TWO-CIRCLE AND THREE-CIRCLE CO-ORDINATE ANGLES

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In the course of a study of calaverite it was found necessary to compare G. F. Herbert Smith's[2] three-circle measurements with other series of measurements made on two-circle instruments. The co-ordinate angles obtained on the two types of goniometers are not directly comparable; but the relations, when found, are simple and either set of angles can be rapidly converted into the other by short logarithmic operations. The necessity of making such conversions may not occur again, but since the relations of the two types of measurements have not been emphasized before the present note may have theoretical interest.

The principle and method of use of Smith's three-circle goniometer are explained in papers in English and German,[3] and the Goldschmidt two-circle goniometer is widely familiar. The following two figures will therefore suffice to show the relations of the two sets of angles in the case, which will be the usual one, where the same zone of reference has been chosen in both modes of measurement. Figure 1 is a stereographic projection on a plane normal to the axis of the zone of reference. Y is the chosen origin-face of the three-circle measurements, and a face-pole P is defined by the angles B and A,[4] respectively, the azimuth and polar distance from Y as pole and the great circle in the plane of the projection as prime meridian. The same face-pole P is defined by the two-circle angles V and ρ, respectively, the azimuth and polar distance from Z as pole and the great circle through Y as prime meridian.

Figure 2 shows the relations in three dimensions. The crystal centre is at C, and CY, CX lie mutually at right angles in the plane normal to the axis of the zone of reference; CY is normal to the origin-face of the three-circle measurements. ZX′, ZY′, respective-

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[2] *Min. Mag.*, **XIII**, pp. 128–133, 1902.

[3] *Min. Mag.*, **XII**, pp. 175–182, 1899; *Zeits. Krist.*, **XXXII**, pp. 209–216, 1900; *Min. Mag.*, **XIII**, pp. 75–76, 1901.

[4] These are named ϕ and ρ in Smith's discussions; but to avoid confusion with the generally accepted significance of ϕ and ρ in two-circle goniometry the letters used by Smith to denote the circles on his instrument are here used for the angles read on those circles.

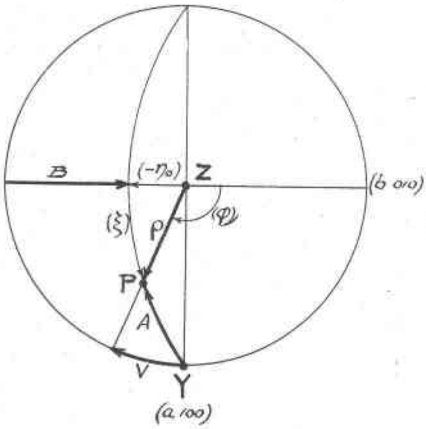

Figure 1

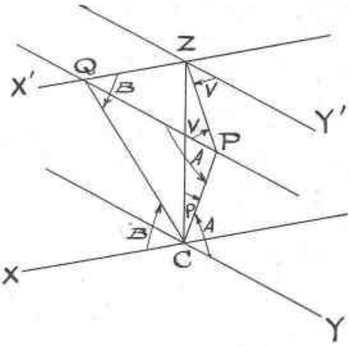

Figure 2

ly, parallel to CX, CY, lie in the gnomonic plane tangent at Z to the sphere with centre C and radius unity. P is the gnomonic face-pole defined by the angles B, A, and by V, ρ. QP, parallel to ZY′, is the projection of the zone containing the face-pole P and the origin-face Y. QZ, QP are the linear co-ordinates of P on the gnomonic plane; by expressing these co-ordinates separately in terms of B, A and V, ρ, and equating the values, B and A can be expressed in terms of V and ρ.

The following triangles are right-angled as indicated:

CZQ	right-angled at	*Z*
CQP	"	*Q*
CZP	"	*Z*
ZQP	"	*Q*

From these triangles:

$$QZ = \cot B \quad \ldots\ldots (1)$$ [5]

$$QP = \cot A \cdot QC$$

$$= \cot A.\ \mathrm{cosec}\ B \quad \ldots\ldots (2)$$ [5]

Again:

$$QZ = \sin V.\ \tan \rho \quad \ldots\ldots (3)$$

$$QP = \cos V.\ \tan \rho \quad \ldots\ldots (4)$$

Hence:

$$\tan V = \tan A.\ \cos B \quad \ldots\ldots (5)$$

$$\cot \rho = \tan B.\ \sin V \quad \ldots\ldots (6)$$

[5] These are essentially the equations given by Smith for plotting three-circle measurements on the gnomonic plane (*Min. Mag.*, **XIII**, p. 312 and fig. 1, 1903).

Thus B and A can be converted in V and ρ, and similar equations for the reverse process can be readily derived.

In the general case B and A each range through two-right-angles while V will range from zero to 360° and ρ from zero to 90°. If the origin-face of the three-circle measurements is the side-pinacoid then V, as obtained above, gives a direct measure of ϕ when V is between 0° and 180°, and $(360° - V) = -\phi$ when V is between 180° and 360°. When a face in the prismatic zone other than the side-pinacoid is the origin-face of the three-circle measurements, V, as derived from B and A, is subject to a systematic addition or subtraction of the angle between the origin-face and the side-pinacoid to give ϕ. In the case of a monoclinic mineral with an orthodome zone of prismatic development, or a triclinic mineral, terminations of opposite ends may have been measured by the observers using the two different instruments; this was the case in the calaverite measurements. By changing the sign of V, as derived from B and A, and correcting for zero, the two sets of measurements were made comparable.

By means of the third circle C a new pole in the zone of reference can be brought to the position of origin-face in the course of three-circle measurements. Such a movement represents a shift of the two-circle prime meridian, affecting V directly by the amount of movement of C.

In the special case where the origin-face of the three-circle measurements is the front pinacoid, in any system except the triclinic, B and A are directly related to Goldschmidt's auxiliary angles η_0 and ξ, as shown in brackets in figure 1.

When B is acute:	$(90 - B) = -\eta_0$
" B " obtuse:	$(B - 90°) = \eta_0$
" A " acute:	$(90° - A) = \xi$
" A " obtuse:	$(A - 90°) = -\xi$
And	$(90° + V) = \phi$

This case is realized in Smith's[6] measurements of krennerite, from which the following angles are taken. The angles η_0 and ξ are taken from Goldschmidt's *Winkeltabellen*. Krennerite is orthorhombic, and therefore the signs of η_0 and ξ can be neglected.

[6] *Min. Mag.*, **XIII**, pp. 266–267, 1903.

	Smith, 3-circle angles; $a(100)$ as pole.				Goldschmidt, 2-circle angles; $c(001)$ as pole.	
	B Azimuth	A Distance	$-\eta_0=$ $(90°-B)$	$\xi=$ $(90°-A)$	η_0	ξ
t 121	44° 35½′	69° 11½′	45° 24½′	20° 48½′	45° 24′	20°45′
o 111	63 07	64 17	26 52½	25 43	26 53	25 42