AUTONOMOUS AND SINGULAR NODES
Victor Goldschmidt, *Heidelberg.*[1]

By a *node* we mean the *projection-point* of a crystal face, as shown either on a gnomonic or stereographic diagram. The point represents the *face-normal*, and this in turn the vector, that is, the direction and intensity of the *force of attraction of the particle (Partikelattraktionskraft)*. The point represents the face itself together with all its properties. The broad significance of the word node is of the greatest value to us (1, pp. 57–59) since it links the above conceptions into a single unity in which the most important properties of the crystal converge.

Nodes are designated by the letters: a, b, c, m, p, x, etc. The letter is the arbitrarily chosen proper name of the node; it represents at once the face, the projection-point, the face-normal and the vector, and includes all their properties.

We can distinguish between principal and subordinate, primary and derived, strong and weak nodes. As a vector, a node has direction and counter-direction, pole and counter-pole, plus and minus direction; as a face, it has face and counter-face. In the gnomonic diagram pole and counter-pole coincide.

A zone connects two nodes and at the intersection of two zones lies a node. Vectorially speaking the *zone-plane* is the field of force between two vectors (nodes). In the zone, derived nodes are developed according to the Law of Complication. Derived zones connect primary and subordinate nodes with differentiation of still other nodes according to the Law of Complication. The primary and the derived nodes and zones taken together constitute the *typical form-system* of the crystal species. This is completed by the addition of vicinal and accessory forms to give the *complete form-system* of the crystal species.

The *development of the form-system* (2) of a crystal species is its derivation from the *primary nodes*. The primary nodes from which the development proceeds are at the same time the *principal nodes*, namely, the most important nodes according to size and frequency, as shown by statistical methods.

In exceptional cases it is observed that a subordinate node, according to size and frequency, acts as a primary node by entering

actively and independently into the development of the form-system. In this case we have a weak (subordinate) primary node. In general, the conceptions of principal node and primary node are confluent, and the two can be interchanged without incurring error. With rare exceptions, principal nodes are at the same time primary nodes, and vice versa.

Only a small group of principal nodes controls the development of forms, twinning and regular intergrowths, cleavage, and other physical properties. This small group can be ascertained for each crystal species by a statistical study. The determination of this group of principal nodes is an important task which must be accomplished for each crystal species.

> *Examples:* Calcite: principal nodes $=\dot{p}, b, c, \dot{\phi}', \dot{\delta}$
> Orthoclase: "   " $=P, M, T$
> Quartz: "   " $=r, \rho, b$

The principal nodes determine the crystal system. We read off the elements, which characterize the crystal system, from the arrangement of the principal nodes in the gnomonic diagram. In doing this, it is immaterial which of the principal nodes we use.

> *Example:* We obtain the same regular (cubic) elements from the octahedron as from the dodecahedron. We succeed in determining the crystal system of every well-developed crystal, as some of the principal nodes are always present.

We can distinguish six crystal systems, and define them as follows: A crystal system is the sum total (Inbegriff) of all crystal species with the same number of variable elements (3). We have the following cases:

	Variable elements
Regular system	0
Hexagonal and tetragonal systems	1
Rhombic system	2
Monoclinic system	3
Triclinic system	5

The elements of length $p_0q_0=1$ and the angular elements λ, μ, $\nu = 60°$, $90°$ are constant.

Node and counter-node. In general, every face has its counter face, and thus direction and counter-direction of the (face) normal to the vector are equivalent; node = counter-node. But there are exceptions in which polar opposition exists between face and counter-face; direction and counter-direction, node and counter-

node are not equivalent. The counter-node then enters differently into the development of the form-system, sometimes not at all. When node and counter-node are equal, we speak of homopolarity; when unequal, of heteropolarity. In the latter case we distinguish between a positive and a negative end, and speak of an analogous and an antilogous (antilog) pole.

Example: In zinc blende the opposition between tetrahedron and counter-tetrahedron is strong; in diamond it is weak.

Analogy: The magnet has a positive and a negative end. If the contrast is strong, we speak of strong magnetism; if weak, of weak magnetism.

Polarity has no influence on the elements, and thus it has none on the crystal system but polarity assists in determining the division (1), namely, hemihedrism, tetartohedrism and hemimorphism.
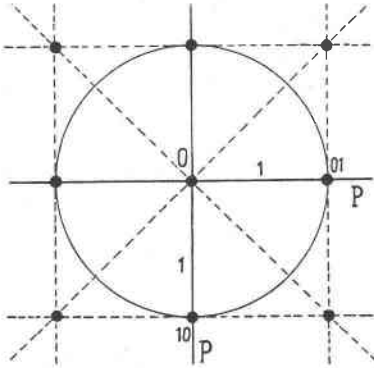

Fig. 1. Regular
24 symmetry fields plus
24 counter-fields.

DIVISION OF GNOMONIC FIELDS

By measurement we determine the crystal system and its elements; these imply a corresponding division into fields in the gnomonic diagram.

Note: We shall restrict ourselves at first to the gnomonic diagram, omitting other representations (stereographic, perspective projection-diagrams, etc.); we employ these when special requirements arise.

Note 1. In the *monoclinic* system we have two equally valid gnomonic normal diagrams:

1. M_1 with projection in the plane $[PQ]$ perpendicular to the prism zone (Fig. 5). We call this the *normal projection*, with the projection elements: $p_0'q_0'h = 1$; $e' = e/h = \cot\mu$.

2. M_2 with projection in the plane $[P\ R]$ ($\perp Q$). We call this *projection on the symmetry plane* (0∞), with the projection elements: $p_0\ r_0$ ($q_0 = h = 1$)μ.

Note 2. In the *rhombic* system we have three equally valid gnomonic diagrams: R_1 in the plane $\infty 0$; R_2 in the plane 0∞. R_3 in the plane 0.

We call R_3 the *normal projection*; it is determined by the choice of the prism zone. $R_1\ R_2$ are employed occasionally. $R_1\ R_2\ R_3$ together with the counter-faces $R_1\ R_2\ R_3$ constitute the surface of the polar form.
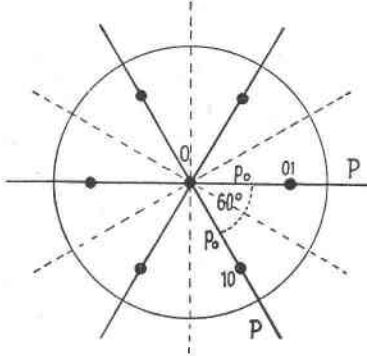

FIG. 2. HEXAGONAL
12 symmetry fields plus
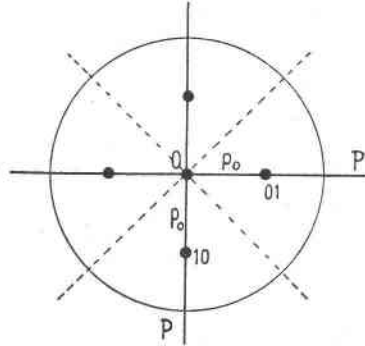12 counter-fields.

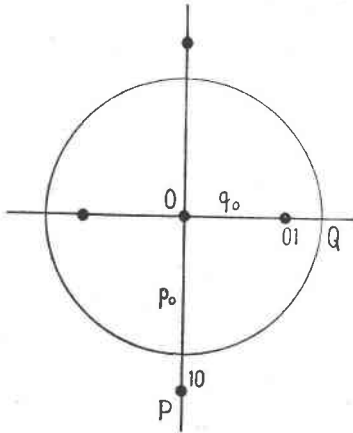FIG. 3. TETRAGONAL
8 symmetry fields plus
8 counter-fields.

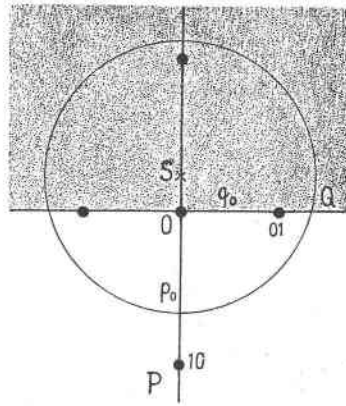

FIG. 4. RHOMBIC
4 symmetry fields plus
4 counter-fields.

FIG. 5. MONOCLINIC (M_1)
2 pairs of symmetry fields plus
counter-fields.

Symmetry of the elements. Symmetry of the crystal system.

The elements of each crystal system show a characteristic sym-

metry in space as they do on the gnomonic diagram. We assume (until we have more information) that this symmetry is due to a symmetry in the arrangement of the component parts of the crystal-building particles, and in the forces of attraction of the particles.

By the symmetry of elements we understand, furthermore, equality of their lengths and angles.
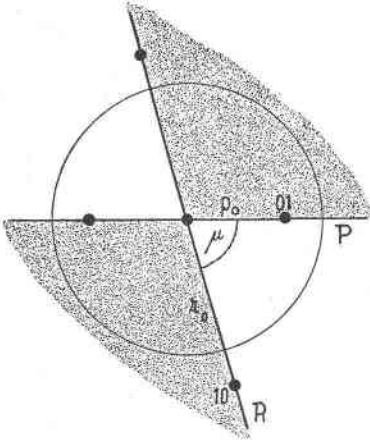

FIG. 6. MONOCLINIC (M_2)
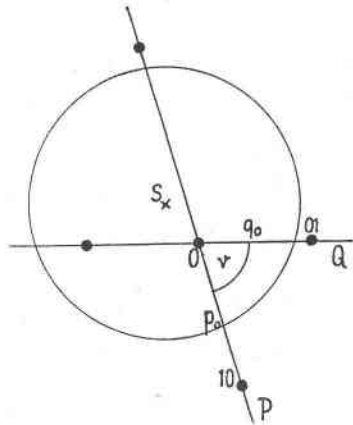2 pairs of symmetry fields plus counter-fields.

FIG. 7. TRICLINIC
4 unsymmetrical fields plus counter-fields.

The symmetry of the polar elements include $p_0 q_0 (r_0 = 1)$ λ, μ, ν. We have the following cases:

Regular: $p_0 = q_0 = r_0 = 1$; $\lambda = \mu = \nu = 90°$.
  Equal: all lengths and angles. Variables: none.
Hexagonal: $p_0 = q_0 \neq r_0 = 1$; $\lambda \mu = 90$, $\nu = 60°$.
  Equal: 2 lengths, 2 angles 90°. Variables: 1 length. 1 angle 60°.
Tetragonal: $p_0 = q_0 \neq r_0 = 1$; $\lambda = \mu = \nu = 90°$.
  Equal: 2 lengths, 3 angles 90°; Variables: 1 length.
Rhombic: $p_0 \neq q_0 \neq r_0 = 1$; $\lambda = \mu = \nu = 90°$.
  Equal: 0 length, 3 angles 90°. Variables: 2 lengths.
Monoclinic: $p_0 \neq q_0 \neq r_0 = 1$; $\lambda \nu = 90°$, $\mu < 90°$.
  Equal: 0 lengths, 2 angles 90°. Variables: 2 lengths, 1 angle.
Triclinic: $p_0 \neq q_0 \neq r_0 = 1$; $\lambda \neq \mu$, $\neq \nu \neq 90°$.
  Equal: 0 length, 0 angle. Variables: 2 lengths, 3 angles.

Symmetrical gnomonic fields, or briefly, gnomonic symmetry. The division into gnomonic fields (Figs. 1–7) shows that for each

crystal we have a definite symmetry of the fields. We distinguish between:

Reflection symmetry: two fields coincide by reflection in a meridian, and rotation symmetry: two fields coincide by rotation about a pole through 30, 45, 60, 90°.

Fields symmetrical by rotation are often also symmetrical by reflection, but not always. Here we consider both types as symmetrical.

Note 1. The symmetry of fields in the regular system does not appear fully in the gnomonic diagram; it does, however, in the projection on the polar form, the cube (Fig. 8). When we speak of symmetrical fields in the regular system, we mean the fields of the polar form.

Note 2: In all the crystal systems, the lower gnomonic plane has the same division into fields as the upper, with the same symmetry of fields.

Number of symmetrical fields: On the upper gnomonic plane we have the following cases:

	Symmetrical fields
Regular.	$24 = 12$ pairs
Hexagonal.	$12 = 6$ pairs
Tetragonal.	$8 = 4$ pairs
Rhombic.	$4 = 2$ pairs
Monoclinic.	$2 = 1$ pairs
Triclinic.	$0 = 0$ pairs

In the paper "*Über Kristallsysteme, deren Definition und Erkennung,*" 1899 (**3**), five different definitions, A, B, C, D, E, were given for the crystal systems; of these D, E, appeared to be the best. They were:

Definition D: A crystal system includes all crystals with the same symmetry of elements.

Definition E: A crystal system includes all crystals with the same number of variable elements.

In addition to the above, we can add a new definition F.

Definition F: A crystal system includes all crystals with the same number of symmetrical gnomonic fields.

Definition F can accompany D and E, or be substituted for them; it has several advantages.

1. F gives precision to the symmetry better than D, and shows the way in which the crystal system is determined on the basis of measurement and projection.

2. The division into fields is the basis for the sub-divisions of the

crystal systems (1), namely, holohedrism, hemihedrism, tetartohedrism and hemimorphism.

Note. Several definitions of the same conception are not objectionable, especially when each definition can be reduced to the other. According to usage, one or the other may be preferred. When considering work of a morphological character, the morphological definition is preferable; when of an optical character, the optical. This is particularly true when the definition, taken rigorously, is only a characterization.

Example. A bird is recognized by its feathers or by its song; a crystal system, by the symmetry of the gnomonic fields or by the number of its variable elements.

Fields of the polar form. The polar form has the half edge-lengths $p_0 q_0 r_0$ and the edge-angles λ, μ, ν. Figures 8–13 give the polar forms of the six crystal systems with the divisions into fields by axes and diagonals.

Symmetrical fields of the polar form. Here, also, we distinguish between reflection and rotation symmetry; reflection symmetry referring to a meridian, rotation symmetry to a pole. We have:

Regular:	3	equal square faces C.
Hexagonal:	1	hexagonal face C; 3 rectangular faces A.
Tetragonal:	1	square face C; 2 rectangular faces A.
Rhombic:	3	rectangular faces C, B, A.
Monoclinic:	2	rectangular faces C, A; 1 parallelogram face B.
Triclinic:	3	parallelogram faces C, B, A.

Each face of a polar form has its division into fields (Figs. 8–13) which are either symmetrical or unsymmetrical. This division into fields is important in the study of the form-system of every crystal species. For our classification it suffices to divide the upper face of the polar form C into fields. Only in the regular system must we consider the three faces C of the polar form to obtain the 24 fields, of which only 8 have the same gnomonic shape, although we may call them all symmetrical. By symmetrical we mean those that are symmetrical on the polar form (cube).

EQUIVALENCE

Under equivalent formations (Gebilde) we would include fields, zones, *Züge*, and nodes. The conception of equivalence was first introduced in 1899 in the following manner. We read (**1**, pp. 60–61, *trans.*):

Let *equivalent* (*äquivalente, gleichwertige*), points be points of equal effect, as points of similar face position, of equal intensity, of equal relation to the form-development, of equal cohesion, of equal refractivity, of like position of the ray in the structure of the particle, and of equal atomic (molecular) position in the particle.
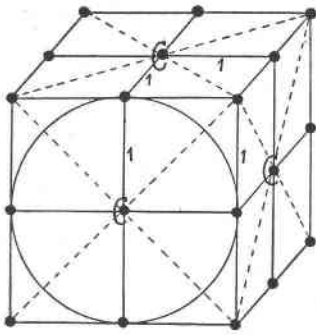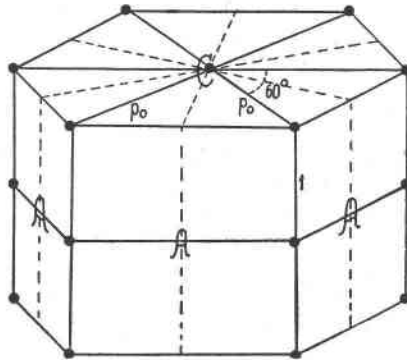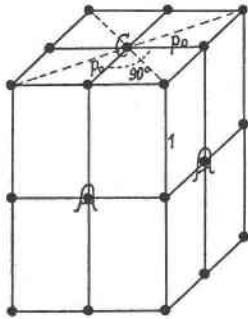
FIG. 8. REGULAR

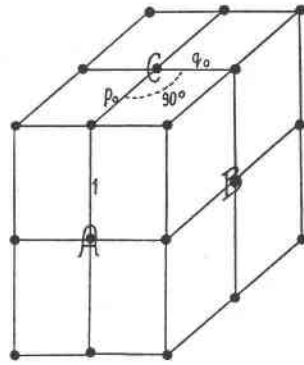FIG. 9. HEXAGONAL

FIG. 10. TETRAGONAL
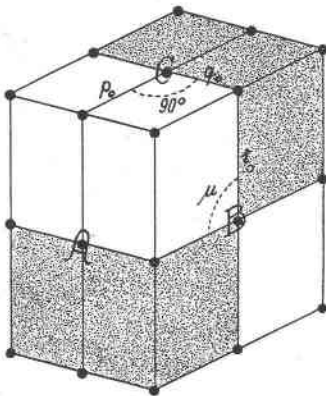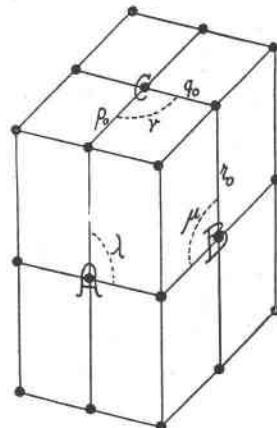

FIG. 11. RHOMBIC

FIG. 12. MONOCLINIC

FIG. 13. TRICLINIC

All these properties are causally connected, so that equality with respect to one of them carries equality with respect to the others. Where this is not the case, equivalence must be specified with respect to a certain property.

Example: Crystallometrically equivalent points, *i.e.* equivalent with respect to position of the faces, are optically equivalent; but the reverse does not hold true.

Equivalent zones (Züge) are those that include equivalent points and equivalent zones connect equivalent nodes.

Let *equivalent fields* be fields of equal shape, bounded by equivalent points (nodes) and zones in the same succession.

In them there takes place at the same time the same processes of complication in the forming of faces, consequently the same distribution of points.

As equivalent fields are crystallometrically, physically and genetically equal in value, we may say: *equivalent fields are fields of equal effect bounded by zone-lines.* A field could also be bounded by lines other than zone-lines.

The Law of Equivalence. The repetition of equivalent points on one crystal is a peculiarity of the crystal. This we shall call the Law of Equivalence; it is the cause of symmetry. A point is, almost without exception, accompanied by one or more equivalents—crystallometrically up to 48 optically entire zones (*Züge*). The repetition of equivalent points causes the repetition of equivalent *Züge*, zones and fields.

The Law of the Complete Form (Gesamtform). The Law of Equivalence appears crystallometrically as the Law of the Complete Form. According to the law that like cause has like effect, faces appear at the same time on a crystal everywhere where the force relations are the same, that is at equivalent points. Therefore, faces do not appear singly but associated in groups. Such groups of associated faces we term a complete form, which we may define as follows: *A complete form (Gesamtform) is a group of associated faces, due to equivalence, and therefore occurring together.*

Division of the crystal systems according to equivalence of fields. We divide the fields, as we did above, according to the elements. We may now express the definition of the divisions so that it has crystallometric, and at the same time, physical and genetic significance. We can say:

Holohedrism is equivalence of all the fields.

Hemimorphism: Upper half of the crystal is not equivalent to the lower half.

Hemihedrism is equivalence of one-half of the fields.

Tetartohedrism is equivalence of one-fourth of the fields.

For the kinds of hemihedrism we have the following grouping:

$$A \begin{cases} F = C, \text{ that is field and counter-field are equivalent.} \\ F \neq C, \text{ that is field and counter-field are not equivalent.} \end{cases}$$

$$B \begin{cases} \text{Equivalent fields alternating in quadrants,} \\ \text{Equivalent fields alternating in pairs,} \\ \text{Equivalent fields alternating singly.} \end{cases}$$

On account of the context, it seemed proper to insert here what was said over thirty years ago (1899). To this I shall add: All

equivalent formations (Gebilde) are symmetrical, but symmetrical formations are not all equivalent.

AUTONOMOUS NODES

The conception of *autonomous* is to be considered as closely related to the term primary. The difference is as follows: primary implies that the development starts from the node; autonomous implies that the node plays an active, independent part in the development. Of the autonomous nodes of a crystal species, one group may, on occasion, initiate the development (that is may be primary); at another time it may be another group. The primary nodes determine the habit.

> *Example:* In alum the octahedron (*p*) and the cube (*c*) are autonomous; but sometimes the development proceeds from the node *p*, and then *p* is primary and determines the habit; sometimes it proceeds from *c*. Thus *p* or *c* may be primary and determinative of the habit and development; both are autonomous.

Autonomous nodes are those making independent, active entry into the form development. Several autonomous nodes may be equivalent. According to their size, direction and symmetry, they determine both the crystal system and division, also the form development and elements of the crystal species.

Each autonomous node has its own peculiarity which is shown by the peculiar way in which each enters actively into the form development. This entry may be one-sided, as for example in quartz (**1,** p. 64).

The search for autonomous nodes and the study of their peculiarities is an important problem of crystal morphology.

Autonomous nodes and the *structure of the particle* are closely related. From the arrangement and peculiarity of autonomous nodes we may infer a peculiarity in the structure of the particle, and, even in the arrangement and peculiarity of the component parts of the particle.

A more detailed exposition is to be found in the author's publication: "Über Entwicklung der Kristallformen" (**2**). There the following hypothesis was introduced: The straight lines from the center of gravity of the particle to the centers of gravity of the individual components of mass (atoms, molecules) composing the particle, are preferred directions.[2] This rests on the hypothesis introduced in

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[2] The context suggests that these "preferred directions" are also the directions given by the autonomous nodes.—*Translator's note.*

1886 (**5,** p. 6) which stated that every plane normal to an attraction force of the particle is crystallonomically a possible face.

Both hypotheses have been repeatedly strengthened and confirmed during the thirty years' development of crystal morphology, and could, therefore, be included in the laws. They open a way of penetrating into the structure of the particle with the help of morphology.

Autonomous nodes and symbols. The elements and the symbols of the single faces depend on the autonomous nodes, since the zones, in which the typical forms are developed according to the Law of Complication, extend from the autonomous nodes. With rare exceptions, autonomous nodes themselves can be expressed in simple symbols, and on this simplicity depends the simplicity of the symbols of the derived forms.

SINGULAR NODES

In addition to the autonomous principal nodes which determine the elements, and thereby the symbols, of the individual forms of a crystal species, and which themselves have simple symbols, autonomous nodes with complicated symbols occur in some crystal species. These we shall call singular nodes with the following definition:

Singular nodes are autonomous nodes with complicated symbols.

Through zonal connection with other nodes, these singular nodes are accompanied by forms having very complicated symbols when referred to the elements of the crystal—so complicated that, in spite of their rational derivation, we hardly like to call them rational.

Series of forms derived from sigular nodes are simple and intelligible only when referred to the appropriate single nodes as points of origin of the form development in the zones.

Such a case is found in calaverite in which occurs a pair of singular nodes C with the symbol $C = -3/29 \cdot 5/29\ (\overline{3}.5.29)$; these enter essentially into the development of the form-system by acting as end-nodes in the zonal development. Details will be given in a forthcoming publication on calaverite from Cripple Creek by V. GOLDSCHMIDT, C. PALACHE AND M. PEACOCK. With the help of the singular nodes C it was possible to understand and derive the form-system of calaverite.

The case is not an isolated one. In wolfsbergite (guejarite) the

form $p = \frac{7}{6} \cdot \frac{3}{2}$ (796) which, with complicated symbol, enters autonomously into the form development, is to be regarded as a singular node (**6,** p. 295).

It is to be expected that singular nodes are not rare. Their determination and the study of their peculiarity and their mode of action might explain some appearances in the development of the forms of crystals hitherto not understood or not observed.

Singular nodes and structures of the particle. Like all autonomous modes, singular nodes are related to the structure of the particle, and may be called upon to give information on this structure. This gives them particular importance. Every singular node corresponds to a singular property in the structure of the particle.

REFERENCES

(1) V. GOLDSCHMIDT: Über Abteilungen der Kristallsysteme, *Zs. f. Krist.*, **32,** pp. 49–65, 1899.

(2) ——— Über Entwicklung der Kristallformen, *ibid*, **28,** 1–35, 414–451, 1897.

(3) ——— Über Kristallsysteme, deren Definition und Erkennung, *ibid*, **31,** pp. 135–159, 1899.

(4) ——— Über Verknüpfung der Kristallpartikel, *ibid*, **29,** pp. 38–53, 1898.

(5) ——— *Index der Kristallformen*, **1,** 1886.

(6) E. ERNEST: Die Kristallformen des Wolfsbergits, *N. Jb. f. Min., Bl. Bd.* **LXI,** A, pp. 275–315, 1928.

PROCEEDINGS OF SOCIETIES

PHILADELPHIA MINERALOGICAL SOCIETY

Academy of Natural Sciences of Philadelphia, November 6, 1930.

A stated meeting of the Philadelphia Mineralogical Society was held on the above date, Mr. Toothaker presiding. There was an attendance of sixty-five persons. Upon favorable recommendation of the council, Mr. George Wilkes was elected a junior member. A committee was appointed by the chair to formulate resolutions on the death of Dr. J. B. S. Egee, one of the oldest members of the society.

Mr. Lester W. Strock spoke of a trip to "Northern Ontario and the Canadian Rockies." Mineral localities visited included Bedford, New York; Portland, Connecticut; Westfield, Massachusetts; Chester, Vermont; Grenville, Perkins Mills, and Cantley, Quebec; Timmons and various mines near Kirkland Lake, Cobalt, Ontario; and Noranda in western Quebec. Common ore minerals were obtained at these localities, but no fine crystallized specimens. The speaker concluded his remarks with an account of a trip to the Columbian icefield. A series of lantern slides was shown. A vote of thanks was given the speaker.

LESTER W. STROCK, *Secretary*