LETTER

Geobarometry from host-inclusion systems: The role of elastic relaxation

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ABSTRACT

Minerals trapped as inclusions within other host minerals can develop residual stresses on exhumation as a result of the differences between the thermo-elastic properties of the host and inclusion phases. The determination of possible entrapment pressures and temperatures from this residual stress requires the mutual elastic relaxation of the host and inclusion to be determined. Previous estimates of this relaxation have relied on the assumption of linear elasticity theory. We present a new formulation of the problem that avoids this assumption. We show that for soft inclusions such as quartz in relatively stiff host materials such as garnet, the previous analysis yields entrainment pressures in error by the order of 0.1 GPa. The error is larger for hosts that have smaller shear moduli than garnet.

Keywords: Inclusions, residual stress, elasticity, elastic relaxation

INTRODUCTION

Minerals trapped as inclusions within other host minerals can develop residual stresses on exhumation as a result of the differences between the thermo-elastic properties of the host and inclusion phases (e.g., Fig. 3. in Howell et al. 2010). Measurement of the residual stress in the inclusions can, in combination with the equations of state (EoS) of the two phases, be used to infer the pressures and temperatures of entrapment if no plastic deformation has occurred (e.g., Zhang 1998; Izraeli et al. 1999; Guiraud and Powell 2006; Howell et al. 2012; Kohn 2014; Kouketsu et al. 2014). The key concept is that when the inclusion was trapped the host and inclusion had the same \( P \) and \( T \), and the inclusion fitted perfectly within the cavity in the host (Fig. 1a), so there were no stress gradients across the host and inclusion.

Consider a soft inclusion in a relatively stiff host recovered from metamorphic conditions to room conditions. The volume change of the host will be less than that expected for a free crystal of the inclusion phase. The inclusion phase is therefore compressed to a smaller volume than expected for the final external \( P \) and \( T \) and is therefore under pressure. The volume change of the host can be calculated from its EoS. The pressure \( P' \) in the inclusion is then calculated from this final host volume and the temperature, using the EoS of the inclusion. At this point, the host is under the external pressure, \( P_{\text{host}} \), but the inclusion is under a stress \( P' \) (Fig. 1b). This is a physically unstable “virtual” state because there is a difference in radial stress at the host/inclusion wall that will force the wall outward because \( P' > P_{\text{host}} \). This expansion leads to compression of the host and thus an increase in the radial stress in the host adjacent to the inclusion, and a relaxation of the pressure inside the inclusion, \( \Delta P_{\text{relax}} \). The resulting expansion of the inclusion continues until the radial stress in the inclusion matches that in the host adjacent to the inclusion (Fig. 1c) with a stress gradient in the host that decreases to the external stress at the outside surface of the host (Goodier 1933; Eshelby 1957; Fig. 1c).

The final observed inclusion pressure \( P_{\text{end}} \) is therefore comprised of two parts,

\[
P_{\text{end}} = P_1' + \Delta P_{\text{relax}}
\]

Since \( P_1' \) can be calculated from the EoS of the two phases, the problem of estimating entrapment conditions from observed inclusion pressures lies in the calculation of the change in pressure upon relaxation. Previous calculations (e.g., Zhang 1998; Izraeli et al. 1999; Guiraud and Powell 2006; Howell et al. 2012; Kohn 2014; Kouketsu et al. 2014) all rely on an estimate of the relaxation as

\[
\Delta P_{\text{relax}} = -\frac{3K}{4G} \left( P_{\text{end}} - P_{1\text{end}} \right).
\]

The derivation of this formula (Zhang 1998) relies on several assumptions including that the inclusion is small and spherical and that both phases are elastically isotropic. We will retain these assumptions. But Zhang’s (1998) derivation also relied on the assumptions of linear elasticity; that the elastic properties of the host and the inclusion do not change with \( P \) or \( T \). This last condition is clearly not valid for changes in pressure and temperature that are geologically relevant. Here we derive a new expression for the relaxation \( \Delta P_{\text{relax}} \) by an approach that does not require this assumption.

METHODOLOGY

We first address the “forward problem” of calculating the final pressure on the inclusion at \( P_{\text{relax}} \), following entrapment at conditions \( P_{\text{in}} \) and \( T_{\text{in}} \). Elastic deformation is reversible by definition. Therefore the stress and strain in the system

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