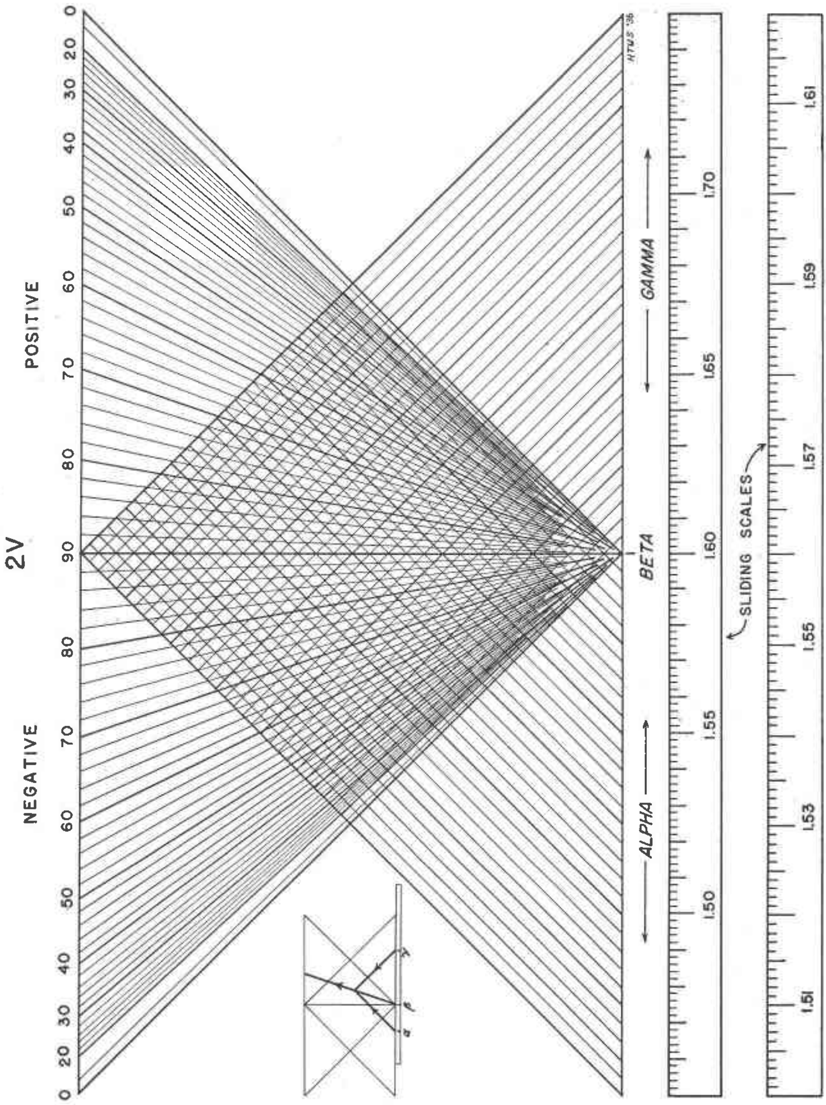
2V
POSITIVE
NEGATIVE
0
20
30
40
50
60
70
80
90
80
70
60
50
40
30
20
0
GAMMA
BETA
ALPHA
1.50
1.55
1.60
1.65
1.70
SLIDING SCALES
1.51
1.53
1.55
1.57
1.59
1.61
α
β
γ

Fig. 1

emphasizing the fundamental interrelation of important optical constants, and thus of encouraging in the student a genetic rather than an empiric approach to laboratory procedure.

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