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Distribution of REE between amphibole and pyroxenes in the lithospheric mantle: An assessment from the lattice strain model

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Appendix A. Parameters in the lattice strain models for REE partitioning between amphibole, pyroxenes, and silicate melts.

Appendix B. Parameters in the semi-empirical models for amphibole temperature

Appendix C. Online Supplementary Text S1–S2, Table S1–2, and Figures S1–S24.

Appendix D. Excel calculator for pyroxene-amphibole REE partition coefficients.
Abstract

Amphibole and pyroxenes are the main reservoirs of rare earth elements (REEs) in lithospheric mantle that has been affected by hydrous metasomatism. In this study, we developed semi-empirical models for REE partitioning between orthopyroxene and amphibole and between clinopyroxene and amphibole. These models were formulated on the basis of parameterized lattice strain models of mineral–melt REE partitioning for orthopyroxene, clinopyroxene, and amphibole, and they were calibrated using major element and REE data of amphibole and pyroxenes in natural mantle samples from intraplate setting. The mineral–melt REE partitioning models suggest that amphibole is not in equilibrium with coexisting pyroxenes in the mantle samples, and that the amphibole crystallized at a lower temperature than that of the pyroxenes. We estimated the apparent amphibole crystallization temperature using major element compositions of the amphibole and established temperature- and composition-dependent models that can be used to predict apparent pyroxene–amphibole REE partition coefficients for amphibole-bearing peridotite and pyroxenite from intraplate lithospheric mantle. Apparent pyroxene–amphibole REE partition coefficients predicted by the models can be used to infer REE contents of amphibole from REE contents of coexisting pyroxenes. This is especially useful when the grain size of amphibole is too small for trace element analysis.

Keywords: amphibole, clinopyroxene, orthopyroxene, peridotite, pyroxenite, REE partitioning, temperature, mineral composition
Introduction

The lithospheric mantle is composed mainly of nominally anhydrous minerals, including olivine, orthopyroxene, clinopyroxene, spinel, and garnet. Hydrous minerals such as amphibole and phlogopite, are also present in regions that have been affected by modal metasomatism. Amphibole can be formed by the interaction between peridotite and hydrous melts (e.g., Sen and Dunn, 1994; Rapp et al., 1999; Wang et al., 2021) and is stable in the lithospheric mantle at temperatures and pressures up to 1150°C and 3.8 GPa (e.g., Green, 1973; Wallace and Green, 1991; Niida and Green, 1999; Fumagalli et al., 2009; Mandler and Grove, 2016). Under these conditions, particularly within the spinel stability field, amphibole and pyroxene are the main reservoirs of incompatible elements. The trace element contents of pyroxene and amphibole have been used to infer geochemical and petrologic processes that occur in the lithospheric mantle. However, amphibole typically occurs as interstitial grains in mantle rocks. Their small sizes often make trace element analysis challenging, and consequently, concentrations of trace element in amphibole have not been frequently reported alongside those of coexisting pyroxenes (e.g., Liu et al., 2010; Matusiak-Małek et al., 2017; Aradi et al., 2020; Zhang et al., 2022). The objective of the present study is to provide an interim solution to this problem by developing semi-empirical models for the distribution of rare earth elements (REEs) between amphibole and pyroxenes.

In general, partitioning of trace elements between a mineral and its coexisting melt depends on pressure, temperature, and mineral and melt composition. The effects of mineral and melt compositions on REE partitioning between pyroxene and melt and between amphibole and melt have been studied using data from laboratory partitioning
experiments and the lattice strain model (e.g., Klein et al., 1997, 2000; Gaetani et al., 2003; Adam and Green, 2003, 2006; Gaetani, 2004; Sun and Liang, 2012, 2013; Yao et al., 2012; Shimizu et al., 2017). The effects of temperature and mineral composition on REE partitioning between orthopyroxene and clinopyroxene have also been investigated using the lattice strain model (e.g., Stosch, 1982; Hellebrand et al., 2005; Witt-Eickschen and O’Neill, 2005; Lee et al., 2007; Witt-Eickschen et al., 2009; Yao et al., 2012; Liang et al., 2013; Sun and Liang, 2014). Yao et al. (2012), Liang et al. (2013), and Sun and Liang (2014) constructed an orthopyroxene–clinopyroxene REE partitioning model from the pyroxene–melt REE partitioning models. Their model suggested that REE partitioning between the two pyroxenes is sensitive to temperature and pyroxene composition and can be used as a thermometer. For two pyroxene-bearing rocks that experienced cooling, the temperatures derived from the REE-in-two-pyroxene thermometer (Liang et al., 2013) are generally higher than temperatures derived from Ca–Mg–Fe based two-pyroxene thermometers (Wells, 1977; Brey and Köhler, 1990; Putirka, 2008) because REEs diffuse more slowly than major elements in pyroxene (Van Orman et al., 2001, 2002; Dimanov and Wiedenbeck, 2006; Cherniak and Liang, 2007; Müller et al., 2013).

Amphibole has more structural sites and a larger range of chemical compositions than pyroxene (e.g., Leake et al., 1997; Hawthorne et al., 2012). The composition of amphibole varies with physical conditions and compositions of the metasomatic melt and peridotite in the lithospheric mantle (e.g., Niida and Green, 1999; Coltorti et al., 2007; Mandler and Grove, 2016; Wang et al., 2021). Variations in the physical conditions and chemical compositions lead to the difference in compositions between amphibole from the supra-subduction zone (S-amphibole of Coltorti et al., 2007) lithospheric mantle, and
amphibole from the intraplate (I-amphibole of Coltorti et al., 2007) lithospheric mantle. In general, the S-amphibole has a higher Mg# \([100 \times Mg/(Mg + Fe), \text{atomic ratio}]\) and lower Ti, Na, and K contents than the I-amphibole (Coltorti et al., 2007) (Fig. 1). According to the lattice strain model of Shimizu et al. (2017), amphibole–melt REE partition coefficients increase with increasing Ti and decreasing Na, K, and Mg contents of amphibole.

REE partitioning between pyroxene and amphibole likely depends on physical conditions and mineral compositions. Klein et al. (1997) compared the parameters of the lattice strain model for clinopyroxene–melt REE partitioning with those for amphibole–melt REE partitioning. They concluded that similarities in the Young’s moduli and between the M4 site in amphibole and the M2 site in clinopyroxene result in nearly identical clinopyroxene and amphibole REE partition coefficients at given physical conditions, and thus subparallel clinopyroxene and amphibole REE patterns. By comparing the REE contents of coexisting clinopyroxenes and amphiboles in mantle xenoliths from West Eifel, Germany, Witt-Eickschen and O’Neill (2005) inferred that clinopyroxene–amphibole REE partition coefficients are controlled by REE ionic radius and Na content of the clinopyroxene. Although significant progress has been made in quantifying REE partitioning in amphibole, there is no predictive model for pyroxene–amphibole REE partition coefficients. In this study, we develop temperature- and mineral composition-dependent models for REE partitioning between clinopyroxene, orthopyroxene, and amphibole in mantle rocks. These semi-empirical models are based on lattice strain models of REE partitioning between pyroxene, amphibole, and silicate melt (Sun and Liang, 2012, 2013;
Yao et al., 2012; Shimizu et al., 2017) and are calibrated using natural amphibole-bearing mantle xenoliths. Because amphibole generally crystallized late than pyroxene in the lithospheric mantle, REE partition coefficients obtained from the semi-empirical models are apparent partition coefficients. Nonetheless, the apparent pyroxene–amphibole REE partition coefficients obtained from our models are especially useful for estimating the REE concentrations of amphibole from the REE contents of pyroxene, especially when the grain size of amphibole is too small for trace element analysis.

**Theoretical basis**

**Parameterized mineral–melt REE partitioning models**

The theoretical basis of this study is the lattice strain model for partitioning of a REE $i$ between a mineral and its coexisting melt (Brice, 1975; Blundy and Wood, 1994; Wood and Blundy, 1997, 2003), which can be written as

$$D_{i}^{\text{min-melt}} = D_0 \exp\left(-\frac{4\pi EN_A}{RT}\left[\frac{r_0}{2}(r_0 - r_i)^2 - \frac{1}{3}(r_0 - r_i)^3\right]\right), \quad (1)$$

where $D_0$ is the partition coefficient for strain-free substitution; $r_0$ is the optimum radius for the lattice site; $E$ is the apparent Young’s modulus for the lattice site; $r_i$ is the ionic radius of the REE; $T$ is the temperature in K; $R$ is the gas constant; and $N_A$ is Avogadro constant. In general, the lattice parameters $D_0$, $r_0$, and $E$ depend on temperature, pressure, and mineral and melt compositions.

Parameterized lattice strain models for amphibole–melt, orthopyroxene–melt, and clinopyroxene–melt REE partitioning were developed by Shimizu et al. (2017), Yao et al. (2012), and Sun and Liang (2012), respectively. These models show that mineral–melt REE partition coefficients depend on temperature and mineral chemistry. With decreasing
temperature, mineral–melt REE partition coefficients increase. In the amphibole model of Shimizu et al. (2017), $D_0$ is positively correlated with Ti in amphibole but negatively correlated with Mg, Na, and K in amphibole, $r_0$ is negatively correlated with the ferromagnesian content of the M4 site in amphibole ($X_{Fm}^{M4}$); and $E$ is a constant. In the low-Ca pyroxene model of Yao et al. (2012), $D_0$ is positively correlated with Ca content of the M2 site ($X_{Ca}^{M2}$) and Al content of the tetrahedral site ($X_{Al}^{T}$) in the pyroxene, and in the clinopyroxene model of Sun and Liang (2012), $D_0$ is positively correlated with $X_{Al}^{T}$ and Mg content of the M2 site ($X_{Mg}^{M2}$) in the pyroxene. $r_0$ and $E$ in the two pyroxene models also depend on pyroxene composition. The lattice strain parameters of the three mineral–melt REE partitioning models are summarized in Appendix A.

The amphiboles used to calibrate the partitioning model of Shimizu et al. (2017) were produced at 780–1100°C and 0.2–2.5 GPa through laboratory experiments. Oxygen fugacities of these experiments are -2 to +3.2 logarithmic unit from the quartz-fayalite-magnetite buffer, which covers a large range of those calculated for spinel peridotite from different tectonic settings (Frost and McCammon, 2008). The experimental amphiboles contain 0.73–6.35 wt% TiO$_2$, 1.20–4.04 wt% Na$_2$O, and 0.03–2.77 wt% K$_2$O, and have Mg#s ranging from 36 to 100 (Fig. 1). Their compositions overlap with a majority (81%) of I-amphiboles but only a small portion (15%) of S-amphiboles from mantle rocks reported in the literature. The latter have lower TiO$_2$ and Na$_2$O, but higher SiO$_2$ and Mg# than amphiboles from the partitioning experiments (Figs. 1a–1c). The Mg#s of clinopyroxenes and low-Ca pyroxenes from the partitioning experiments for calibrating the parameterized models of Sun and Liang (2012, 2013) and Yao et al. (2012) are 54–100 and 70–100, respectively, covering the main range of
pyroxene compositions in mantle rocks.

**Pyroxene–amphibole REE partitioning models**

When two minerals (α and β) and the melt are in thermodynamic equilibrium, it is possible to calculate the mineral–mineral REE partition coefficients by taking the ratio of the two mineral–melt partition coefficients, viz.,

$$D_{i}^{\beta-\alpha} = \frac{D_{i}^{\beta-\text{melt}}}{D_{i}^{\alpha-\text{melt}}} = \frac{D_{0}^{\beta}}{D_{0}^{\alpha}} \exp \left\{ -\frac{4\pi E_{\beta}N_{A}}{RT} \left[ \frac{r_{0}^{\beta}}{2} \left( r_{0}^{\beta} - r_{i} \right)^{2} - \frac{1}{3} \left( r_{0}^{\beta} - r_{i} \right)^{3} \right] + \frac{4\pi E_{\alpha}N_{A}}{RT} \left[ \frac{r_{0}^{\alpha}}{2} \left( r_{0}^{\alpha} - r_{i} \right)^{2} - \frac{1}{3} \left( r_{0}^{\alpha} - r_{i} \right)^{3} \right] \right\}. \quad (2)$$

Yao et al. (2012), Liang et al. (2013), and Sun and Liang (2014) demonstrated that Eq. (2) reproduces the measured orthopyroxene–clinopyroxene REE partition coefficients in well-equilibrated peridotites and can be used under magmatic and subsolidus conditions.

The REE-in-two-pyroxene thermometer of Liang et al. (2013) was developed by rearranging Eq. (2). Temperatures obtained from the REE thermometer ($T_{\text{REE}}$) for well-equilibrated peridotites are similar to those obtained from major element-based pyroxene thermometers such as those of Brey and Köhler (1990), Wells (1977), and Putirka (2008) (gray fields in Fig. 2).

Applying Eq. (2) to the mineral pairs of pyroxene and amphibole, we have the orthopyroxene–amphibole and clinopyroxene–amphibole REE partitioning models:

$$D_{i}^{\text{pyx-amp}} = \frac{D_{i}^{\text{pyx-melt}}}{D_{i}^{\text{amp-melt}}} = \frac{D_{0}^{\text{pyx}}}{D_{0}^{\text{amp}}} \exp \left\{ -\frac{4\pi E^{\text{pyx}}N_{A}}{RT} \left[ \frac{r_{0}^{\text{pyx}}}{2} \left( r_{0}^{\text{pyx}} - r_{i} \right)^{2} - \frac{1}{3} \left( r_{0}^{\text{pyx}} - r_{i} \right)^{3} \right] + \frac{4\pi E^{\text{amp}}N_{A}}{RT} \left[ \frac{r_{0}^{\text{amp}}}{2} \left( r_{0}^{\text{amp}} - r_{i} \right)^{2} - \frac{1}{3} \left( r_{0}^{\text{amp}} - r_{i} \right)^{3} \right] \right\}, \quad (3)$$

where pyx is either orthopyroxene (opx) or clinopyroxene (cpx), and the lattice strain parameters are presented in Appendix A. Eq. (3) is valid so long as pyroxene and
amphibole are in chemical equilibrium.

Major element and REE data of pyroxenes and amphibole in amphibole-bearing mantle rocks from the literature can be used to test the pyroxene-amphibole REE partitioning models (e.g., Gregoire et al., 2000; Witt-Eickschen and O’Neill, 2005; Ishimaru et al., 2007; Xu et al., 2010; Bénard et al., 2013, 2021; Zhou, 2014; Pintér et al., 2015; Matusiak-Małek et al., 2017; Aradi et al., 2020; Belousov et al., 2021; Nishio et al., 2022). In order to apply these models, a mantle sample must meet the following two requirements: (1) amphibole and pyroxene compositions are within the calibration ranges of the mineral–melt partitioning models, and (2) amphibole and pyroxenes crystallized from the same melt, i.e., formed from the same metasomatic event. The calibration ranges of pyroxene-melt partitioning models of Sun and Liang (2012, 2013) and Yao et al. (2012) cover pyroxene compositions of mantle rocks, including the collected amphibole-bearing samples. Twenty-eight of the amphibole-bearing mantle rocks from the literature cited above have amphibole compositions within the calibration range of amphibole–melt partitioning model of Shimizu et al. (2017). They include 26 samples with I-amphibole from Western Pannonian Basin, Hungary (Aradi et al., 2020), Nyos Lakes, Cameroon (Pinter et al., 2015), Wilcza Góra, Southwestern Poland (Matusiak-Małek et al., 2017), Huadian, northeastern China (Xu, unpublished), and 2 samples with S-amphibole from Laiwu, North China Craton (Zhou, 2014). To check if requirement (2) is met, we examined the texture and mineral chemistry for the 28 samples (See Text S1 of online Appendix C for details.) Among these samples, 2 peridotites with I-amphibole from Western Pannonian Basin and the 2 websterites with S-amphibole from Laiwu show significant disequilibrium textures. Additionally, their constituent pyroxenes have
significantly higher Mg#s (by 3.5–8.1 units) than amphibole Mg#s (Fig. 3a, Table S1). These samples are excluded from the test. The remaining 24 samples all contain I-amphibole. Their orthopyroxene Mg#s are similar to the clinopyroxene Mg#s, but both pyroxene Mg#s are slightly higher (by 1.0–3.0 units) than amphibole Mg# (Fig. 3b, Table S1). Figure 2 compares the temperatures calculated using the REE-in-two-pyroxene thermometer ($T_{REE}$, Liang et al., 2013) with those calculated using the major-element based two-pyroxene thermometers of Brey and Köhler (1990, $T_{BKN}$), Wells (1977, $T_{W77}$), Putirka (2008, his Eq. (37), $T_{P37}$), and average of the three temperatures ($T_{avg}$) for the 24 amphibole-bearing mantle rocks. Temperatures of these samples are mostly within the range of well-equilibrated mantle xenoliths, suggesting that the REEs (at least the heavy REEs) and major elements in orthopyroxene are in equilibrium with those in clinopyroxene (Liang et al., 2013).

Figure 4a shows the inversion diagram based on the REE-in-two-pyroxene thermometer of Liang et al. (2013) for an amphibole-bearing lherzolite from Wilcza Góra, SW Poland (Matusiak-Malek et al., 2017, sample WLK30). Using the inverted temperature ($T_{REE} = 1010 \pm 14^\circ$C), the mineral–mineral REE partitioning model of Eq. (2) well reproduced orthopyroxene–clinopyroxene partition coefficients for most of the REEs (Fig. 4b). The exceptions are La and Ce, which may be attributed to their higher closure temperatures (Liang, 2015). Interestingly, the pyroxene–amphibole REE partition coefficients calculated using Eq. (3), $T_{REE}$, and major element compositions of the amphibole and pyroxenes differ markedly from the measured values (Fig. 4c and 4d). The isotherms established by the orthopyroxene–amphibole REE partitioning model indicate that orthopyroxene and amphibole would be equilibrated at a lower temperature (between
750°C and 925°C, Fig. 4c). In addition, the model-derived clinopyroxene–amphibole REE partition coefficients are well above the measured values, suggesting REEs in amphibole and clinopyroxene are not in equilibrium at any temperature (Fig. 4d). A likely explanation is that amphibole crystallized at a temperature lower than $T_{REE}$, and that chemical equilibria between amphibole and the pyroxenes have not been established. This is consistent with the observation from hydrous melt–peridotite reactive crystallization experiments of Wang et al. (2021), where the interstitial and overgrowth occurrences of amphibole in the product peridotite and pyroxenite suggest a later crystallization of amphibole than pyroxenes. Below, we present a semi-empirical model that can be used to estimate the crystallization temperature of amphibole in mantle rocks.

**Semi-empirical models for crystallization temperature of amphibole**

Since the amphibole crystallizes later than the pyroxenes, the pyroxene–melt and amphibole–melt REE partition coefficients are defined at their crystallization temperatures ($T_{pyx}$ and $T_{amp}$, respectively). Taking the ratio of $D_{pyx\text{-melt}}$ at $T_{pyx}$ and $D_{amp\text{-melt}}$ at $T_{amp}$, we have, from the lattice strain model (Eq. (1)), the pyroxene-amphibole apparent partition coefficient:

$$D_{i}^{pyx\text{-amp}} = \frac{D_{i}^{pyx\text{-melt}}}{D_{i}^{amp\text{-melt}}} = \frac{D_{i}^{pyx\text{-melt}}}{D_{i}^{amp\text{-melt}}} \frac{c_{i}^{melt\text{amp}}}{c_{i}^{melt\text{pyx}}}$$

$$= \frac{D_{0}^{pyx}}{D_{0}^{amp}} \exp \left\{ -\frac{4\pi E^{pyx}_{N_{A}}}{RT_{pyx}} \left[ \frac{r_{0}^{pyx}}{2} \left( r_{0}^{pyx} - r_{i} \right)^{2} - \frac{1}{3} \left( r_{0}^{pyx} - r_{i} \right)^{3} \right] \right\}$$

$$+ \frac{4\pi E^{amp}_{N_{A}}}{RT_{amp}} \left[ \frac{r_{0}^{amp}}{2} \left( r_{0}^{amp} - r_{i} \right)^{2} - \frac{1}{3} \left( r_{0}^{amp} - r_{i} \right)^{3} \right] + \ln \frac{c_{i}^{melt\text{pyx}}}{c_{i}^{melt\text{amp}}}, \quad (4)$$

where $melt_{pyx}$ and $melt_{amp}$ are melts in equilibrium with pyroxene and amphibole,
respectively, and $\frac{C_{i}^{\text{meltpyx}}}{C_{i}^{\text{meltamp}}}$ is the ratio of melt REE concentrations at the two equilibrium temperatures. Given the generally small amount of amphibole crystallized after pyroxenes in the mantle samples and the moderate incompatibility of REEs in amphibole (partition coefficients mostly 0.1–1) at a range of temperatures (780–1100°C) (Shimizu et al., 2017), we infer that the ratio $\frac{C_{i}^{\text{meltpyx}}}{C_{i}^{\text{meltamp}}}$ is on the order of one, and thus $\ln \frac{C_{i}^{\text{meltpyx}}}{C_{i}^{\text{meltamp}}}$ is negligible compared to contributions from lattice strains in pyroxene and amphibole. Hence, Eq. (4) can be simplified as:

$$D_{i}^{\text{pyx-amp}} = \frac{D_{0}^{\text{pyx}}}{D_{0}^{\text{amp}}} \exp \left\{ -\frac{4\pi E_{\text{pyx}} NA}{R T_{\text{pyx}}} \left[ \frac{r_{i}^{\text{pyx}}}{2} \left( r_{0}^{\text{pyx}} - r_{i} \right)^{2} \right] + \frac{4\pi E_{\text{amp}} N A}{R T_{\text{amp}}} \left[ \frac{r_{0}^{\text{amp}}}{2} \left( r_{0}^{\text{amp}} - r_{i} \right)^{2} \right] \right\} , \quad (5)$$

where $T_{\text{amp}}$ is the apparent amphibole–melt REE partitioning temperature or amphibole crystallization temperature while neglecting the difference in melt REE concentration ($\ln \frac{C_{i}^{\text{meltpyx}}}{C_{i}^{\text{meltamp}}}$ in Eq. (4)). Rearranging Eq. (5), we have

$$\ln D_{i}^{\text{pyx-amp}} = A_{i}^{\text{pyx,amp}} + \frac{B_{i}^{\text{amp}}}{T_{\text{amp}}} , \quad (6)$$

The terms $A_{i}^{\text{pyx,amp}}$ and $B_{i}^{\text{amp}}$ are presented in Appendix B. Eq. (6) can be written in the linear form:

$$B_{i}^{\text{amp}} = T_{\text{amp}} \left( \ln D_{i}^{\text{pyx-amp}} - A_{i}^{\text{pyx,amp}} \right) . \quad (7)$$

In the absence of additional information, we can approximate $T_{\text{pyx}}$ using the temperature derived from the REE-in-two-pyroxene thermometer of Liang et al. (2013, $T_{\text{REE}}$). The apparent amphibole temperature can be calculated using an inversion diagram constructed by plotting $\left( \ln D_{i}^{\text{opx-amp}} - A_{i}^{\text{opx,amp}} \right)$ or $\left( \ln D_{i}^{\text{cppx-amp}} - A_{i}^{\text{cppx,amp}} \right)$ against

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The slope of regression line passing through the origin in the diagram is the apparent temperature.

Figures 5a and 5b display the inversion diagrams of the apparent amphibole temperatures for the amphibole-bearing lherzolite shown in Fig. 4 (sample WLK30 from Matusiak-Malek et al., 2017). Using the robust linear least-squares regression method, we fit a line through the origin and data in the diagram, and obtain the temperature from the slope of this line. Similar apparent temperatures are obtained from the orthopyroxene–amphibole REE partitioning data ($T_{\text{amp,opx-amp}} = 856 \pm 10^\circ C$) and the clinopyroxene–amphibole REE partitioning data ($T_{\text{amp,cpx-amp}} = 861 \pm 5^\circ C$). In Figures S1–S24 of online Appendix C, we present the $T_{\text{amp}}$ inversion diagrams for the 24 mantle samples included in this study. For most of the samples, data of light and some middle REEs are off the regression line, and these outliers were excluded from the inversion. Most of these elements are also outliers in the inversion diagrams when applying the REE-in-two-pyroxene thermometer of Liang et al. (2013). This is attributed in part to the low analytical precision for the depleted light REEs (Liang et al., 2013) and their higher closure temperatures (Liang, 2015). Substituting the inverted amphibole temperature ($856^\circ C$ or $861^\circ C$) and $T_{\text{REE}} (1010^\circ C)$ into Eq. (5), we obtain orthopyroxene–amphibole and clinopyroxene–amphibole REE partition coefficients. These partition coefficients reproduce the measured values for sample WLK30 (Figs. 5c and 5d). The inverted amphibole temperatures for the amphibole-bearing mantle xenoliths range from 754$^\circ C$ to 959$^\circ C$, which are 69–205$^\circ C$ lower than $T_{\text{REE}}$ (Figs. S1–S24). The amphibole temperatures inverted from the orthopyroxene–amphibole model ($T_{\text{amp,opx-amp}}$) and those from the clinopyroxene–amphibole modal ($T_{\text{amp,cpx-amp}}$) are generally in agreement with
each other (Figs. S1–S24), suggesting the internal consistency of our semi-empirical models.

**Partition coefficients of REE between pyroxene and amphibole**

*T*-X-dep**endent pyroxene-amphibole REE partition coefficients**

To predict pyroxene-amphibole partition coefficients through temperature (*T*) and mineral major element composition (*X*), we need the amphibole crystallization temperature (*T*<sub>amp</sub>) without relying on its REE data. For this purpose, we fit the inverted *T*<sub>amp</sub> (average value of *T*<sub>amp,opx–amp</sub> and *T*<sub>amp,cpx–amp</sub>) with amphibole major element compositions for the 24 samples using the least squares method, and obtain the empirical expression:

\[
T_{\text{amp}}(°C) = 258(±33)X_{\text{Si}} + 822(±134)X_{\text{Ti}} - 743(±208)X_{\text{Na}} - 507(±213)X_{\text{K}} + 755(±131)X_{\text{Fm}}^{M4} - 273.15.
\]

(8)

The amphibole components are calculated using the method of Shimizu et al. (2017) which is described in Text S2 of online Appendix C. Numbers in parentheses are standard error of the coefficients. We also attempted to include pyroxene components in the regression, such as Al, Mg, Ca, and Na. The prediction is not significantly improved, and large uncertainties are introduced. *T*<sub>amp</sub> calculated using Eq. (8) is generally within ±50°C deviation from the *T*<sub>amp</sub> inverted using the partitioning models of Eq. (7) (Fig. 6a). The calculated *T*<sub>amp</sub> are commonly lower than temperatures calculated from the two-pyroxene thermometers (*T*<sub>REE</sub>, *T*<sub>BKN</sub>, *T*<sub>W77</sub>, and *T*<sub>P37</sub>, Figs. 6b-6f). The empirical Eq. (8) for *T*<sub>amp</sub> allows us to predict *D*<sub>opx–amp</sub> and *D*<sub>cpx–amp</sub> for amphibole-bearing mantle samples using Eq. (5).
Figure 7 compares the predicted pyroxene–amphibole REE partition coefficients with measured values for the amphibole-bearing mantle rocks. $T^{\text{amp}}$ in the models is calculated using Eq. (8). The models are capable of reproducing the partition coefficients of Dy–Lu and Y. The orthopyroxene–amphibole model underestimates some of the La–Tb partition coefficients (Fig. 7a and 7b). Out of 341 orthopyroxene-amphibole REE partition coefficients, seventy-eight (mostly La–Sm) plot below the 1:2 correlation line (Fig. 7a). These outliers are also the outliers on the inversion diagrams for $T_{\text{REE}}$ and $T^{\text{amp}}$ (Figures S1-S24). The clinopyroxene–amphibole model does a better job for reproducing REE partition coefficients for the mantle samples than the orthopyroxene–amphibole model (Figs. 7c and 7d). Out of 356 orthopyroxene-amphibole REE partition coefficients, twenty (mostly La–Nd) plot above the 2:1 correlation line, and 94% of the data plot between the 1:2 and 2:1 correlation lines (Fig. 7c).

**Effect of amphibole crystallization temperature**

We have demonstrated that, in the pyroxene-amphibole REE partitioning models, the amphibole temperature ($T^{\text{amp}}$) is generally lower than $T^{\text{pyx}}$ (Fig. 6). To test the effect of amphibole crystallization temperature on pyroxene–amphibole REE partitioning, we calculated pyroxene–amphibole REE partition coefficients for the lherzolite from Wilcza Góra (Matusiak-Małek et al., 2017, sample WLK30) and a lherzolite from the Western Pannonian Basin, Hungary (Aradi et al., 2020, sample AUB1407), using the models presented above and assuming $T^{\text{amp}}$s that are 50–300°C lower than $T^{\text{pyx}}$ (assigned as $T_{\text{REE}}$). Although major element compositions and modal mineral abundances vary during cooling, no model is available to account for these changes, and we neglected these...
variations for the purpose of this exercise. As shown in Fig. 8, the calculated pyroxene–amphibole REE partition coefficients decrease systematically with increasing deviation of $T^{\text{amp}}$ from $T^{\text{pyx}}$. This is attributed to the negative correlation of amphibole–melt REE partition coefficients with temperature (Shimizu et al., 2017). In addition, with increasing deviation of $T^{\text{amp}}$ from $T^{\text{pyx}}$, the pyroxene-amphibole mid REE partition coefficients decrease more dramatically than heavy and light REE partition coefficients. Since amphibole generally crystallizes after pyroxenes during hydrous melt–peridotite reactions (e.g., Wang et al., 2021), the preceding exercise underscores the importance of crystallization sequence on the REE distributions between pyroxenes and amphibole.

Field applications

Conditions of application and a program for calculation

The semi-empirical partitioning models presented in the preceding sections (Eqs. (5)-(8)) can be used to calculate apparent pyroxene–amphibole REE partition coefficients for amphibole-bearing mantle rocks that have mineral compositions comparable to those from experiments used to calibrate the mineral–melt REE partitioning models (Sun and Liang, 2012, 2013; Yao et al., 2013; Shimizu et al., 2017). The pyroxene–melt models cover the main range of pyroxene compositions in mantle rocks, whereas the amphibole–melt model covers mostly the I-amphiboles (Fig. 1). Hence it is important to check amphibole chemistry before an application. Another caveat is that the constituent amphibole and pyroxenes were formed or affected by the same metasomatic event. This can be assessed from texture and mineral composition (e.g., Mg#s). As we have shown in Fig. 2b, the amphibole Mg# is slightly lower than the pyroxene Mg#s, and the differences
between pyroxene Mg#s and amphibole Mg# in samples for calibration are less than 3
units.

In the online Appendix D, we provide an Excel worksheet that can be used to
calculate pyroxene-amphibole REE partition coefficients for amphibole-bearing mantle
rocks. It requires inputs of pyroxene and amphibole major element compositions and \( T_{\text{pyx}} \).
The pyroxene temperature can be estimated using the REE-in-two-pyroxene thermometer
of Liang et al. (2013). For mantle rocks from well-equilibrated lithospheric mantle, major
element-derived temperatures are similar to \( T_{\text{REE}} \) (Fig. 2). Temperatures calculated using
pyroxene thermometers such as \( T_{\text{BKN}} \), \( T_{p37} \), and \( T_{w77} \) can also be used as \( T_{\text{pyx}} \) when \( T_{\text{REE}} \) is
inaccessible.

**Applications**

To test and validate the pyroxene–amphibole REE partitioning models, we calculate
the REE contents of amphibole-bearing mantle xenoliths and compare them with the
measured amphibole REE contents. The harzburgite xenoliths WG5 and WG6 from
Wilcza Góra, Poland (Matusiak-Małek et al., 2017), consist of coarse grains of olivine,
orthopyroxene (± clinopyroxene), and fine-grained intergranular aggregates. The
aggregates contain secondary amphibole and clinopyroxene (± phlogopite).
Matusiak-Małek et al. (2017) reported major element and REE compositions of the
orthopyroxenes for WG5, clinopyroxenes for WG6, and major element compositions of
amphiboles for both samples. Mg#s in pyroxene and amphibole are similar (91 in
orthopyroxene and 89 in amphibole for WG5, and 88 in clinopyroxene and 85 in
amphibole for WG6). They calculated pyroxene temperatures using the
Al-in-orthopyroxene thermometer of Witt-Eickschen and Seck (1991) for WG5 (990°C) and the REE-in-two-pyroxene thermometer of Liang et al. (2013) for WG6 (1023°C). Using the available data and our models, we obtained $T_{amp}$s for WG5 (888°C) and WG6 (852°C), and calculated pyroxene–amphibole REE partition coefficients and REE contents of the amphiboles. The chondrite-normalized REE patterns are presented in Fig. 9a. The calculated amphibole REE patterns of samples WG5 and WG6 are similar to those of the genetically related samples (Matusiak-Małek et al., 2017). The predicted REE contents of the amphibole in WG5 are higher than those of the other samples, and the predicted REE pattern for the amphibole in WG6 shows less fractionation between light and heavy REEs than those of the other samples (pink field in Fig. 8a, Matusiak-Małek et al., 2017). These are consistent with differences in the REE patterns of the pyroxene in WG5 and WG6, and in the other samples (Figures 6c and 7e in Matusiak-Małek et al., 2017).

Another set of examples are the lherzolite xenoliths KPFS0402 and PST1403 from the Western Pannonian Basin, Hungary (Aradi et al., 2020). KPFS0402 has a protogranular texture and contains 0.2% amphibole, and PST1403 has a porphyroclastic texture with 0.02% amphibole. The amphibole crystals in both xenoliths grew on the rims of clinopyroxene and spinel grains. The two samples have uniform orthopyroxene, clinopyroxene, and amphibole Mg#s (90/89/88 and 91/91/89). Using the reported mineral compositional data, we calculated $T_{REE}$ (1025°C and 1018°C) and $T_{amp}$ (927°C and 929°C) for the two samples, as well as the pyroxene–amphibole REE partition coefficients. The REE contents of amphibole in the two samples calculated using the partition coefficients and the measured clinopyroxene and orthopyroxene REE data are plotted in Fig. 9b. The
amphibole REE patterns are comparable to measured patterns of amphiboles in xenoliths that have similar petrologic features and sample locations (green field in Fig. 8b, Aradi et al., 2020). Amphibole REE patterns calculated from orthopyroxene–amphibole partitioning are perturbed for REEs from La to Gd. This perturbation is in correspondence with those in the orthopyroxene REE patterns (Aradi et al., 2020), which is likely a result of the low concentrations of the light to middle REEs in orthopyroxene relative to the analytical precision.

Recently, Puziewicz et al. (2023) reported two amphibole-bearing lherzolites from Cameroon volcanic line. Sample H71 has a porphyroclastic texture, and small amphiboles (1%) are dispersed among olivine and pyroxene grains. It has similar \( T_{\text{REE}} \) (901°C) and \( T_{\text{BKN}} \) (911°C). Orthopyroxene, clinopyroxene, and amphibole have similar Mg#s (90, 92, and 89). Using the reported mineral major element data and \( T_{\text{REE}} \), we calculated \( T_{\text{amp}} \) (778°C) and pyroxene–amphibole REE partition coefficients. The predicted REE patterns are compared with the measured pattern in Fig. 9c. The models mimic middle to heavy REE pattern of the amphibole, but slightly overestimate concentrations of light REEs. The elevated La, Ce, and Pr in amphibole follow that of orthopyroxene REEs.

The other Cameroon lherzolite (sample H69 from Puziewicz et al., 2023) has a sheared texture, and contains more abundant amphibole (3.9%). The amphibole forms asymmetric aggregates around spinels and unevenly distributed in the sample. \( T_{\text{BKN}} \) (952°C) of this sample is considerably higher than \( T_{\text{REE}} \) (872°C), which suggests a possible heating event after the formation of pyroxenes. \( T_{\text{amp}} \) obtained from Eq. (8) are 652°C. Our models overestimate amphibole REE contents, even for the heavy REEs (Fig.9d). This suggests that Eq. (8) is not capable of estimating amphibole temperature
from compositions of pyroxene and amphibole that experienced heating. $T_{\text{amp}}$ is likely higher than the calculated values since $D_{\text{pyx-amp}}$s increase with decreasing difference between $T_{\text{pyx}}$ and $T_{\text{amp}}$ (Fig. 8). This can be testified by the $T_{\text{amp}}$ inverted using the REE partitioning models of Eq. (7). Both the orthopyroxene-amphibole and clinopyroxene-amphibole models yield an $T_{\text{amp}}$ of 745°C (Fig. 9d).

Summary

Based on the parameterized lattice strain models for pyroxene–melt and amphibole–melt REE partitioning (Sun and Liang, 2012, 2013; Yao et al., 2012; Shimizu et al., 2017), we developed semi-empirical models for REE partitioning between orthopyroxene and amphibole and between clinopyroxene and amphibole. The models were calibrated using 24 amphibole-bearing mantle samples that have major element and REE compositions of pyroxenes and amphibole reported in the literature. These amphibole-bearing mantle rocks have similar REE-derived temperatures (Liang et al., 2013) and major element-derived two-pyroxene temperatures (e.g., Wells, 1977; Brey and Köhler, 1990; Putirka, 2008). However, the mineral–melt REE partitioning models indicate that the amphibole–melt equilibrium temperatures are generally lower than the two-pyroxene temperatures, supporting the experimental observation that amphibole crystallizes later than pyroxene during hydrous melt–peridotite reaction (e.g., Wang et al., 2021). Therefore, we use the term “apparent temperature” for the amphibole–melt equilibrium temperature, and empirically regressed the apparent temperature against amphibole major element compositions (Eq. (8)). The partitioning models (Eq. (5)), combined with the empirical model for the apparent temperature, can be used to predict orthopyroxene–
amphibole and clinopyroxene–amphibole REE partition coefficients for mantle rocks containing I-amphibole (Coltorti et al., 2007).

Implications

The temperature- and composition-dependent REE partitioning models presented in this study make it possible to estimate REE concentrations in amphibole in mantle rocks using REE contents of coexisting orthopyroxene or clinopyroxene. The new partitioning models require major element compositions of amphibole and pyroxenes and the temperature at which the pyroxenes equilibrated. This temperature can be obtained from pyroxene thermometers, including those of Wells (1977), Brey and Köhler (1990), Putirka (2008), and Liang et al. (2013).

The amphibole crystallization temperature has been quantified using thermometers based on amphibole composition, the coexisting melt composition, or a combination of the two (e.g., Ridolfi and Renzulli, 2012; Molina et al., 2015; Putirka, 2016). However, these thermometers were not designed for mantle rocks. The REE partitioning models (Eq. (7)) or the empirical Eq. (8) can be used to invert or calculate amphibole crystallization temperature. Hence, the semi-empirical and empirical models for amphibole crystallization temperature presented in this study have potential implications for constraining the thermal property of the amphibole-bearing lithospheric mantle.

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Appendix A. Parameters in the lattice strain models for REE partitioning between amphibole, pyroxenes and silicate melts

The following equations summarize the parameters used in the lattice strain models for REE partitioning between mantle minerals (amphibole, clinopyroxene, and low-Ca pyroxene) and silicate melts of Eq. (3):

**Amphibole model (Shimizu et al., 2017)**

\[
\ln D_0 = -4.21 + \frac{7.27 \times 10^4}{RT} + 1.52X_{\text{Ti}} - 0.35X_{\text{Mg}} - 1.83X_{\text{Na}} - 2.95X_{\text{K}}, \quad (A1a)
\]

\[
r_0(\text{Å}) = 1.043 - 0.039X_{\text{Fm}}^M, \quad (A1b)
\]

\[
E(\text{GPa}) = 337. \quad (A1c)
\]

**Clinopyroxene model (Sun and Liang, 2012)**

\[
\ln D_0 = -7.14 + \frac{7.19 \times 10^4}{RT} + 4.37X_{\text{Al}}^T + 1.98X_{\text{Ca}}^M - 0.91X_{H_2O}^melt, \quad (A2a)
\]

\[
r_0(\text{Å}) = 1.066 - 0.104X_{\text{Al}}^M - 0.212X_{\text{Mg}}^M, \quad (A2b)
\]

\[
E(\text{GPa}) = (2.27r_0 - 2.00) \times 10^3. \quad (A2c)
\]

**Low-Ca pyroxene model (Yao et al., 2012; Sun and Liang, 2013)**

\[
\ln D_0 = -5.37 + \frac{3.87 \times 10^4}{RT} + 3.54X_{\text{Al}}^T + 3.56X_{\text{Ca}}^M - 0.84X_{\text{Ti}}^melt, \quad (A3a)
\]

\[
r_0(\text{Å}) = 0.693 + 0.432X_{\text{Ca}}^M + 0.228X_{\text{Mg}}^M, \quad (A3b)
\]

\[
E(\text{GPa}) = (1.85r_0 - 1.37 - 0.53X_{\text{Ca}}^M) \times 10^3. \quad (A3c)
\]
In Eqs. (A1a) and (A1b), $X_{Ti}$, $X_{Mg}$, $X_{Na}$, and $X_{K}$ are cation numbers of Ti, Mg, Na, and K per 23-oxygen, and $X^M_{Fm}$ is the total cation number of Fe$^{2+}$ + Mn$^{2+}$ + Mg$^{2+}$ on the M4 site. The amphibole formula is calculated using the method described in Shimizu et al. (2017). This method is simplified from that in Leake et al. (1997) by assuming all Fe to be ferrous for parameterizing the amphibole-melt REE partitioning model. The calculation procedure is described in Text S2 of online Appendix C. In Eqs. (A2a), (A2b), and (A3a–c), $X^T_{Al}$ is the cation content of the tetrahedral Al in pyroxene per 6-oxygen; $X^M_{Al}$ is the cation number of Al on the M1 site in pyroxene per 6 oxygen; $X^M_{Ca}$ and $X^M_{Mg}$ are cation numbers of Ca and Mg, respectively, on the M2 site in pyroxene per 6 oxygen; $X^m_{H_2O}$ is the molar fraction of H$_2$O in the melt calculated following Wood and Blundy (2002); and $X^m_{Ti}$ is the cation fraction of Ti in the melt per 6 oxygen. $X^m_{Ti}$ is added to correct the effect of melt TiO$_2$ for REE partitioning between low-Ca pyroxene and lunar basalts which have high TiO$_2$ concentrations, and it can be neglect in cases of Earth’s mantle conditions (Yao et al., 2012; Sun and Liang, 2013). Pyroxene formulae are calculated by assuming a random distribution of Fe$^{2+}$ and Mg$^{2+}$ over the M1 and M2 sites (Wood and Banno, 1973) and that all iron is present as ferrous iron (Sun and Liang, 2012, 2013; Yao et al., 2012).

Appendix B. Parameters in the semi-empirical models for amphibole temperature

The following equations present the expressions of terms and in the semi-empirical models for amphibole temperature of (Eq. (5)): $A^{opx,amp}_i = -5.37 + \frac{3.87 \times 10^4}{RT_{pyx}} + 3.54 X^T_{Al} + 3.56 X^M_{Ca} + 4.21 - 1.52 X^m_{Ti}$
$$0.35X_{\text{Mg}}^\text{amp} + 1.83X_{\text{Na}}^\text{amp} + 2.95X_{\text{K}}^\text{amp} - \frac{4\pi E_{\text{opx}}^\text{NA}}{R_{\text{opx}}} \left[ \frac{r_{0}^\text{opx}}{2} (r_{0}^\text{opx} - r_{l})^2 - \frac{1}{3} (r_{0}^\text{opx} - r_{l})^3 \right],$$

(B1a)

$$A_{l}^\text{cpx,amp} = -7.14 + \frac{7.19 \times 10^4}{R_{\text{cpx}}} + 4.37X_{\text{Al}}^\text{cpx} + 1.98X_{\text{Mg}}^\text{cpx} + 4.21 - 1.52X_{\text{Ti}}^l +$$

$$0.35X_{\text{Mg}}^\text{amp} + 1.83X_{\text{Na}}^\text{amp} + 2.95X_{\text{K}}^\text{amp} - \frac{4\pi E_{\text{cpx}}^\text{NA}}{R_{\text{cpx}}} \left[ \frac{r_{0}^\text{cpx}}{2} (r_{0}^\text{cpx} - r_{l})^2 - \frac{1}{3} (r_{0}^\text{cpx} - r_{l})^3 \right],$$

(B1b)

$$B_{l}^\text{amp} = -\frac{7.27 \times 10^4}{R} + \frac{4\pi E_{\text{amp}}^\text{NA}}{R} \left[ \frac{r_{0}^\text{amp}}{2} (r_{0}^\text{amp} - r_{l})^2 - \frac{1}{3} (r_{0}^\text{amp} - r_{l})^3 \right].$$

(B2)

The lattice strain parameters $r_{0}^\text{cpx}$, $E_{\text{cpx}}$, $r_{0}^\text{opx}$, $E_{\text{opx}}$, $r_{0}^\text{amp}$, and $E_{\text{amp}}$ are given in Appendix A.

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**Figure captions**

**Figure 1.** Compositions of amphiboles (oxides in wt%, on volatile-free basis) from amphibole-bearing mantle rocks. Red and blue plus signs are intraplate (I)-amphiboles and supra-subduction (S)-amphiboles, respectively, classified after Coltrotti et al. (2007). Green dots are amphiboles from the amphibole–melt REE partitioning experiments used to calibrate the model of Shimizu et al. (2017). Closed symbols are amphiboles in mantle samples that have compositions within the range of those used for calibration. The calculation of chemical formulae and the compositional boundaries in (e) follow those of Leake et al. (1997). Tr, tremolite; Ath, anthophyllite; Fac, ferro-actinolite; Gru, grunerite.

**Figure 2.** Correlations of temperature derived from the REE-in-two-pyroxene thermometer of Liang et al. (2013, \(T_{\text{REE}}\)) with those from the major element-based two-pyroxene thermometers of Brey and Köhler (1990, \(T_{\text{BKN}}\)), Wells (1977, \(T_{\text{W77}}\)), Putirka (2008, his Eq. (37), \(T_{\text{P37}}\)), and average of the three (\(T_{\text{avg}}\)) for the amphibole-bearing mantle rocks. Symbols are the same as in Fig. 1. Error bars are 1σ uncertainties in \(T_{\text{REE}}\). The solid gray lines in denote the 1:1 correlation, and the dashed gray lines denote the ±100°C deviations. Gray fields show the ranges of temperatures of well-equilibrated peridotite xenoliths from Liang et al. (2013).

**Figure 3.** Comparisons of amphibole Mg# with orthopyroxene and clinopyroxene Mg#s for the amphibole-bearing mantle samples. Symbols with cross denote samples that display obvious textual and compositional disequilibrium. See text for details.

**Figure 4. (a)** Inversion diagram for the REE-in-two-pyroxene temperature (\(T_{\text{REE}}\)) of an amphibole-bearing lherzolite (sample WLK30 from Wilcza Góra, SW Poland,
Matusiak-Malek et al., 2017). (b–d). Comparisons of the measured orthopyroxene–clinopyroxene and pyroxene–amphibole REE partition coefficients (solid red patterns) with those calculated using the mineral–mineral REE partitioning model of Eq. (2) at the inverted $T_{\text{REE}}$ (solid blue curves). Partition coefficients calculated at 750°C, 925°C, and 1100°C are also shown for comparison (dashed blue isotherms). The partition coefficients are plotted against 8-fold ionic radii of REEs from Shannon (1976).

**Figure 5.** (a and b). Inversion diagrams for the apparent amphibole temperatures ($T_{\text{amp,opx–amp}}$ and $T_{\text{amp,cpx–amp}}$) of sample WLK30 (the lherzolite sample shown in Fig. 4) using our semi-empirical models. (c and d). Comparisons of the measured pyroxene–amphibole REE partition coefficients (solid red patterns) with those calculated using the REE partitioning models of Eq. (5) with amphibole temperature at the inverted $T_{\text{amp,opx–amp}}$ or $T_{\text{amp,cpx–amp}}$ (solid green curves). Partition coefficients calculated assuming the amphibole temperature at $T_{\text{REE}}$, $T_{\text{REE}} - 100^\circ$C, and $T_{\text{REE}} - 200^\circ$C are plotted for comparison (dot-dashed green isotherms). The partition coefficients are plotted against 8-fold ionic radii of REEs from Shannon (1976).

**Figure 6.** Comparisons of the $T_{\text{amp}}$ calculated using Eq. (8) with (a) the inverted $T_{\text{amp}}$ (average value of $T_{\text{amp,opx–amp}}$ and $T_{\text{amp,cpx–amp}}$), (b) $T_{\text{REE}}$, and (c–f) major-element derived temperatures for the amphibole-bearing mantle rocks. Symbols are the same as in Fig. 1, and the element-based two-pyroxene thermometers are identical to those in Fig. 2. The solid gray lines in denote the 1:1 correlation, and the dashed gray lines denote the ±50°C deviations.

**Figure 7.** Comparison between the measured pyroxene-amphibole REE partition
coefficients with those predicted using the partitioning models of Eq. (5) and $T_{\text{amp}}$ calculated using Eq. (8). The solid gray lines in (a and c) denote the 1:1 correlation, and the dashed gray lines denote the 1:2 and 2:1 correlations. Data in (a and c) are color-coded with 8-fold ionic radii of REEs from Shannon (1976).

**Figure 8.** Patterns of pyroxene–amphibole REE partition coefficients calculated from the partitioning model of Eq. (5) with $T_{\text{amp}}$ assumed to be 50–300°C lower than the $T_{\text{pyx}}$ (assigned as $T_{\text{REE}}$) for the lherzolites WLK30 from Wilcza Góra (Matusiak-Małek et al., 2017) and AUB1407 from the Western Pannonian Basin, Hungary (Aradi et al., 2020). See text for discussions.

**Figure 9.** Chondrite-normalized REE patterns of amphiboles calculated using the semi-empirical pyroxene-amphibole REE partitioning models and REE contents in the coexisting pyroxenes for amphibole-bearing peridotite xenoliths (a) WG5 and WG6 from Wilcza Góra (Matusiak-Małek et al., 2017), (b) KPFS0402 and PST1403 from Western Pannonian Basin (Aradi et al., 2020), and (c) H71 and (d) H69 from Cameroon volcanic line (Puziewicz et al., 2023). Gray patterns in (d) were calculated using $T_{\text{amp}}$ inverted from Eq. (7), which testifies the effect of heating. Also plotted for comparison are measured REE patterns of amphiboles in the Wilcza Góra xenoliths genetically related to WG5 and WG6 (Matusiak-Małek et al., 2017), Western Pannonian Basin xenoliths genetically related to KPFS0402 and PST1403 (Aradi et al., 2020), and Cameroon xenoliths H71 and H69 (Puziewicz et al., 2023). The chondrite REE abundances are from Anders and Grevesse (1989).
Figure 1

(a) Mg# vs. SiO₂ (wt%) plot with various mineral phases indicated.
(b) TiO₂ (wt%) vs. SiO₂ (wt%) plot.
(c) Na₂O (wt%) vs. SiO₂ (wt%) plot.
(d) K₂O (wt%) vs. SiO₂ (wt%) plot.
(e) $A(Na + K)_{apfu}$ vs. Si_{apfu} with mineral phase boundaries.
(f) Ca₇Si₈O₂₂OH₂ vs. Al content plot with mineral phase boundaries.
Figure 2
Figure 3
Figure 4

(a) B/1000 vs. ln(D) - A

(b) Ionic radius (Å)

(c) opx-cpx

(d) opx-amp
Figure 5

(a) $T_{\text{amp,opx-amp}} = 856 \pm 5^\circ C$

(b) $T_{\text{amp,cpx-amp}} = 861 \pm 5^\circ C$

(c) Ionic radius (Å)

(d) Ionic radius (Å)

Measurement
Figure 6
Figure 7
Figure 8
Figure 9

The diagram shows the distribution of rare earth elements (REE) in amp occurring in genetically linked samples and in the same sample.

- **Calculated amp REE** from opx-amp model and from cpx-amp model.
- **Measured amp REE** in the same sample.

The plots depict the REE abundance in different samples and conditions, with axes showing Amp/chondrite ratios on a logarithmic scale.

Sample abbreviations and conditions:
- WG5, WG6
- KPFS0402, PST1403
- H71, H69 (Heated)
- $T_{\text{amp}} = 652^\circ\text{C}$ (Eq. (8))
- $T_{\text{amp}} = 745^\circ\text{C}$ (Eq. (7))