

## Appendix I

A multi-step image analysis protocol, customized for the investigated samples, was adopted to separate melt and fluid inclusions from other non-relevant features within the garnets. A segmentation based only on gray values (GV) thresholding for the separation of melt and fluid inclusions was not possible because a large fraction of undesired features would have been included, in particular large voids, fractures or portions of other mineral phases (e.g. plagioclase and sillimanite, typically in large crystals). The adopted method for the separation of inclusions was based on a suitable combination of gray values segmentation, regions of interest (ROIs) and other image processing operations. The entire procedure was carried out using the Skyscan CT-Analyser software (Bruker Micro-CT). At first, a GV threshold was applied to separate the lighter materials (including the coeval melt and fluid inclusions, voids, fractures and a minor fraction of mineral inclusions) from the host garnet and other higher density phases. The lower limit of the GV-based segmentation was set to zero while the minimum point between the two main peaks of the GV histogram was selected as a convenient upper limit. This latter was intentionally chosen slightly higher than the actual upper GV limit of melt inclusions, in order to include also a significant fraction of undesired features (e.g. sillimanite and plagioclase) in the selection. The reason for this was to obtain a sufficient spatial continuity in the binarized datasets among the unwanted features (in particular large sillimanite and plagioclase crystals) that, being significantly larger in size than the inclusions of interest (i.e. those with an equivalent sphere diameter lower than 50  $\mu\text{m}$ ), can be differentiated on the basis of their size. The objects with a

volume lower than a certain value, i.e. the regions potentially containing melt inclusions, were then excluded from this selection. An appropriate volume limit for this operation was found between 3500 and 4000 voxels (in monochromatic synchrotron data) after repeated attempts. At this point, the entire garnet, excluding the previously separated portions occupied by cracks or large mineral grains, was used as a 3D region of interest. Within the selected ROI, a second, narrower GV segmentation was applied to isolate only the relevant inclusions. A satisfactory upper limit for the segmentation was chosen in this case on the mid point of the first peak of the GV histogram in phase-contrast datasets. At the end of the procedure, the volume and the coordinates of the barycenter of the inclusions were calculated using the 3D object analysis tool included in the CT-Analyser software.

The 3D spatial distribution of heavy mineral inclusions (e.g. ilmenite, monazite, zircon, and apatite), as identified by SEM-EDS chemical mapping) was investigated as well. The main issue for an appropriate segmentation was the presence of bright phase-contrast fringes in the areas characterized by strong density gradients (e.g. on edges around the garnets and in proximity of cracks and voids). The high gray values of the phase-contrast fringes largely overlapped with those of heavy minerals, therefore it was not possible to separate all the heavy mineral inclusions only using a simple GV threshold. The use of datasets processed using a phase-retrieval approach allowed to easily overcome this limitation. Only a moderate voxel erosion operation was applied to eliminate some remnants of phase-contrast fringes from the external surface of the garnets and then heavy mineral segmentation based on GV thresholding was straightforward.