

THE AMERICAN MINERALOGIST

JOURNAL OF THE MINERALOGICAL SOCIETY OF AMERICA

Vol. 21

FEBRUARY, 1936

No. 2

CLEAVAGE OF IONIC MINERALS*

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ABSTRACT

Mineral cleavage can be resolved into two components: cleavability and optical effect. The electrical theory of matter in the solid state leads to a quantitative expression for the cleavability of ionic minerals

$$C_{\{hkl\}} = \frac{A_{(hkl)}}{\sum_i n_i s_i \cos \theta_i}$$

Approximate values for s are obtained by using the electrostatic bond strength. Systematic application to minerals whose constituent atoms have inert-gas cores gives good agreement with observation.

CONTENTS

1. Introduction	75
2. Previous work on cleavage	76
3. Derivation of a quantitative expression for cleavability	78
4. Selection of ionic minerals	84
5. Structure and cleavage	85
6. Discussion of cleavage	97
7. Conclusions	102

1. INTRODUCTION

It has long been recognized that a close interrelationship must exist between the geometrical arrangement of the constituent particles of a mineral and the cleavage it exhibits; as a result the development of the theory of crystal structure has led to attempts to explain cleavage while at the same time work on cleavage has of necessity been required to gain an understanding of structure. Nevertheless in spite of its long standing the problem of cleavage has continued as one of great difficulty.

Cleavage has been defined¹ as "the natural fracture of a crystallized

* This research was greatly aided by a Storrow Fellowship in Geology of the National Research Council. I wish to thank Professor Linus Pauling for his kindly interest and advice throughout the investigation. I am also indebted to Gates Chemical Laboratory for research facilities.

¹ Dana, J. D. (Sixth ed. by E. S. Dana.) System of Mineralogy, John Wiley, New York, 1892.

mineral yielding more or less smooth surfaces; it is due to minimum cohesion."

It is evident that the term cleavage should be resolved into two distinct parts, first a comparison of the work of separation, i.e. "cleavability," and second the nature of the surface fracture and its interaction with light. For example the goniometric signal from the cleavage form {111} of diamond is strong—hence the cleavage is said to be perfect. The necessary distinction between these two phases of cleavage has not always been made. Cleavability is the more fundamental and the present investigation is largely restricted to a study of this property.

Experimentally cleavage is obtainable by several methods, of which the following are the more important:

1. Blow. This is likely to result in a general shattering of the material with the process largely one of shearing.
2. Wedge-action, e.g. with a knife. It can be either static or dynamic; in the former case pressure on the wedge is increased until fracture occurs while in the latter a number of blows on the wedge is made.
3. Bending. Analogous to the deflection of a beam.
4. Direct pull normal to the cleavage plane, i.e. the stress is a tension. The second method is the one usually applied.² Up to the present time the experimental data on cleavage have been chiefly of a qualitative nature.

2. PREVIOUS WORK ON CLEAVAGE

Cleavage investigations have taken in general two distinct directions; either that cleavage can be accounted for by the point geometry of the crystal or that it depends on the breaking of bonds between the atoms.

The first method is a development of the fundamental researches of Bravais.³ He held that cleavage is most easily obtained parallel to the plane of greatest net density, and that in case cleavage is obtained parallel to several forms, the ease of obtaining it decreases in the order of the decreasing net densities. However, since the net densities are inversely proportional to the lattice face areas, he used the latter for calculating relative values. Moreover since net density and interplanar distance are proportional, he gave as an alternative condition for most complete cleavage that the interplanar distance be greatest. Sohncke⁴ used the interplanar

² Tertsch, H., Einfache Kohäsionsversuche I, II, III: *Zeit. Krist.*, vol. 74, pp. 476-500, 1930; vol. 78, pp. 53-75, 1931; vol. 81, pp. 264-274, 1932.

³ Bravais, A., Études Cristallographiques: *Jour. d. l'Ecole Poly.*, vol. 22, pp. 101-278, 1851.

⁴ Sohncke, L., Ueber Spaltungsflächen und naturliche Kristallflächen: *Zeit. Krist.*, vol. 13, pp. 214-235, 1887.

distance condition but because of lattice interpenetration added to it the further condition that the cleavage planes be parallel to planes where the tangential cohesion is greatest. This is because interpenetration often causes, in a sequence of parallel planes, several of them to be grouped closer together into a layer which is repeated at regular intervals. The assumption being made that such layers are more strongly held together the result is greater tangential cohesion. When Bravais' theory was systematically applied to crystals of the trigonal and hexagonal system, Tertsch⁵ found numerous exceptions. The application of Sohncke's condition requires knowledge of the crystal structure. Basing their conclusions on *x*-ray determinations, Ewald and Friedrich⁶ independently came to the same condition as Sohncke's. Stark⁷ considering crystals in which the atoms can be considered to exist as ions, related cleavage to the repulsion of like ions of adjacent net planes approaching each other during a shearing process; however the possible cleavage faces are too numerous for this to be a unique condition. Scharizer⁸ postulated that the adjoining planes of two layers must be similar, the same holding true for a non-layer sequence; this condition evidently permits the application of Stark's relation. Niggli,⁹ by summing the number of electrons of the atoms at the lattice points, converted Bravais' net density into planes of electron density. Beckenkamp's¹⁰ treatment is essentially a combination of the conditions of Sohncke and Stark. Parker¹¹ has applied the method of net density to the structure of octahedrite.

The concept of conditioning cleavage on the breaking of bonds between particles was used by Barlow¹² as a result of his studies on the close-packing of spherical particles; the bonds broken in the process are not necessarily those which under static conditions have least strength. He gave as a probable condition that cleavage planes separate opposed

⁵ Tertsch, H., Spaltbarkeit und Struktur im trigonalen und hexagonalen Systeme: *Zeit. Krist.*, vol. 47, pp. 56-74, 1909.

⁶ Ewald, P. P., and Friedrich, W., Röntgenaufnahmen von kubischen Kristallen, insbesondere Pyrit: *Ann. der Phys.*, vol. 44, pp. 1183-1184, 1914.

⁷ Stark, J., Neuere Ansichten über die zwischen- und innermolekulare Bindung in Kristallen: *Jahrb. der Rad. u. Elek.*, vol. 12, pp. 279-296, 1915.

⁸ Scharizer, R., Die Bragg'schen Kristallgitter und die Spaltbarkeit: *Zeit. Krist.*, vol. 55, pp. 440-443, 1916.

⁹ Niggli, P., Geometrische Kristallographie des Diskontinuums. Borntraeger, Leipzig, 1919.

¹⁰ Beckenkamp, J., Atomanordnung und Spaltbarkeit: *Zeit. Krist.*, vol. 58, (Festband Groth) pp. 7-39, 1923.

¹¹ Parker, R. L., Zur Kristallographie von Anatase und Rutil. II Teil. Die Anatasstruktur: *Zeit. Krist.*, vol. 59, pp. 1-54, 1923.

¹² Barlow, W., Geometrische untersuchung über eine mechanische Ursache der Homogenität der Struktur und der Symmetrie . . . : *Zeit. Krist.*, vol. 29, pp. 433-588, 1898.

unlike particles, a condition directly opposite to that of Stark. Ewald¹³ postulated that cleavage will take place where the fewest bonds are broken. Relative calculated values per unit area for diamond gave agreement with observation; however discrepancies arising in a later study¹⁴ caused this postulate to be questioned and the condition of the previous paragraph to be set up as essential. Huggins¹⁵ came to the conclusion that new crystal surfaces should be left electrically neutral, that weak bonds would be ruptured in preference to strong bonds but that where all bonds are equally strong, the cleavage plane would break fewest bonds per unit area. He considered that the inclination of the bond to the cleavage normal could be neglected. Tertsch^{16,18a} considering the problem as one of attractive and repulsive forces between ions, calculated a value for the force across various possible cleavage planes, the minimum indicating the most cleavable. The inclination of the individual force directions to the cleavage normal is taken into account by using its direction cosine. Pauling¹⁷ calculated the density (bonds per unit area) of Al-O-Si bonds for cleavage in several aluminosilicates; he found that ease of cleavage decreases as bond-density increases. Wooster^{18b} divided crystals into groups depending on the types of bond and gave rules to summarize the cleavage associated with the various structures; electrical neutrality of the cleavage surfaces is not considered to be necessarily required.

3. DERIVATION OF A QUANTITATIVE EXPRESSION FOR CLEAVABILITY

Cleavage in minerals is one phase of the phenomenon of the cohesion of matter in the solid state; it accordingly is concerned with the interactions of the constituent particles when disturbed by mechanical forces from the configuration taken by them at equilibrium. The necessary and sufficient condition for equilibrium in a conservative system is that¹⁹

$$(\delta S)_E = 0,$$

¹³ Ewald, P. P., Die Intensität der Interferenzflecke bei Zinkblende und das Gitter der Zinkblende: *Ann. der Phys.*, vol. **44**, pp. 257-282, 1914.

¹⁴ See reference 6.

¹⁵ Huggins, M. L., Crystal cleavage and crystal structure: *Am. Jour. Sci.*, vol. **5**, pp. 303-313, 1923.

¹⁶ Tertsch, H., Bemerkungen zur Spaltbarkeit: *Zeit. Krist.*, vol. **65**, pp. 712-718, 1927.

¹⁷ Pauling, L., The structure of some sodium and calcium aluminosilicates: *Proc. Nat. Acad. Sci.*, vol. **16**, pp. 453-459, 1930.

^{18a} Tertsch, H., Wie erfolgt der Spaltungsvorgang bei Kristallen?: *Zeit. Krist.*, vol. **81**, pp. 275-284, 1932.

^{18b} Wooster, N., The correlation of cleavage and structure: *Science Progress*, vol. **26**, pp. 462-473, 1932.

¹⁹ Gibbs, J. W., On the equilibrium of heterogeneous substances. (1876). Collected Works. Longmans, New York, 1928.

where S is the entropy and E the energy. A more workable criterion is gained however by using the free energy, F , which is a function of S . The condition then becomes in the commonly used notation of Lewis and Randall²⁰

$$dF=0. \quad (1)$$

In the wave equation of quantum mechanics,²¹

$$\nabla^2\psi + \frac{8\pi^2\mu}{h^2} (w-v)\psi = 0,$$

the product of the eigenfunction ψ by its complex conjugate $\bar{\psi}$ may be interpreted as the electron density ρ , i.e. the probability of the configuration of the system. The distribution of ρ has been shown to be spherically symmetrical about the nucleus for atoms and ions having completed subgroups of electrons.^{22,23} It follows that the n electrons in an ion can be considered in effect as if located at the center of symmetry.²⁴ Under the assumption that polarization can be neglected, the center of symmetry for both the resultant positive and negative charges coincide, permitting a system of ions to be treated mathematically as discrete point charges (plus a repulsive term to be taken into account later), of value

$$\pm z = Z - n \quad (2)$$

where z denotes the valence, Z the charge on the nucleus and n the number of electrons. Since the atoms are in thermal agitation about a mean position it is unnecessary to refer the system to a temperature of absolute zero and accordingly the lattice constants as determined at ordinary temperatures can be used.

In the equilibrium condition the quantity F is given by

$$F = E + PV - TS,$$

where E is the total energy, P the pressure, V the volume, T the absolute temperature and S the entropy of the system. The term containing the entropy drops out at the absolute zero, and if the region surrounding the system is void of matter the PV term also disappears. Hence under these conditions

²⁰ Lewis and Randall, *Thermodynamics*. McGraw-Hill, New York, 1923.

²¹ Condon and Morse, *Wave mechanics*. McGraw-Hill, New York, 1929.

²² Unsöld, A., *Beiträge zur Quantenmechanik der Atome: Ann. der Phys.*, vol. 82, pp. 355-393, 1927.

²³ Pauling, L., The sizes of ions and the structure of ionic crystals: *Jour. Am. Chem. Soc.*, vol. 49, pp. 765-790, 1927.

²⁴ Jeans, J. H., *The mathematical theory of electricity and magnetism*. 5th ed. University Press, Cambridge, 1925.

$$F = E. \quad (3)$$

It is convenient to consider the crystal as at absolute zero with a volume energy content E obtained by using the ordinary lattice constants without extrapolating to zero, since the difference of energy is small.²⁵

A method of evaluating E for ionic crystals is known,^{26,27} the energy, expressed in either ergs or calories per mole, being designated as the lattice energy. The results of crystal structure determinations, while giving the configuration of ions in a crystal, say nothing as to the interactions between them.²⁸ It is known from Earnshaw's Theorem²⁹ that in classical electrostatics a system of electric charges alone cannot be in equilibrium; a repulsive term is accordingly required. The potential law between two ions is assumed as a first approximation to be

$$\phi = -\frac{z_1 z_2 e^2}{r} + \frac{b}{r^n} \quad (4)$$

where z_1 and z_2 are the charges on the ions obtained from Equation 2, e the electron charge, r the distance between ions, b a proportionality constant and n the repulsive exponent, which ordinarily can be taken as approximately equal to nine. The equilibrium energy of a unit cell is given by

$$\Phi = -\frac{z_1 z_2 e^2 A}{r_0} \left(1 - \frac{1}{n}\right) \quad (5)$$

where b has been eliminated, r_0 is the equilibrium distance between adjacent ions, and A is the Madelung constant. The energy due to the repulsive term is evidently small. Dividing Φ by p , the number of molecules in the unit cell, and multiplying by N , Avogadro's number, gives

$$E = \frac{N}{p} \Phi. \quad (6)$$

Accordingly if a system of ions, e.g., a crystallized mineral, satisfies the condition

$$dE = 0 \quad (7)$$

the system is in equilibrium and its energy is given by equation 6.

²⁵ Around 1 per cent or about 2 large calories per mole for crystals of the halite type.

²⁶ Born, M., *Atomtheorie des Festen Zustandes: Ency. d. Math. Wiss.*, vol. 5, pp. 527-789. Teubner, Leipzig, 1923.

²⁷ Born, M. and Bollnow, O. F., *Der Aufbau der festen Materie: Hdbh. d. Phy.*, vol. 24, pp. 370-465. Springer, Berlin, 1927.

²⁸ See ref. 13.

²⁹ See ref. 24.

When cleavage takes place the crystal is separated along a surface into two parts, each containing a volume energy, E_1 and E_2 , and in addition if A is the area of the cleavage plane the area of surface has been increased by $2A$. On separation the interaction energy E_{12} will equal zero since the ions have only a small radius of influence; there enters^{30,31,32,33} however a surface free energy, $2\sigma A$, whose absolute specific value is given by

$$\sigma = \frac{E_{12}}{2A}. \quad (8)$$

The value of E_{12} can be obtained by a method of a similar nature to that used in calculating E . The change in free energy then becomes

$$\Delta F = F_1 - F_2 = E_{12} = 2\sigma A. \quad (9)$$

For a sequence of variously oriented planes through a crystal, a series of values is obtained for ΔF and the plane of easiest separation is given by

$$\Delta F = \text{minimum}. \quad (10)$$

Accordingly the cleavability C of a mineral is defined as the reciprocal of the change in free energy

$$C = \frac{1}{\Delta F}. \quad (11)$$

In terms of unit area equation 11 becomes

$$C_{\{hkl\}} = \frac{1}{2\sigma}. \quad (12)$$

If the volume remains constant, which is approximately correct³⁴, equation 10 becomes for the cleavage form the surface energy law of Gibbs³⁵ and Curie^{36,37}

³⁰ Langmuir, I., The constitution and fundamental properties of solids and liquids: *Jour. Am. Chem. Soc.*, vol. **38**, pp. 2221-2295, 1916.

³¹ Harkins, W. D. et al., The structure of the surfaces of liquids, etc.: *Jour. Am. Chem. Soc.*, vol. **39**, pp. 354-364, 1917.

³² Born, M. and Stern, O., Über die Oberflächenenergie der Kristalle und ihren Einfluss auf die Kristallgestalt: *Sitzber. d. Pr. Akad. d. Wiss.*, Berlin, pp. 901-913, 1919.

³³ Harkins, W. D. and Cheng, Y. C., The orientation of molecules in surfaces: *Jour. Am. Chem. Soc.*, vol. **43**, pp. 35-53, 1921.

³⁴ Lennard-Jones, J. E. In Fowler, R. H., *Statistical Mechanics*. Univ. Press, Cambridge, 1929.

³⁵ See ref. 19.

³⁶ Curie, P., Sur la formation des cristaux et sur les constantes capillaires de leurs différentes faces: *Bull. Soc. Miner. Fr.*, vol. **8**, pp. 145-150, 1885.

³⁷ Ehrenfest, P., Zur Kapillaritätstheorie der Kristallgestalt: *Ann. der Phy.*, vol. **48**, pp. 360-368, 1915.

$$\Sigma\sigma s = \text{minimum} \quad (13)$$

where s is the face area. It may be mentioned that a fictitious "surface tension" is often conveniently used in the mathematical calculations instead of surface energy since the dimensions are the same for both.^{38,39,40} A 3-dimensional method to exhibit the value of σ for various orientations of the separation surface in a crystal is known.^{41,42,43,44,45} If normals to the plane are drawn from a center of coordinates within the crystal⁴⁶ proportional to σ , their extremities form a surface in which the minima are depressions. These will be symmetrically placed according to the symmetry of the crystal and may consist of secondary as well as primary minima, the secondary being due to a lesser degree of cleavability.

The calculation of the surface energy σ needed for equation 12 is involved and has only been carried out for the most simple ionic configurations; moreover it is desirable to express the cleavability in terms more closely related to the mechanical properties of the crystal rather than in terms of energy. This can be done by identifying E_{12} of equation 8 with the mechanical work W done on the system.^{47,48,49,50,51} Equation 12 then becomes

$$C_{(hkl)} = \frac{A_{(hkl)}}{W} \quad (14)$$

Since the maximum force per unit area is the tensile strength, i.e. the breaking strength of the crystal for the direction of the normal to the

³⁸ See ref. 19.

³⁹ Bakker, G., Kapillarität und Oberflächenspannung: *Hdbh. d. Exper. Phy.*, vol. 6, Akad. Verlags, Leipzig, 1928.

⁴⁰ Adams, N. K., The physics and chemistry of surfaces. Clarendon Press, Oxford, 1930.

⁴¹ Wulff, G., Zur Frage der Geschwindigkeit des Wachstums und der Auflösung der Kristallflächen: *Zeit. Krist.*, vol. 34, pp. 449-530, 1901.

⁴² Hilton, H., Mathematical Crystallography. Clarendon Press, Oxford, 1903.

⁴³ Yamada, M., Über die Oberflächenenergie der Kristalle und die Kristallformen: *Phy. Zeit.*, vol. 24, pp. 364-372, 1923.

⁴⁴ Yamada, M., Anhang . . . : *Phy. Zeit.*, vol. 25, pp. 52-56, 1924.

⁴⁵ Biemüller, J., Über die Oberflächenenergie der Alkalihalogenide: *Zeit. f. Phy.*, vol. 38, pp. 759-771, 1926.

⁴⁶ Usually coinciding with the center of symmetry.

⁴⁷ Griffith, A. A., The phenomena of rupture and flow in solids: *Trans. Roy. Soc., London*, vol. A221, pp. 163-198, 1921.

⁴⁸ Polanyi, M., Über die Natur des Zerzeissvorganges: *Zeit. f. Phy.*, vol. 7, pp. 323-327, 1921.

⁴⁹ See ref. 30.

⁵⁰ See ref. 31.

⁵¹ Smekal, A., Kohäsion der Festkörper: In Auerbach und Hort, *Hdbh. d. phy. u. tech. Mechanik*, vol. 4, 2, pp. 1-153, 1931.

cleavage plane, $W = kS$, where k is a parameter which without loss of generality can be placed equal to unity. Cleavability is the reciprocal of the tensile strength,

$$C_{(hkl)} = \frac{1}{S_{(hkl)}}. \tag{15}$$

The force between pairs of ions having the cleavage surface interposed between them can be considered as in the nature of a bond. On stressing the crystal until S is reached each of these bonds will have a value s giving as its normal component $s \cos \theta$ (see Fig. 1). Let the number of

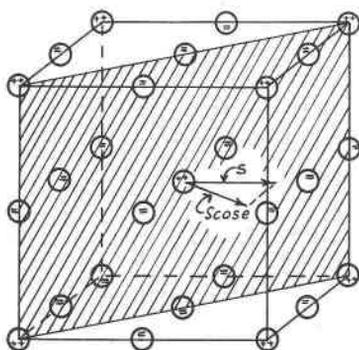


FIG. 1. The (110) plane for periclase showing the relationship of s and θ to the cleavage normal.

such bonds for each i^{th} ion be denoted by n , then the maximum force normal to the face area is given by

$$F_M = \sum_i n_i s_i \cos \theta_i.$$

Since $S_{(hkl)} = F_M/A_{(hkl)}$, equation 15 now becomes

$$C_{(hkl)} = \frac{A_{(hkl)}}{\sum_i n_i s_i \cos \theta_i} \tag{16}$$

A simple estimate of the bond strength s is obtainable from the coordination theory of ionic structures.⁵² It is evident from crystal geometry that the strongest bonds are those between cations and anions of the coordinated polyhedra, hence the electrostatic valence bond strength equal to the charge on the cation z divided by the coordination number ν ,

⁵² Pauling, L., The principles determining the structure of complex ionic crystals: *Jour. Am. Chem. Soc.*, vol. 51, pp. 1010-1026, 1929.

$$s = \frac{z}{\nu}$$

can be used as a first approximation.⁵³ The angular change in θ from its equilibrium value is usually small and in general may be neglected. The expression for cleavability, equation 16, may now be applied to ionic minerals.

4. SELECTION OF IONIC MINERALS

Minerals are a heterogeneous group of substances and a classification is accordingly necessary in order to have some basis of selection for those likely to be amenable to a simple treatment.

A rational basis for grouping is to be found in an examination of the structure of the elements. If the valence electrons are stripped from the atoms of the periodic table, the elements can be divided into two main groups, (1) Ionic, (2) Covalent.

The covalent group is composed largely of elements whose cores have an outer shell of eighteen electrons. The bond between such elements is predominately due to the sharing of electrons requiring treatment by quantum mechanical methods. Moreover due to their ease of deformation, polarization effects further complicate the cohesive phenomena of minerals composed of such atoms. The elements of this group are those with atomic numbers 26 (Fe) to 35 (Br), 44 (Ru) to 53 (I), and 76 (Os) to 84 (Po). Minerals consisting essentially of atoms from this group will be eliminated from this investigation.

The ionic group have cores whose electron configuration is that of the inert gases, hence with an outer shell of eight electrons. As previously shown the bonds between such ions in the solid state can be considered as of an electrostatic nature. The superimposed effects of a dipole field can be greatly reduced by restricting the anions to those of the smallest atomic radii, namely oxygen O^{2-} and fluorine F^{-} . The hydroxyl ion, OH^{-} , where it is a subordinate constituent as in the amphiboles, will be assumed to give no appreciable polarization effect.

The fundamental nature of this two-fold grouping has been shown by several recent geochemical investigations.^{54,55,56}

As a basis for systematic application of the cleavability expression, ionic minerals treated in this study are classified as follows:

⁵³ For a physical justification of the electrostatic valence bond see Bragg, W. L., The architecture of the solid state: (Kelvin lecture.) *Nature*, vol. 128, pp. 210-212; pp. 248-250, 1931.

⁵⁴ Washington, H. S., Chemistry of the earth's crust: *Jour. Franklin Inst.*, vol. 190, pp. 757-815, 1920.

⁵⁵ Niggli, P., Ore deposition of magmatic origin. Murby, London, 1929.

⁵⁶ Goldschmidt, V. M., Geochemische Verteilungsgesetze und kosmische Häufigkeit der Elemente: *Naturw.*, vol. 18, pp. 999-1013, 1930.

A. Simple ions

I. Binary system, $A-X$.Class 1. $A:X=1:1$.

- Villiaumite, NaF. (Halite structure.)
- Bromellite, BeO. (Wurtzite structure.)
- Periclase, MgO. (Halite structure.)
- Lime, CaO. (Halite structure.)

Class 2. $A:X=2:3$.

- Corundum, Al_2O_3 .

Class 3. $A:X=1:2$.

- Sellaite, MgF_2 . (Rutile structure.)
- Fluorite, CaF_2 .
- Quartz, SiO_2 .
- Cristobalite, SiO_2 .
- Tridymite, SiO_2 .
- Rutile, TiO_2 .
- Octahedrite, TiO_2 .
- Brookite, TiO_2 .

II. Ternary system, $A-B-X$.

- Spinel, $MgAl_2O_4$.

B. Complex radical, ARX_3 .

- Calcite, $CaCO_3$.

C. Silicates.^{57,58}

I. Independent tetrahedral groups.

Class 1. Single SiO_4 groups.

- Phenacite, Be_2SiO_4 .
- Kyanite, Al_2OSiO_4 .
- Topaz, $Al_2SiO_4F_2$.
- Zircon, $ZrSiO_4$.
- Grossularite (garnet group), $Ca_3Al_2(SiO_4)_3$.

Class 2. Si_2O_7 groups.

- Melilite, $Ca_2MgSi_2O_7$.

II. Tetrahedral chains.

Class 1. Single chains.

- Diopside (pyroxene group), $CaMgSi_2O_6$.

Class 2. Double chains.

- Tremolite (amphibole group), $Ca_2Mg_5(Si_4O_{11})_2(OH)_2$.

III. Tetrahedral planes.

- Muscovite, $KAl_2(Si_3Al)O_{10}(OH, F)_2$.

IV. Three-dimensional network of tetrahedra.

- Sodalite, $Na_4Al_3Si_3O_{12}Cl$.

5. STRUCTURE AND CLEAVAGE

For most of the structural arrangements given below reference is made to Ewald, P. P. and Hermann, C., "Strukturbericht, 1913-1928,"

⁵⁷ Bragg, W. L., The structure of the silicates: *Zeit. Krist.*, vol. 74, pp. 237-305, 1930.

⁵⁸ Náráy-Szabó, St., Ein auf der Kristallstruktur basierendes Silicatsystem: *Zeit. Phy. Chem.*, vol. 9B, pp. 356-377, 1930.

Zeit. Krist. Ergänzungsband, 1931. In the comparison of calculated relative values with observation the work of Dana (see ref. 1) is used. It is found that electrical neutrality of cleavage surfaces, where it does not follow as a result of a plane of cleavage, is obtainable by a non-planar cleavage surface; possible cleavage surfaces are considered to be electrically neutral. In cases where a cleavage form has several alternative cleavage surfaces the most probable is taken to be the one giving the highest value for the cleavability. Cleavability values marked by a star denote cases where it is probable that the angular change in θ cannot be neglected.

Villiaumite, Periclase and Lime

Structure characteristics: Cubic, Γ_c' , O_h^5 . $Z=4$. Villiaumite, NaF, $a=4.62\text{\AA}$; periclase MgO, $a=4.20\text{\AA}$; lime, CaO, $a=4.80\text{\AA}$. Coordination: Octahedra of anions.

Areas: Villiaumite, $A_{(100)}=21.15\text{\AA}^2$, $A_{(110)}=33.65\text{\AA}^2$, $A_{(111)}=36.65\text{\AA}^2$. Periclase, $A_{(100)}=17.63\text{\AA}^2$, $A_{(110)}=24.93\text{\AA}^2$, $A_{(111)}=30.54\text{\AA}^2$. Lime, $A_{(100)}=23.00\text{\AA}^2$, $A_{(110)}=32.52\text{\AA}^2$, $A_{(111)}=39.84\text{\AA}^2$.

Bond strengths: Villiaumite, $s_{\text{Na-F}}=\frac{1}{6}$. Periclase, $s_{\text{Mg-O}}=\frac{1}{3}$. Lime, $s_{\text{Ca-O}}=\frac{1}{3}$.

Summations: Villiaumite, $\Sigma(100)=4\times\frac{1}{6}\times 1.00(\theta=0^\circ)=0.67$, $\Sigma(110)=8\times\frac{1}{6}\times 0.71(\theta=45^\circ)=0.95$, $\Sigma(111)=12\times\frac{1}{6}\times 0.82(\theta=35^\circ)=1.64$. Periclase, $\Sigma(100)=4\times\frac{1}{3}\times 1.00(\theta=0^\circ)=1.33$, $\Sigma(110)=8\times\frac{1}{3}\times 0.71(\theta=45^\circ)=1.89$, $\Sigma(111)=12\times\frac{1}{3}\times 0.82(\theta=35^\circ)=3.28$. Lime, same as periclase.

TABLE 1. CLEAVABILITIES OF VILLIAUMITE, PERICLASE AND LIME

Form		{100}	{110}	{111}
Type		Closed cubic	Closed dodecahedral	Closed octahedral
Number of faces		6	12	8
Villiaumite				
Cleavability	Calc.	31.5	31.5*	22.3
	Obs. ⁵⁹	Complete	—	—
Periclase				
Cleavability	Calc.	13.2	13.2*	9.3
	Obs.	Perfect	—	Less distinct

* The areas, bond strengths and summations are omitted for bromellite and the minerals following. These data are given in Shappell, M. D., Doctorate dissertation, California Institute of Technology, Pasadena, 1933.

⁵⁹ Hintze, C., *Handbuch der Mineralogie*. Veit, Leipzig, 1915.

Lime

Cleavability	Calc.	17.2	17.2*	12.1
	Obs. ⁶⁰	Complete	Possibly	—

Remark: The separation surfaces for {100} are coplanar ions.

*Bromellite**

Structure characteristics: Hexagonal, Γ_h , C_{6v}^4 . $Z=4$. Lattice constants: $a=2.69\text{\AA}$, $c=4.37\text{\AA}$. Composition: BeO. Coordination: Be-O tetrahedra.

TABLE 2. CLEAVABILITY OF BROMELLITE

Form		{10 $\bar{1}$ 0}	{0001}	{11 $\bar{2}$ 0}	{10 $\bar{1}$ 1}
Type		Open prismatic	Open basal	Open prismatic	Closed bipyramidal
Number of faces		6	2	6	12
Cleavability	Calc.	12.5	12.5*	11.7	11.1
	Obs. ⁶¹	Distinct	Doubtful	—	—

Remark: According to Groth (see Ref. 60) there is no distinct cleavage.

Corundum

Structure characteristics: Trigonal, Γ_{rh} , D_{3d}^6 . $Z=2$. Lattice constants: $a=5.12\text{\AA}$, $\alpha=55^\circ 17'$. Composition: Al_2O_3 . Coordination: Al-O octahedra.

TABLE 3. CLEAVABILITY OF CORUNDUM

Form	x-ray	{110}	{2 $\bar{1}$ 1}	{111}	{10 $\bar{1}$ }	{100}
		Dana	{10 $\bar{1}$ 1}	{10 $\bar{1}$ 0}	{0001}	{11 $\bar{2}$ 0}
Type		Closed rhombohedral	Open prismatic	Open basal	Open prismatic	Closed rhombohedral
Number of faces		6	6	2	6	6
Cleavability	Calc.	12.4	10.2	9.5	9.2	8.6
	Obs.	—	—	—	—	—

Remark: Parting on {110} often prominent.

⁶⁰ Groth, P., *Chemische Krystallographie*. Engelmann, Leipzig, 1906.

⁶¹ Aminoff, G., *Über Berylliumoxyd als Mineral und dessen Kristallstruktur: Zeit. Krist.*, vol. 62, pp. 113-122, 1925; vol. 63, p. 175, 1926.

Fluorite

Structure characteristics: Cubic, T_c' , O_h^5 . $Z=4$. Lattice constant: $a=5.45\text{\AA}$. Composition: CaF_2 . Coordination: Ca-F hexahedra.

TABLE 4. CLEAVABILITY OF FLUORITE

Form		{110}	{111}	{100}
Type		Closed dodecahedral	Closed octahedral	Closed cubic
Number of faces		12	8	6
Cleavability	Calc.	29.7*	25.7	25.7*
	Obs.	Occasionally distinct ⁶²	Perfect	—

Quartz

Structure characteristics: Hexagonal, T_h , D_6^5 or D_6^4 . $Z=3$. Lattice constants: $a=5.01\text{\AA}$, $c=5.47\text{\AA}$. Composition: SiO_2 . Coordination: Si-O tetrahedra. High temperature modification.

TABLE 5. CLEAVABILITY OF QUARTZ

Form		{10 $\bar{1}$ 1}	{11 $\bar{2}$ 0}	{0001}	{10 $\bar{1}$ 0}
Type		Closed bipyramidal	Open prismatic	Open basal	Open prismatic
Number of faces		12	6	2	6
Cleavability	Calc.	18.7	17.4	17.0	16.2
	Obs.	Difficult and seldom observed	More difficult	More difficult	—

Remark: According to Rogers⁶³ imperfect {10 $\bar{1}$ 1} cleavage is rather common, especially in thin section.

⁶² See ref. 59.

⁶³ Rogers, A. F., Cleavage and parting in quartz: (Abstract.) *Am. Mineral.*, vol. 18, pp. 111-112, 1933.

Cristobalite

Structure characteristics: Cubic, Γ_o' , O_h^7 . $Z=8$. Lattice constant: $a=7.12\text{\AA}$. Composition: SiO_2 . Coordination: Si-O tetrahedral framework. High temperature modification.

TABLE 6. CLEAVABILITY OF CRISTOBALITE

Form		{100}	{111}	{110}
Type		Closed cubic	Closed octahedral	Closed dodecahedral
Number of faces		6	8	12
Cleavability	Calc.	22.2	22.0	21.9
	Obs.	—	—	—

Tridymite

Structure characteristics: Hexagonal, Γ_h , D_{6h}^4 . $Z=4$. Lattice constants: $a=5.03\text{\AA}$, $c=8.22\text{\AA}$. Composition: SiO_2 . Coordination: Framework of Si-O tetrahedra. High temperature modification.

TABLE 7. CLEAVABILITY OF TRIDYMITE

Form		{10 $\bar{1}$ 0}	{0001}	{1120}	{10 $\bar{1}$ 1}
Type		Open prismatic	Open basal	Open prismatic	Closed bipyramidal
Number of faces		6	2	6	12
Cleavability	Calc.	22.0	21.9	20.6	20.4
	Obs.	Not distinct	—	—	—

Remark: Parting sometimes observed parallel to {0001}.

Sellaite and Rutile

Structure characteristics: Tetragonal, Γ_t , D_{4h}^{14} . $Z=2$. Lattice constants: Sellaite, MgF_2 , $a=4.64\text{\AA}$, $c=3.06\text{\AA}$. Rutile, TiO_2 , $a=4.58\text{\AA}$, $c=2.95\text{\AA}$. Coordination: Octahedra of anions.

TABLE 8. CLEAVABILITY OF SELLAITE AND RUTILE

Form		{100}	{110}	{001}	{111}
Type		Open prismatic	Open prismatic	Open basal	Closed bipyramidal
Number of faces		4	4	2	8

Sellaite

Cleavability	Calc.	30.1	30.1	23.2	16.7
	Obs.	Perfect	Perfect	—	—

Rutile

Cleavability	Calc.	14.4	14.4	11.1	8.1
	Obs.	Distinct	Distinct	—	Traces

Octahedrite

Structure characteristics: Tetragonal, Γ_1' , D_{4h}^{19} . $Z=4$. Lattice constants: $a=3.73\text{\AA}$, $c=9.37\text{\AA}$. Composition: TiO_2 . Coordination: Ti-O octahedra.

TABLE 9. CLEAVABILITY OF OCTAHEDRITE

Form	x -ray	{101}	{001}	{100}	{110}
	Dana	{111}	{001}	{110}	{100}
Type		Closed bipyramidal	Open basal	Open prismatic	Open prismatic
Number of faces		8	2	4	4
Cleavability	Calc.	17.2	15.5	13.1	12.9
	Obs.	Perfect	Perfect	—	—

Brookite

Structure⁶⁴ characteristics: Orthorhombic, Γ_0 , V_h^{15} . $Z=8$. Lattice constants: $a=9.16\text{\AA}$, $b=5.44\text{\AA}$, $c=5.14\text{\AA}$. Composition: TiO_2 . Coordination: Ti-O octahedra.

TABLE 10. CLEAVABILITY OF BROOKITE

Form	x -ray	{210}	{110}	{111}	{001}	{010}	{100}
	Dana	{110}	{120}	{121}	{001}	{010}	{100}
Type		Open prismatic	Open prismatic	Closed bipy.	Open basal	Open pinacoidal	Open pin.
Number of faces		4	4	8	2	2	2
Cleavability	Calc.	18.3	16.1	11.0	10.6	10.1	9.3
	Obs.	Indistinct	—	—	More indistinct	—	—

⁶⁴ Pauling, L., and Sturdivant, J. H., The crystal structure of brookite: *Zeit. Krist.*, vol. 68, pp. 239-256, 1928.

Spinel

Structure characteristics: Cubic, Γ_6' , O_h^7 . $Z=8$. Lattice constant: $a=8.09\text{\AA}$. Composition: $MgAl_2O_4$. Coordination: Mg-O tetrahedra, Al-O octahedra.

TABLE 11. CLEAVABILITY OF SPINEL

Form		{111}	{110}	{100}
Type		Closed octahedral	Closed dodecahedral	Closed cubic
Number of faces		8	12	6
Cleavability	Calc.	15.3	12.8	12.7
	Obs.	Imperfect	—	—

Remark: The reflection surface for {111} is poor.

Calcite

Structure characteristics: Trigonal, Γ_{rh} , D_3^6 . $Z=2$. Lattice constants: $a=6.36\text{\AA}$, $\alpha=46^\circ 7'$. Composition: $CaCO_3$. Coordination: C-O triangles, Ca-O octahedra.

TABLE 12. CLEAVABILITY OF CALCITE

Form	x -ray unit	{211}	{111}	{211}	{101}
	Dana	{10 $\bar{1}$ 1}	{0001}	{10 $\bar{1}$ 0}	{11 $\bar{2}$ 0}
Type		Closed rhombohedral	Open basal	Open prismatic	Open prismatic
Number of faces		6	2	6	6
Cleavability	Calc.	35.4	34.4	31.8	24.6
	Obs.	Highly perfect	—	—	—

Remark: Ions of the {211} surface are coplanar.

Phenacite

Structure characteristics: Trigonal, Γ_{rh} , C_{3i}^2 , $Z=6$. Lattice constants: $a=7.68\text{\AA}$, $\alpha=108^\circ 1'$.
Composition: Be_2SiO_4 . Coordination: Be-O tetrahedra, Si-O tetrahedra.

TABLE 13. CLEAVABILITY OF PHENACITE

Form	α -ray	{100}	{111}	{10 $\bar{1}$ }	{2 $\bar{1}\bar{1}$ }
	Dana	{10 $\bar{1}\bar{1}$ }	{0001}	{11 $\bar{2}$ 0}	{10 $\bar{1}$ 0}
Type		Closed rhombohedral	Open basal	Open prismatic	Open prismatic
Number of faces		6	2	6	6
Cleavability	Calc.	20.4	20.4	19.0	18.9
	Obs.	Imperfect	—	Distinct	—

Remark: According to Niggli⁶⁵ {100} very imperfect, {111} perhaps, {10 $\bar{1}$ } not very distinct.

Kyanite

Structure^{66,67,68} characteristics: Triclinic, Γ_{tr} , C_1^1 , $Z=4$. Lattice constants: $a=7.09\text{\AA}$, $b=7.72\text{\AA}$, $c=5.56\text{\AA}$, $\alpha=90^\circ 5.5'$, $\beta=101^\circ 2'$, $\gamma=105^\circ 44.5'$. Composition: Al_2OSiO_4 . Coordination: Al-O octahedra, Si-O tetrahedra.

TABLE 14. CLEAVABILITY OF KYANITE

Form		{100}	{010}	{001}
Type		Open pinacoidal	Open pinacoidal	Open basal
Number of faces		2	2	2
Cleavability	Calc.	15.1	11.7	11.7
	Obs.	Very perfect	Less perfect	—

Remarks: Ions of the {100} surface are coplanar. Parting parallel to {001}.

⁶⁵ Niggli, P., *Lehrbuch der Mineralogie*. Second ed., Borntraeger, Berlin, 1926.

⁶⁶ Bragg, W. L. and West, J., The structure of certain silicates: *Proc. Roy. Soc.*, vol. A114, pp. 450-473, 1927.

⁶⁷ See ref. 52.

⁶⁸ Náray-Szabó, S., Taylor, W. H., and Jackson, W. W., The structure of kyanite: *Zeit. Krist.*, vol. 71, pp. 117-130, 1929.

Topaz

Structure^{69,70} characteristics: Orthorhombic, Γ_0 , V_h^{16} , $Z=4$. Lattice constants: $a=4.64\text{\AA}$, $b=8.78\text{\AA}$, $c=8.37\text{\AA}$. Composition: $\text{Al}_2\text{SiO}_4\text{F}_2$. Coordination: Si-O tetrahedra, Al-O octahedra.

TABLE 15. CLEAVABILITY OF TOPAZ

Form		{001}	{100}	{010}	{110}
Type		Open basal	Open pinacoidal	Open pinacoidal	Open prismatic
Number of faces		2	2	2	4
Cleavability	Calc.	28.6	22.4	18.1	16.4
	Obs.	Highly perfect	—	—	—

Remark: Ions of {001} surface are coplanar.

Zircon

Structure characteristics: Tetragonal, Γ_1' , D_{4h}^{19} , $Z=4$. Lattice constants: $a=6.58\text{\AA}$, $c=5.93\text{\AA}$. Composition: ZrSiO_4 . Coordination: Si-O tetrahedra, Zr-O hexahedra.

TABLE 16. CLEAVABILITY OF ZIRCON

Form	x-ray	{100}	{110}	{101}	{001}	{111}
	Dana	{110}	{100}	{111}	{001}	{221}
Type		Open prismatic	Open prismatic	Closed bipyramidal	Open basal	Closed bipyramidal
Number of faces		4	4	8	2	8
Cleavability	Calc.	19.9	19.4	14.6	11.7	9.8
	Obs.	Imperfect	—	Less distinct	—	—

⁶⁹ Pauling, L., The crystal structure of topaz: *Proc. Nat. Acad. Sci.*, vol. 14, pp. 603-606, 1928.

⁷⁰ Alston, N. A., and West, J., The structure of topaz, etc.: *Zeit. Krist.*, vol. 69, pp. 149-167, 1928.

Grossularite

Structure characteristics: Cubic, Γ_2'' , O_h^{10} . $Z=8$. Lattice constant: $a=11.83\text{\AA}$. Composition: $\text{Ca}_3\text{Al}_2(\text{SiO}_4)_3$. Coordination: Si-O tetrahedra, Al-O octahedra, Ca-O hexahedra.

TABLE 17. CLEAVABILITY OF GROSSULARITE

Form		{110}	{100}	{111}
Type		Closed dodecahedral	Closed cubic	Closed octahedral
Number of faces		12	6	8
Cleavability	Calc.	22.4	16.5	Probably less than {100}
	Obs.	Sometimes rather distinct	—	—

Remarks: It is uncertain whether {110} is cleavage or parting. The limit of error for the calculated {111} cleavability value is large.

Melilite

Structure⁷¹ characteristics: Tetragonal, Γ_1 , D_{2d}^8 . $Z=2$. Lattice constants: $a=7.73\text{\AA}$, $c=5.01\text{\AA}$. Composition: $(\text{Ca}, \text{Na})_2(\text{Mg}, \text{Al})_1(\text{Si}, \text{Al})_2\text{O}_7$. Coordination: Si-O tetrahedra, Mg-O tetrahedra, Ca-O hexahedra.

TABLE 18. CLEAVABILITY OF MELILITE

Form	α -ray	{001}	{110}	{100}
	Dana		{001}	{100}
Type		Open basal	Open prismatic	Open prismatic
Number of faces		2	4	4
Cleavability	Calc.	35.5	26.5	17.6
	Obs.	Distinct	Indistinct	—

Remark: Ions of {001} surface are coplanar.

⁷¹ Warren, B. E., The structure of melilite . . . : *Zeit. Krist.*, vol. 74, pp. 131-138, 1930.

Diopside

Structure⁷² characteristics: Monoclinic, Γ_m' , C_{2h}^6 . $Z=4$. Lattice constants: $a=9.71\text{\AA}$, $b=8.89\text{\AA}$, $c=5.24\text{\AA}$, $\beta=74^\circ 10'$. Composition: $\text{CaMgSi}_2\text{O}_6$. Coordination: Si-O tetrahedra, Mg-O octahedra, Ca-O hexahedra.

TABLE 19. CLEAVABILITY OF DIOPSIDE

Form		{110}	{001}	{100}	{010}
Type		Open prismatic	Open basal	Open pinacoidal	Open pinacoidal
Number of faces		4	2	2	2
Cleavability	Calc.	18.5	17.8	17.2	16.3
	Obs.	Rather perfect (sometimes) but interrupted	—	—	—

Remarks: {110} often only observed in thin sections $\perp c$. Parting on {001} often very prominent, on {100} less distinct and less common.

Tremolite

Structure⁷³ characteristics: Monoclinic, Γ_m' (oriented as a body centered lattice in order to agree with the usual crystallographic axes), C_{2h}^3 . $Z=2$. Lattice constants: $a=9.78\text{\AA}$, $b=17.8\text{\AA}$, $c=5.26\text{\AA}$, $\beta=73^\circ 58'$. Composition: $\text{Ca}_2\text{Mg}_5(\text{Si}_4\text{O}_{11})_2(\text{OH})_2$. Coordination: Si-O tetrahedra, Mg-O octahedra, Ca-O hexahedra.

TABLE 20. CLEAVABILITY OF TREMOLITE

Form		{110}	{100}	{010}	{001}
Type		Open prismatic	Open pinacoidal	Open pinacoidal	Open basal
Number of faces		4	2	2	2
Cleavability	Calc.	44.1	39.5	31.0	16.6
	Obs.	Highly perfect	Sometimes distinct	Sometimes distinct	—

Remarks: See discussion.

⁷² Warren, B. E., and Bragg, W. L., The structure of diopside, . . . : *Zeit. Krist.*, vol. 69, pp. 168–193, 1928.

⁷³ Warren, B. E., The structure of tremolite . . . : *Zeit. Krist.*, vol. 72, pp. 42–57, 1929.

Muscovite

Structure^{74,76} characteristics: Monoclinic, Γ_m' , C_{2h} ⁸, $Z=4$. Lattice constants: $a=5.19\text{\AA}$, $b=9.00\text{\AA}$, $c=20.04\text{\AA}$, $\beta=95^\circ 30'$. Composition: $\text{KAl}_2(\text{Si}_3\text{Al})\text{O}_{10}(\text{OH}, \text{F})_2$. Coordination: (Si, Al)—O tetrahedra, Al—(O, OH) octahedra, $\nu_K=12$.

TABLE 21. CLEAVABILITY OF MUSCOVITE

Form	x-ray	{001}	{100}	{010}
	Dana		{001}	{201}
Type		Open basal	Open pinacoidal	Open pinacoidal
Number of faces		2	2	2
Cleavability	Calc.	73.1	25.0	21.4
	Obs.	Eminent	—	—

Remark: Fourier analysis indicates that the ions of the separation surfaces for {001} are exactly coplanar.

Sodalite

Structure^{77,78} characteristics: Cubic, Γ_c (closely approximates a body-centered lattice), T_d^4 (possibly T_d^4). $Z=2$. Lattice constant: $a=8.87\text{\AA}$. Composition: $\text{Na}_4\text{Al}_3\text{Si}_3\text{O}_{12}\text{Cl}$. Coordination: Si—O tetrahedra, Al—O tetrahedra, Na—O hexahedra.

TABLE 22. CLEAVABILITY OF SODALITE

Form		{110}	{100}	{111}
Type		Closed dodecahedral	Closed cubic	Closed octahedral
Number of faces		12	6	8
Cleavability	Calc.	45.5	32.7	32.2
	Obs.	More or less distinct	—	—

⁷⁴ Mauguin, C., Étude du mica muscovite au moyen rayons X: *Comp. Rend.*, vol. **185**, pp. 288–291, 1927.

⁷⁵ Pauling, L., The structure of the micas and related minerals: *Proc. Nat. Acad. Sci.*, vol. **16**, pp. 123–129, 1930.

⁷⁶ Jackson, W. W., and West, J., The crystal structure of muscovite . . . : *Zeit. Krist.*, vol. **76**, pp. 211–227, 1931.

⁷⁷ Pauling, L., The structure of sodalite . . . : *Zeit. Krist.*, vol. **74**, pp. 213–225, 1930.

⁷⁸ Barth, T. F., The structures of the minerals of the sodalite family: *Zeit. Krist.*, vol. **83**, pp. 405–414, 1932.

6. DISCUSSION OF CLEAVAGE

Comparative cleavabilities and data on the cohesive properties of the ionic minerals studied are given in Table 23. The twenty-five minerals listed are arranged in the order of their highest cleavability. The calculated cleavabilities for the different forms, taken from the foregoing tables, are contained in the second column, observed cleavage being denoted by underlines, three for very good, two for good, and one for poor. The table shows that the general agreement between calculated cleavability values and observed cleavage is good.

The best observed cleavage for each species is for the form with the highest calculated cleavability value, except those of fluorite, lime, periclase and bromellite. These anomalies are probably due to the neglect of angular change in θ during the process of rupture; since the treatment is in the nature of a first order approximation it is to be expected that such cases may occur. In the case of fluorite there may exist the additional effect of a good reflecting surface for a form of lower cleavability in the sequence. The lack of observed cleavage for cristobalite and corundum is discussed later (see Table 24). Second order effects due to impurities, growth conditions and mechanical strains such as gliding set up during the process of cleavage, although difficult to evaluate, do not seem to materially affect the results.

The agreement of muscovite is excellent; its high cleavability is due to the comparatively weak K-O bonds which hold strong layers together; the statistical alternation of K^+ ions across the cleavage surface results in electrical neutrality. An extreme case of this nature is that of talc whose structure consists of neutral layers held together by second order electrical effects; since there are no direct bonds the cleavability value is very high. In the case of diopside although there are single chains of Si-O tetrahedra parallel to the c -axis the numerical values for the possible cleavage forms are fairly close together indicating a compact structure (see Table 24); this is confirmed by the observed frequent failure of the prismatic $\{110\}$ cleavage except in thin section. Tremolite with double chains of tetrahedra gives an interesting contrast. The high cleavability (44.1) for $\{110\}$ together with the low cleavability (16.6) for $\{001\}$ clearly indicates a structure capable of yielding fibres; the probable cleavage path for $\{110\}$ is shown in Fig. 2; a suggested⁷⁹ cleavage giving the very low value of 15.9 for the cleavability of $\{110\}$ is erroneous, if the calculations of this paper are trustworthy. The cleavage of that portion of the path shown in the figure parallel to b is analogous to the mica $\{001\}$ cleavage with its accompanying high cleavability;

⁷⁹ See ref. 73.

TABLE 23. COMPARATIVE CLEAVABILITIES AND COHESIVE PROPERTIES OF IONIC MINERALS

Species	Sequence of Cleavabilities		Total number planes	Hardness	Fracture	Tenacity
Muscovite	<u>73.1</u>	25.0 21.4	6	2.00-2.25	—	—
Sodalite	<u>45.5</u>	32.7 33.2	26	5.50-6.00	Uneven to subc.	Brittle
Tremolite	<u>44.1</u>	<u>39.5</u> 31.0 16.6	10	5.00-6.00	Uneven to subc.	Brittle
Melilite	<u>35.5</u>	26.5 17.6	10	5.00	Uneven to conch.	Brittle
Calcite	<u>35.4</u>	34.4 31.8 24.6	20	3.00	Conchoidal	—
Villiaumite	<u>31.5</u>	31.5* 22.3	26	<3.00	—	—
Sellaite	<u>30.1</u>	30.1 23.2 16.7	18	5.00-6.00	Conchoidal	Brittle
Fluorite	<u>29.7*</u>	<u>25.7</u> 25.7*	26	4.00	Flat-conchoidal	Brittle
Topaz	<u>28.6</u>	22.4 18.1 16.4	10	8.00	Uneven to subc.	Brittle
Grossularite	<u>22.4</u>	16.5	18	6.50-7.50	Uneven to subc.	Brittle
Cristobalite	<u>22.2</u>	22.0 21.9	26	6.00-7.00	—	—
Tridymite	<u>22.0</u>	21.9 20.6 20.4	26	7.00	Conchoidal	Brittle
Phenacite	<u>20.4</u>	20.4 19.0 18.9	20	7.50-8.00	Conchoidal	Brittle
Zircon	<u>19.9</u>	19.4 14.6 11.7 9.8	26	7.50	Conchoidal	Brittle
Quartz	<u>18.7</u>	17.4 17.0 [?] 16.2	26	7.00	Subc. to conchoidal	Brittle
Diopside	<u>18.5</u>	17.8 17.2 16.3	12	5.00-6.00	Uneven to conchoidal	Brittle
Brookite	<u>18.3</u>	16.1 11.0 10.6 10.1 9.3	22	5.50-6.00	Subconchoidal	Brittle
Octahedrite	<u>17.2</u>	<u>15.5</u> 13.1 12.9 [?]	18	5.50-6.00	Subconchoidal	Brittle
Lime	<u>17.2</u>	17.2* 12.1	26	—	—	—
Spinel	<u>15.3</u>	12.8 12.7	26	8.00	Conchoidal	Brittle
Kyanite	<u>15.1</u>	11.7 11.7	6	5.00, 7.25	—	—
Rutile	<u>14.4</u>	14.4 11.1 8.1 <u>—</u> [?]	18	6.00-6.50	Uneven to subc.	Brittle
Periclase	<u>13.2</u>	<u>13.2*</u> 9.3	26	6.00	—	—
Bromellite	<u>12.5</u>	12.5* 11.7 11.1	26	9.00	—	—
Corundum	<u>12.4</u>	10.2 9.5 9.2 8.6	26	9.00	Uneven to conchoidal	Brittle

Note: Observed cleavage is denoted by underlining. ===== very good ===== good ===== poor

since two double Si-O chains are held together by strong Mg-O bonds the resulting columnar units have a nearly square cross-section of greatly increased strength. The silica modifications, namely quartz, cristobalite and tridymite, have their cleavabilities calculated from the high temperature forms since it is probable that these configurations are approximate to those of the low temperature forms; the agreement obtained is good; the compact structures indicated by their cleavabilities is reflected in the poor or no cleavage (see Table 24). Sellaite, rutile and octahedrite

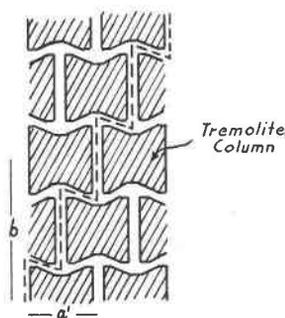


FIG. 2. Basal projection of tremolite columns. The $\{110\}$ cleavage path is indicated.

give excellent agreement. The form $\{111\}$ of spinel has the highest cleavability but probably due to a poor reflecting surface the observed cleavage instead of being good is imperfect; this is in marked contrast to the effect of surface on the $\{211\}$ form of calcite (see Table 25). The agreement of kyanite, topaz and melilite is excellent. Sodalite, a mineral with a three-dimensional network of tetrahedra, gives very good agreement. It is seen that the cleavage of a mineral is predominately determined by

TABLE 24. RESTRICTED RANGE OF CLEAVABILITY VALUES AND CLEAVAGE

Species	Number of planes	Range of cleavability values	Cleavage
Cristobalite	26	22.2-21.9	None
Tridymite	26	22.0-20.4	Not distinct
Phenacite	20	20.4-18.9	Poor
Quartz	26	18.7-16.2	Difficult
Diopside	12	18.5-16.3	Often only in thin section
Spinel	26	15.3-12.7	Poor
Bromellite	26	12.5-11.1	Doubtful
Corundum	26	12.4- 8.6	None

the relative magnitudes of the calculated cleavabilities for the crystal. In species where pronounced cleavage occurs the calculated cleavability

value for that form is appreciably greater than the values for other forms of the same crystal; an example is furnished by muscovite whose eminent {001} cleavage has a cleavability 200% greater than that of any other form. That the change in observed cleavage from mineral to mineral is not determined by the relative values of the highest calculated cleavabilities is shown by the form {110} of tremolite having the same cleavability as {110} of sodalite, while the observed cleavage for tremolite is highly perfect in marked contrast to the more or less distinct cleavage of sodalite.

If the cleavabilities for a large number of forms were calculated, it is probable that the values would approach an asymptotic lower limit. Such a tendency is shown by the data of Table 23 as for example in the cases of brookite whose last four calculated values are 11.0, 10.6, 10.1, 9.3, and corundum with values of 10.2, 9.5, 9.2, 9.6. Since the cleavability range of a mineral is the interval between the greatest and least of its cleavabilities, these may be replaced by the greatest and least of the calculated values and the approximate range so obtained may be designated as the "range of cleavability values." The lower asymptotic limit being unknown, the significance of this range is to some extent determined by the total number of planes in the forms considered. This number is given in the third column. There seems to be a correlation between the number of planes in a form and the degree of cleavage as evidenced by the excellent cleavage of muscovite, topaz and kyanite, each having two planes in the cleavage form while sodalite and grossularite with twelve planes for {110} have poor cleavage.

It follows that minerals having a restricted range of cleavability values should not show pronounced cleavage on any form. As shown by the minerals listed in Table 24, this is in agreement with the results of observation. The cleavability values given in column three for the forms considered change by only a few per cent. Cristobalite and corundum show no cleavage, while for tridymite, phenacite, quartz, etc., the reported cleavage is poor.

The observational data on hardness (according to the scale of Mohs) is given in column four of Table 23. It is noteworthy that the difference in cleavability between the two end species muscovite and corundum is large, the former having six times the cleavability of the latter, and is correlative with the least and greatest of the hardness values, and further that in a general way the intermediate cleavability and hardness values tend to be inverse to each other. These data suggest that hardness increases as the cleavability decreases. To establish the exact nature of this inverse relationship requires more detailed work.

Departure of the rupture surface from a planar condition with its

accompanying good cleavage results in the irregular surface of conchoidal fracture. As indicated by columns five and six of Table 23, the observed fracture range is uneven to conchoidal with the tenacity uniformly brittle.

TABLE 25. EFFECT OF PLANE OPTICAL SURFACE ON CLEAVAGE

Species	Cleavage form	Cleavability	Surface	Cleavage
Muscovite	{001}	73.1	Exactly plane	Eminent
Calcite	{211}	35.4	Plane	Highly per.
Villiumite	{100}	31.5	Plane	Complete
Topaz	{001}	28.6	Plane	Highly per.
Lime	{100}	17.2	Plane	Complete
Kyanite	{100}	15.1	Plane	Very per.
Periclase	{100}	13.2	Plane	Perfect

The relatively less important component of cleavage is the effect of the optical properties of the cleavage surface. There are listed in Table 25 mineral species whose highest calculated cleavabilities range from 73.1 (muscovite) to 13.2 (periclase), being the maximum and nearly the minimum of Table 23. All these cleavage forms have planar surfaces, with

TABLE 26. CLEAVABILITY AND PARTING

Species	Form	Cleavability	Range of cleavability values of species	Observation
Grossularite	{110}	22.4	22.4-16.5	Uncertain whether cleavage or parting.
Tridymite	{0001}	21.9	22.0-20.4	Parting sometimes observed.
Diopside	{001}	17.8	18.5-16.3	Parting often very prominent.
Corundum	{110}	12.4	12.4- 8.6	Parting often prominent.

that of muscovite known to be exactly so. In strong contrast to the variation in cleavability, the degree of cleavage given in the last column of the table is observed to be much better than average. It is evident that the sequence of cleavabilities for a species need not be the same as that of observed cleavage (cleavability plus optical effect) although in general they correspond.

The cleavabilities of parting forms for grossularite, tridymite, diopside and corundum are given in Table 26. Comparison of the data of

columns three and four show that the cleavabilities for these forms are at or near the maximum limit of the range of cleavability values, indicating that cleavage may frequently occur on such forms rather than parting.

The use of equation 16 gives not only calculated cleavabilities for various forms of the same species but also permits comparing cleavabilities of different minerals. Although restricted in this study to ionic minerals it is probable that the expression for cleavability is of more general scope.

7. CONCLUSIONS

1. The dominant component of the phenomenon of mineral cleavage is cleavability.
2. Ionic cleavability is given by the expression

$$C_{\{hkl\}} = \frac{A_{(hkl)}}{\sum_i n_i s_i \cos \theta_i}$$

3. A limited range of cleavability values indicate absence of cleavage.
4. The higher degrees of cleavage are due to high relative cleavability plus plane optical surfaces.
5. The data suggest that hardness increases as cleavability decreases.
6. High relative cleavabilities for parting forms indicate cleavage rather than parting.