

REVISION 1

A model for the kinetics of high temperature reactions between polydisperse volcanic ash and SO₂ gas

Fabian B. Wadsworth^{1,*}, Jérémie Vasseur², Ana Silvia Casas², Pierre Delmelle³, Kai-Uwe Hess², Paul M. Ayris², Donald B. Dingwell²

¹Earth Sciences, Durham University, Science Labs, Durham, DH1 3LE, United Kingdom, ²Earth and Environmental Sciences, Ludwig-Maximilians-Universität, Theresienstr. 41, 80333 Munich, Germany. ³Earth and Life Institute, Environmental Sciences, Université catholique de Louvain, L7.05.10, 1348 Louvain-la-Neuve, Belgium.

*Corresponding author: fabian.b.wadsworth@durham.ac.uk

Abstract

Rapid calcium diffusion occurs in rhyolitic volcanic ash particles exposed to hot SO₂ atmospheres. Such chemical transport is important immediately following fragmentation, during proximal transport in eruption plumes, and during percolative gas transport through a permeable volcanic edifice. Here we analyze published results of experiments designed to constrain the kinetics of this process. The experiments involve crushed rhyolitic glass particles tumbled in SO₂-bearing atmospheres at a wide range of relevant temperatures. We find that the particle-gas reaction is fed by calcium diffusion from the bulk to the particle surfaces where calcium-sulfate crystals grow. The calcium flux is accommodated by local iron oxidation state changes. This process results in time-dependent concentrations of surface calcium that are leachable in aqueous solutions. Those leachate concentrations represent a proxy for the diffusive flux of Ca²⁺ out of the particle to form the surface deposits. We formulate a mathematical framework to convolve the starting particle size distributions with the solution to Fickian 1-dimensional diffusion, to find a weighted polydisperse result. Using this framework, we minimize for a temperature-dependent calcium diffusivity and compare our results with published calcium diffusivity data. We demonstrate that calcium diffusivity in rhyolite can be decomposed into two regimes: (1) a high temperature regime in which the diffusivity is given by the Eyring equation, and (2) a low-temperature regime more relevant to rhyolite volcanism and these gas-ash reactions. As a further test of our model, we compare the output against spatially-resolved data for the calcium gradients in the experimental particles. Our analysis suggests that surface reaction rates are rapid compared with the diffusion of calcium from the particle to the surface such that full diffusion models must be solved to predict the rhyolite-SO₂ reaction. We conclude by suggesting how this framework could be used to make quantitative predictions of sulfur budgets and iron oxidation during rhyolitic eruptions.

Keywords: scavenging; scrubbing; gas-ash reactions; volcanic eruption

37

1. Introduction

38 The release of SO₂ during large volcanic eruptions can affect Earth's climate over timescales of
39 months to years (Bluth et al. 1997; Robock 2000) or perhaps even millennia (Baldini et al. 2015). For
40 recent or ongoing eruptions that occurred during the satellite age, the SO₂ input into the atmosphere
41 can be monitored and quantified directly (e.g. Carn et al. 2017). However, for eruptions that have
42 occurred deeper in Earth's history, the SO₂ budget is more difficult to estimate. The so-called 'sulfur
43 excess' problem is one example of how the sulfur budget of the trans-crustal magma system prior to
44 eruption cannot be derived simply from observed erupted SO₂, and that other processes must play a
45 significant role (Shinohara 2008; Edmonds et al. 2010; Wallace and Edmonds 2011). Here we draw on
46 the experimental observation that hot ash particles at magmatic temperatures can scrub SO₂ from the
47 initial eruptive gas phase (Ayrís et al. 2013; Delmelle et al. 2018) – a process for which there remains
48 no general model. Thus, SO₂ scrubbing represents a reaction pathway in the volcanic sulfur cycle that
49 warrants further investigation and which potentially further impacts the relationship between pre-
50 eruptive sulfur and the atmospheric sulfur loading from eruptions.

51 As magmas fragment, they become a rapidly moving and evolving mixture of ash and volatile gases (a
52 'dusty gas') and react chemically over relatively short timescales (Ayrís et al. 2013, 2014; Delmelle et
53 al. 2018; Renggli and King 2018). Initial experimental results, and their scaling to volcanic conditions,
54 have indicated that reactions between volcanic ash and SO₂ may be sufficiently rapid to result in
55 significant modification of erupted SO₂ budgets during in-conduit and proximal ash transport (Ayrís et
56 al. 2013; Delmelle et al. 2018) or during percolation through volcanic domes (Casas et al. 2019).
57 Before these insights can be used to compute the effect of gas-ash reactions on total sulfur budgets in
58 volcanic eruptions quantitatively and reliably, experimental data has to be tested against detailed
59 constitutive models for the kinetics of this reaction.

60 In natural systems, high temperature SO₂-ash reactions involve the formation and growth of calcium-
61 bearing salt crystals on the surfaces of ash particles (Ayrís et al. 2013; Renggli and King 2017, 2018;
62 Renggli et al. 2019). At the range of conditions that have been explored, the dominant salt formed on
63 natural glass is calcium sulfate (Renggli and King 2018), whose generation is rate-limited by the
64 diffusion of calcium to the particle surfaces (Ayrís et al. 2013; Delmelle et al. 2018). In turn, that
65 diffusive flux of calcium is affected by both the temperature and the particle size(s) – where high
66 temperatures and small particles render gas-ash reactions more efficient compared with low
67 temperatures or larger particles. These conditions of small particles and high temperatures could be
68 favored in particularly large explosive eruptions on Earth. During eruptions, small particles are
69 produced when fragmentation is vigorous (Kueppers et al. 2006) and high temperatures can be
70 maintained when fragmentation occurs deep, when conduits are wide, or when mass eruption rates are
71 high, such that there is a long insulated transport pathway before cooling in the plume (Ayrís et al.
72 2013). In Fig. 1 we sketch the portion of the volcanic system in which these high-temperature gas-ash
73 reactions may be operative.

74 Cognate processes involving the interaction between SO₂ gas and silicate glass or rocks have been
75 explored for synthetic glasses (Douglas and Isard 1949; Johnson and Burnett 1993; Burnett et al. 1997;
76 Renggli and King 2018; Renggli et al. 2019), for calcite-SO₂ reactions (Mavrogenes and Blundy
77 2017a, 2017b; Saadatfar et al. 2020), and as a mechanism for porphyry copper deposit formation
78 (Henley et al. 2015). In all cases, it is proposed that these reaction pathways represent a key
79 component of the volcanic- or crustal- sulfur cycle on Earth and other planets, and yet there is no clear
80 way to make quantitative predictions of the net contribution of these reaction pathways to the volcanic
81 sulfur budgets across all relevant conditions and input parameters. Here, we aim to present a general
82 formulation for the kinetics of these gas-ash reactions for the case where iron is present in the glass.
83 We validate a mathematical model using a large published dataset, which is specific to SO₂ reactions
84 with rhyolite particles (Casas et al. 2019), yet is general in principle and therefore may be easily
85 adapted to other cases of ash and gas composition.

86

87

88

2. Developing a framework for high-temperature gas-ash reactions

89 Here we present a mathematical framework in which the rates of gas-ash reaction can be predicted.
90 Throughout, while we focus on the case of iron-bearing rhyolite particles interacting with SO₂ gas, we
91 note that the framework we present is general and where relevant we highlight how it could be adapted
92 for other compositions of ash or gas for which the same reaction mechanisms are appropriate.

93

94

a. Phenomenology: Iron-bearing ash/glass and SO₂ gas

95 Sulfate compounds are found on the surface of freshly erupted natural volcanic ash particles, which is
96 inferred to indicate that SO₂-ash reactions have taken place (Rose 1977; Varekamp et al. 1984;
97 Gerlach and McGee 1994; De Hoog et al. 2001; de Moor et al. 2005). Initially, it was proposed that
98 these compounds precipitate from sulfur-bearing liquid droplets or liquid layers on the surface of ash
99 such as H₂SO₄ liquid (Rose 1977), which would then represent a low-temperature leaching and
100 precipitation process. However, Ayris et al. (2013) questioned the importance of the low-temperature
101 process(es) given that most plumes reach altitudes where liquid droplets are less likely to form and
102 where ice-based SO₂ scavenging is less efficient (Textor et al. 2003). As an alternative, a high
103 temperature mechanism of SO₂ scavenging can also explain the presence of sulfate salt crystals on ash
104 surfaces and is instead driven by the diffusion of cations to the surface, feeding the reaction and salt
105 deposition (Douglas and Isard 1949; Ayris et al. 2013, 2014; Delmelle et al. 2018; Casas et al. 2019).
106 Therefore, while low-temperature adsorption or acid droplet leaching and precipitation may be
107 important down-plume and during or after deposition, the in-plume relevant process of interest here is
108 a high temperature one and thus occurs in the presence of significant cation diffusion in the ash
109 particles.

110 By examining the phenomenological results from previous work on high-temperature reactions
111 between glass and SO₂ (Ayris et al. 2013; Renggli and King 2018; Casas et al. 2019; Renggli et al.
112 2019), a few key starting points can be identified. In most cases where an experimental glass used was
113 iron-bearing, exposure of glass surfaces to high temperatures results in a time-dependent formation of
114 surface phases of a wide range of compositions: CaSO₄, K₂SO₄, FeSO₄, Na₂SO₄, MgSO₄,
115 Na₂Ca(SO₄)₂, and Al₂(SO₄)₃, as well as minor other phases (for a review see Renggli and King 2018).
116 Most of this complexity is found in experimental reactions between mafic glasses and SO₂, such as
117 tholeiitic basalts (Johnson and Burnett 1993; Burnett et al. 1997; Palm et al. 2018; Renggli et al.
118 2019), not only in terms of the diversity of surface phases that crystallize but also a strong dependence
119 on oxygen fugacity. We posit that more silica-rich rhyolitic particles reacting with SO₂ is a
120 comparatively simple system. For the purposes of this work, we focus on experiments between an
121 iron-bearing rhyolitic glass and SO₂ (Ayris et al. 2013; Casas et al. 2019) for which the dominant
122 surface phase is CaSO₄.

123 Ayris et al. (2013) showed that the time-dependent precipitation of CaSO₄ on phonolite particle
124 surfaces was coupled with a spatial gradient of calcium in the underlying glass substrate, depleted
125 toward the interface and inferred that similar profiles develop in the interior of particles of other
126 compositions. This implies that diffusion is the dominant mechanism supplying calcium to the reaction
127 sites at the interface, confirming conclusions made by Douglas and Isard (1949) that cation diffusion
128 in the glass is the rate-limiting mechanism controlling the time-evolution of this family of gas-glass
129 reactions. Ayris et al. (2013) applied simplified diffusion models to their experimental data, treating
130 the time-dependent mass of calcium in the surficial calcium sulfate deposits as a proxy for the
131 diffusive flux of calcium out of the particles. The rate of increase of mass of the salts deposited scales
132 non-linearly with temperature, and particle size (Ayris et al. 2013; Casas et al. 2019). Taken together,
133 these observations suggest that a full diffusion model for calcium fluxes driven by the chemical
134 potential exerted by the presence of SO₂ can be a reasonable way to build a predictive model for the
135 rates of gas-ash reactions.

136 While experimental evidence appears to confirm that calcium diffusion rates are the limiting processes
137 controlling the kinetics in rhyolite-SO₂ reactions, it clearly also takes a finite time for the nucleation
138 and growth of the crystalline surface phases (Douglas and Isard 1949; Ayris et al. 2013; Delmelle et
139 al. 2018; Renggli and King 2018; Renggli et al. 2019). Therefore, there are two processes at play: (1)
140 calcium moves from the particle bulk to the surface, and (2) the surface reaction(s) must occur.
141 Presumably overall, the process of gas-ash reactions will occur at the rate of whichever of these two
142 processes is the slowest; which remains untested. Ayris et al. (2013) proposed that the surface reaction
143 first forms CaSO₃, which then oxidizes to form CaSO₄, requiring additional molecular oxygen.
144 However, no direct evidence for surface CaSO₃ has been found by experiment. Instead, CaSO₄ could
145 form directly, consistent with observations and without the need for additional second-step oxidation
146 of the surface (Henley et al. 2015; Palm et al. 2018; King et al. 2018). Either way, the relative
147 importance of these surface reactions to the overall rate of the process is poorly understood in
148 rhyolites.

149

150 **b. Diffusion in monodisperse particles**

151 Diffusion acts on timescales $\lambda_D \propto L^2/D$ or, equivalently over lengthscales $L \propto \sqrt{Dt}$, where λ_D is the
152 characteristic diffusion time, L is a characteristic diffusion length, t is the time since the onset of
153 diffusion, and D is the diffusivity that is variably dependent on temperature, pressure, and the total
154 local concentration of the diffusing species (Crank 1975; Zhang and Cherniak 2010). These
155 relationships provide a useful approximation for scaling diffusion problems and are especially accurate
156 when D can be taken to be constant. Such cases of constant D only strictly exist when the system of
157 concern is both isothermal and isobaric, and when the absolute variation in concentration across the
158 concentration profile is small. However, in most applied cases of interest, magmas are non-isothermal,
159 polybaric, and the concentration gradients are large compared with the absolute concentration at
160 equilibrium, making D a function of both time and spatial position, and potentially invalidating the
161 simplest scaling tools. In such cases, a full diffusion solution is needed along with a relationship
162 between D and the system temperature, pressure, and local concentration. To meet this need, there has
163 been a large body of work invested in the determination and systemization of diffusivities of many
164 cations of magmatic relevance in a wide variety of liquid or glass compositions and conditions
165 (Jambon 1982; Dingwell 1990; Mungall et al. 1999; Zhang et al. 2010).

166 For the purposes of understanding diffusion in volcanic particles, Fick's law for diffusive mass
167 transport applies which is $\partial C/\partial t = \nabla(D\nabla C)$ where C is the concentration of the species of interest
168 (throughout this work unless otherwise stated, C refers to the wt.% value of calcium). For practical
169 purposes, a spherical approximation for geometry is often used for volcanic particles (Wadsworth et
170 al. 2017b). When cast in spherical coordinates Fick's general law becomes (Crank 1975)

171

$$\frac{\partial C}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 D \frac{\partial C}{\partial r} \right), \quad \text{Eq. 1}$$

172

173 where r is the radial spatial position in the sphere measured from the centre. In order to compare
174 solutions to Eq. 1 across a wide range of conditions, it is useful to render Eq. 1 dimensionless by
175 making the following substitutions where a bar above a parameter denotes its dimensionless form

$$\bar{r} = \frac{r}{R}; \quad \bar{C} = \frac{C}{C_r}; \quad \bar{t}_r = \frac{t}{\lambda}; \quad \bar{D} = \frac{D}{D_r}; \quad \text{Fi}_r = \frac{D_r}{R^2} \lambda \quad \text{Eq. 2}$$

176

177 where R is the radius of the particle sphere, C_r is a reference concentration, λ is a reference time such
178 that \bar{t}_r is the normalization of time relative to that reference and \mathcal{D}_r is a reference diffusivity. Fi_r is a
179 reference Fick number which is the value of the Fick number at a time of λ and using the reference
180 coefficient of diffusion. The general Fick number is then a dimensionless diffusion time and is given
181 by $Fi = t/\lambda_D = \mathcal{D}t/R^2$ where λ_D is the diffusion timescale. We pick λ to be an arbitrarily long time –
182 longer than the time window of interest, which does not affect the result. Now Eq. 1 can be written in
183 dimensionless form:

$$\frac{\partial \bar{C}}{\partial \bar{t}_r} = \frac{Fi_r}{\bar{r}^2} \frac{\partial}{\partial \bar{r}} \left(\bar{r}^2 \bar{\mathcal{D}} \frac{\partial \bar{C}}{\partial \bar{r}} \right). \quad \text{Eq. 3}$$

184

185 The diffusivity \mathcal{D} is typically dependent on temperature T via an Arrhenian relationship that is linear
186 when $\ln(\mathcal{D})$ is compared with $1/T$. Equivalently

$$\mathcal{D}(T) = \mathcal{D}_\alpha \exp\left(-\frac{\alpha}{T}\right) \quad \text{Eq. 4}$$

187

188 and where \mathcal{D}_α and α are empirical coefficients that are usually determined by experiment. For some
189 chemical components that diffuse through volcanic materials, \mathcal{D} is also a function of the absolute
190 concentration C at all local spatial positions. This is the case, for example, for the diffusivity of
191 molecular or total water in rhyolites (Zhang and Ni 2010). When this has been shown to be the case, a
192 different form of $\mathcal{D}(T)$ is often used, which is still Arrhenian but which depends on the total
193 concentration of the diffusing species. The dependence on C is either linear (e.g. for the case of water
194 diffusion in rhyolites; Zhang and Ni 2010), or exponential (e.g. in the case of magnesium diffusion in
195 olivine; Morioka 1981). Here, we acknowledge that this may be the case for calcium diffusion in
196 rhyolite and so explore functional forms for $\mathcal{D}(T, C)$. These variations on Eq. 4 for the case of a
197 dependence of \mathcal{D} on C are

$$\mathcal{D}(T, C) = \mathcal{D}_\beta \exp\left(-\frac{\beta}{T}\right) C, \quad \text{Eq. 5a}$$

$$\mathcal{D}(T, C) = \mathcal{D}_\gamma \exp\left(-\frac{\gamma}{T}\right) \exp(C) \quad \text{Eq. 5b}$$

198

199 where, like \mathcal{D}_α and α in Eq. 4, \mathcal{D}_β , \mathcal{D}_γ , β , and γ are coefficients that are usually determined by
200 experiment. Eqs 4-5 can then be rendered dimensionless by taking the following substitutions

$$\bar{T} = \frac{T}{T_r}; \quad \bar{\alpha} = \frac{\alpha}{T_r}; \quad \bar{\beta} = \frac{\beta}{T_r}; \quad \bar{\gamma} = \frac{\gamma}{T_r} \quad \text{Eq. 6}$$

201
202 where T_r is a reference temperature. Now \mathcal{D}_r in Eq. 2 can be defined as the diffusivity at a temperature
203 of T_r as $\mathcal{D}_r = \mathcal{D}_\alpha \exp(-\alpha/T_r)$, $\mathcal{D}_r = \mathcal{D}_\beta \exp(-\beta/T_r) C_r$, or $\mathcal{D}_r = \mathcal{D}_\gamma \exp(-\gamma/T_r) \exp(C_r)$ in the case
204 of Eq. 4 or Eq. 5, respectively. Eqs 4-5 can be cast in dimensionless form as:

205

$$\bar{\mathcal{D}} = \exp \left[\bar{\alpha} \left(1 - \frac{1}{\bar{T}} \right) \right], \quad \text{Eq. 7a}$$

$$\bar{\mathcal{D}} = \exp \left[\bar{\beta} \left(1 - \frac{1}{\bar{T}} \right) \right] \bar{C}, \quad \text{Eq. 7b}$$

$$\bar{\mathcal{D}} = \exp \left[\bar{\gamma} \left(1 - \frac{1}{\bar{T}} \right) \right] \exp(\bar{C}). \quad \text{Eq. 7c}$$

206
207 If the temperature is constant, then Eq. 7a reduces to $\bar{\mathcal{D}} = 1$, Eq. 7b reduces to $\bar{\mathcal{D}} = \bar{C}$ and Eq. 7c
208 reduces to $\bar{\mathcal{D}} = \exp(\bar{C})$.

209 For any diffusion problem in a particle sphere, we can now solve Eq. 3 using either Eq. 7a
210 (temperature dependent diffusion), Eq. 7b (temperature and linear concentration dependent diffusion),
211 or Eq. 7c (temperature and exponential concentration dependent diffusion) for different scenarios. We
212 do this by using a finite difference implicit numerical scheme (a so-called ‘backwards time, centered
213 space’ scheme) with a relaxed fixed-point method to ensure convergence at each time step. The
214 dimensionless temporal and spatial resolutions we use are 10^{-3} and 10^{-2} , respectively. To convert the
215 solution for $d\bar{C}/d\bar{t}_r$ back to a dimensional answer, C_r and T_r would have to be defined, which is
216 specific to the problem in question and is therefore discussed later. Specific numerical approaches for
217 incorporating the concentration dependence of diffusivity are given for the problem of hydrating
218 legumes during soaking in Hsu (1983).

219
220 **c. Initial conditions, boundary conditions, and reaction rates**

221 To solve Eq. 3 (or Eq. 1), we must define boundary and initial conditions. Our initial condition is
222 taken such that $C = C_i$ for all r , where C_i is the initial concentration in the glass before diffusion
223 begins, and that the concentration outside the particle in the gas phase is zero. At the particle centre
224 $r = 0$, we take Neumann boundary $DdC/dr = 0$ for all t . At the particle rim $r = R$, for $t > 0$ we
225 have two possible choices

226

$$C = C_x \quad \text{Eq. 8a}$$

$$\mathcal{D} \frac{dC}{dr} = H(C_\infty - C) \quad \text{Eq. 8b}$$

227
228 where Eq. 8a represents a constant boundary value and assumes that the concentration immediately
229 falls to C_x at $r = R$ for all $t > 0$. Eq. 8b represents a boundary reaction term, where H is a reaction
230 rate constant (in $\text{m} \cdot \text{s}^{-1}$). From a physico-chemical perspective, it is reasonable to assume that there is
231 a timescale associated with the reaction between calcium being diffused to the surface of the particles

232 and the SO₂ gas. Previous work has not proved the extent to which this reaction rate is strictly
233 important in these problems of gas-ash reactions, or if it can be neglected. In order to determine its
234 importance, this reaction timescale can be scaled for by $\lambda_{\text{Bi}} \approx R/H$ where λ_{Bi} is a Biot timescale. A
235 Biot number $\text{Bi} = \lambda_{\mathcal{D}}/\lambda_{\text{Bi}} = HR/\mathcal{D}$ is a measure of the relative importance of boundary mass transfer
236 (limited by H) and the internal mass transfer (limited by \mathcal{D}). We anticipate that $\text{Bi} \gg 1$ is the regime
237 where the reaction rate at the surface is sufficiently rapid to not impede the process and therefore can
238 be neglected, whereas $\text{Bi} \ll 1$ is the regime where the reaction rate is sluggish and must be accounted
239 for via Eq. 8b.

240 As before, we render the boundary and initial conditions dimensionless so that initially $\bar{C} = 1$ for all \bar{r} ,
241 and the boundaries are: (1) at $\bar{r} = 0$, we have $\bar{\mathcal{D}}d\bar{C}/d\bar{r} = 0$ for all \bar{t} , and (2) at $\bar{r} = 1$, Eq. 8a is $\bar{C} = 0$
242 or Eq. 8b is $\bar{\mathcal{D}}d\bar{C}/d\bar{r} = \text{Bi}_r(\bar{C}_\infty - \bar{C})$, where Bi_r is the reference Biot number at the reference
243 temperature T_r .

244

245 **d. Gradients of diffusivity and bulk fluxes**

246 A model solution to Eq. 3 is explicitly solvable as described above. If experimental data is spatially
247 resolved – for example in the case of electron microprobe analysis transects along a quenched
248 diffusion profile– then no conversion for bulk fluxes would be required and the data can be directly
249 compared with model solutions. However, it is useful to understand bulk flux data and not just
250 spatially-resolved data – for example, in the case of time-dependent extraction of an element from the
251 surface of a bulk sample particle (Ayriss et al. 2013; Casas et al. 2019), as used here. In this case, the
252 bulk data need converting before they can be directly compared to a diffusion model.

253 To non-dimensionalize time, we give the diffusion timescale in Eq. 2 as $\lambda_{\mathcal{D}} = R^2/\mathcal{D}$ which is strictly
254 valid when used to normalize time in spatially resolved diffusion solutions. However, to scale
255 experimental data in which the diffusivity varies spatially, such as in the case of a concentration-
256 dependent diffusivity, or in non-isothermal conditions, we have to redefine $\lambda_{\mathcal{D}}$.

257 If we take the case where diffusivity is dependent on temperature but not on concentration, and if we
258 take a scenario where any given test-case is isothermal, then $\bar{\mathcal{D}} = 1$ and $\lambda_{\mathcal{D}}$ is as given in Eq. 2.
259 However, in the case where the diffusivity is dependent on concentration (i.e. Eqs 7b & 7c), then
260 diffusion will induce a spatial gradient of C and therefore of \mathcal{D} , requiring that we recast $\lambda_{\mathcal{D}}$. To do this
261 we simply convert the spatial gradient of C into a spatial gradient of \mathcal{D} via Eq. 5 and then average \mathcal{D}
262 over the spatial positions in the sphere giving $\langle \mathcal{D} \rangle$ by:

263

$$\langle \mathcal{D} \rangle = \frac{\int_0^1 \bar{r}^2 \mathcal{D} d\bar{r}}{\int_0^1 \bar{r}^2 d\bar{r}} \quad \text{Eq. 9}$$

264

265 so that for any isothermal monodisperse sphere diffusion problem, $\lambda_{\mathcal{D}} = R^2/\langle \mathcal{D} \rangle$. For isothermal
266 problems, because $\bar{\mathcal{D}} = \bar{C}$, then $\langle \mathcal{D} \rangle/\mathcal{D}_r$ – termed $\langle \bar{\mathcal{D}} \rangle$ – is the same as $\langle C \rangle/C_r$ – termed $\langle \bar{C} \rangle$ – implying
267 that it does not matter whether the concentration or diffusivity is integrated.

268 For non-isothermal conditions, we could additionally integrate the change in $\langle \mathcal{D} \rangle$ with the evolution in
269 T to get a dimensionless time $\bar{t} = 1/R^2 \int_{t_i}^t \langle \mathcal{D} \rangle dt$ where t_i is the initial time of diffusion. However, in
270 the present work we only deal with isothermal datasets so that, while the temperature varies from
271 experiment to experiment, it does not vary with time in a single experiment.

272

273 **e. Polydisperse distributions of particles**

274 In general, the polydispersivity of a distribution of spheres is captured by $S = \langle R^2 \rangle \langle R \rangle / \langle R^3 \rangle$ where
275 $\langle R^n \rangle$ is the n th moment of the distribution (Torquato 2013; Wadsworth et al. 2017a; Vasseur et al.
276 2020). Here $S = 1$ is the monodisperse end member and $S = 0$ is infinitely polydisperse. It is useful to
277 compute S and, for distributions $S < 1$, to account for the polydispersivity in diffusion scaling. This is
278 not necessary when concentration gradients are measured within individual particles, but it is
279 necessary when we are using bulk metrics, such as the total leached amount of a diffusing species in a
280 population of particles of different sizes.

281 In monodisperse populations of spheres where there is a defined single R , $\lambda_{\mathcal{D}}$ is defined as described
282 above. However, for polydisperse distributions of spheres, there is not a single R . This has two effects,
283 first, it means that any bulk signal, such as the bulk amount of extraction of calcium from a
284 distribution of spheres requires a weighted scaling, and second, $\lambda_{\mathcal{D}}$ needs redefining.

285 First, $\langle \bar{C} \rangle$ can be weighted as a function of the probability density function of the particle size
286 distribution $F(R)$. The result of such a weighting, which we call $\langle \bar{\bar{C}} \rangle$, can be found by

287

$$\langle \bar{\bar{C}} \rangle = \int_0^{\infty} F(R) \langle \bar{C} \rangle dR \quad \text{Eq. 10}$$

288

289 The value $\langle \bar{\bar{C}} \rangle$ then gives a bulk average concentration signal for a population of spherical particles of
290 any distribution of radii. Radii data are more commonly cast as discrete binned measured data and not
291 as a continuous function, in which case

292

$$\langle \bar{\bar{C}} \rangle = \sum_{j=1}^{j=n} \phi(R_j) \langle \bar{C} \rangle_j \quad \text{Eq. 11}$$

293

294 where $\langle \bar{C} \rangle_j$ and R_j are the j th bin of $\langle \bar{C} \rangle$ and R , respectively, and ϕ is the volume fraction of the j th bin
295 of R . In the definition of $\lambda_{\mathcal{D}}$, we take the characteristic lengthscale as $\langle R^3 \rangle^2 / \langle R^2 \rangle^2$ giving $\lambda_{\mathcal{D}} =$
296 $\langle R \rangle^2 / [S \langle \mathcal{D} \rangle]$.

297

298

3. Materials and methods

299 We use the published results of experiments from Casas et al. (2019) in which rhyolitic glass particles
300 are exposed to hot SO₂ gas. In this section, we document how the experimental material was prepared,
301 the conditions of the experiments, and the analysis of the experimental products undertaken.

302

303 a. Preparation of experimental material

304 To test how diffusion of calcium evolves in distributions of particles exposed to a gas phase, we re-
305 analyze a suite of data presented in Casas et al. (2019). For completeness, we repeat the experimental
306 details here. The data were produced using a natural rhyolitic obsidian from Hrafninnuhryggur, Krafla
307 (Iceland), the source location for which is described in detail in Tuffen and Castro (2009).
308 Specifically, the material was collected from the approximate AO locality given in Tuffen and Castro
309 (2009) and which has a bulk composition given in Table 1, measured using milled powder of the same
310 material by electron microprobe spot analyses (Ludwig-Maximilians-Universität). For the electron
311 microprobe analysis, we used a Cameca SX-100 with LaB₆ cathode and five spectrometers;
312 measurement conditions were 10 μm diameter defocused beam at 5 nA and acceleration voltage of 15
313 kV. For all elements, peak and background measurement times were 10 and 5 s, respectively.

314 As described in Casas et al. (2019), the sample was crushed gently into powder using a Retsch GmbH
315 centrifugal ball mill S1000 under dry conditions at rates sufficiently slow to ensure that no appreciable
316 heating occurred. The material was then sieved to achieve three distinct particle size distributions
317 confirmed using a Beckman-Coulter LS230 laser particle diffraction analyzer. The particle size
318 distributions are given in Fig. 2. Throughout this contribution, we refer to these three size distribution
319 samples by the mean radius $\langle R \rangle$, such that the distributions are $\langle R \rangle = 10.6 \mu\text{m}$, $\langle R \rangle = 19.5 \mu\text{m}$, and
320 $\langle R \rangle = 127.7 \mu\text{m}$ (Fig. 2).

321

322 b. Gas-ash reaction experiments and reaction products

323 The powder samples were loaded into the gas-solid reactor described in Ayriss et al. (2015). This
324 reactor is composed of a hand-blown quartz glass (SiO₂) tube with a sample bulb at one end, and open
325 at both ends. The sample is placed in the bulb which has internal paddles that keep the sample in
326 motion as the bulb and tube rotate. The tube sits in the central hot zone of a 3-zone furnace in which
327 temperature is monitored by a K-type thermocouple accurate to within ± 2 K. Once the sample powder
328 is heated to an isothermal working temperature, an SO₂-Ar mixture for which 1 mol. % is SO₂ and
329 99 mol. % is Ar, is pumped into the furnace and into the glass tube at a constant volumetric flow rate,
330 regulated by flow controllers. While Casas et al. (2019) produce and analyze a larger dataset using
331 other gas mixtures (e.g. SO₂+H₂O), we focus on the SO₂-Ar experiments. The pre- and post-
332 experimental sample surfaces were analysed using a Hitachi field-emission scanning electron
333 microscope SU5000.

334

335 c. Bulk leachate data collection and treatment

336 After each experiment, the samples were taken out of the reactor and left to cool to room temperature.
337 The collected samples were stored in vials. We leached the samples with deionized water for 1 h at a
338 1/250 solid/water ratio, followed by filtration through 0.22 μm mixed cellulose ester membrane. Ion
339 analysis were performed with a Metrohm Ion Chromatography system (733 separation center and 732
340 conductivity detector). This method results in a measurement of the composition of any surface
341 compounds that can be dissolved, such as salts of interest here. Leaching of components from the
342 underlying glass is negligible at these conditions. This allowed the measurement of calcium ion (Ca²⁺)

343 concentration as a function of time and temperature of exposure to SO₂. The value of Ca²⁺ is recorded
344 as a mass fraction of the total mass of the sample. For high temperature volcanic particles in the
345 presence of SO₂ gas, this leachable component has been shown to be the Ca²⁺ that diffused to the
346 surface sites of the particles and formed solid CaSO₄ crystals, which dissolve in the deionized water
347 (Ayris et al. 2013). The average initial calcium concentration in the obsidian is 1.66 ± 0.04 wt.% as
348 CaO, or 1.19 ± 0.04 wt.% as Ca (Table 1), which provides a reference value for use in our analysis.

349 The values of Ca²⁺ concentration leached by our analysis represent the total amount of calcium that
350 has been removed from the rhyolitic glass and deposited on the surface as salts, which we can call C_T .
351 Therefore, taking C_r to be the initial amount, we can define $\langle \bar{C} \rangle = C_r - C_T$ as the concentration
352 remaining in the glass. In Fig. 3 we report the values of C_T relative to C_r , which we term \bar{C}_T . The data
353 demonstrate that the evolution of \bar{C}_T is non-linearly dependent on time of exposure to SO₂ gas, and is
354 also dependent on the temperature and the particle size distribution (Ayris et al. 2013; Casas et al.
355 2019).

356 We convert t into \bar{t} using several different methods, rendering the data of \bar{C}_T a function of
357 dimensionless time. Then we compare these methods with model results for the process. We note that
358 for any of the results using Eq. 3, $\bar{C}_T = 1 - \langle \bar{C} \rangle$, for the monodisperse case, or, additionally when
359 using Eq. 11, $\bar{C}_T = 1 - \langle \bar{C} \rangle$ for the polydisperse case. In Fig. 4, we present the monodisperse solution
360 for $\bar{C}_T = 1 - \langle \bar{C} \rangle$ and the polydisperse solution for $\bar{C}_T = 1 - \langle \bar{C} \rangle$ for each of the three particle size
361 distributions given in Fig. 2. Using the non-dimensionalization of time we describe for diffusion in
362 monodisperse spheres, the solution is universal and invariant of the conditions used. Using the non-
363 dimensionalization of time we describe for diffusion in polydisperse spheres, there is not a unique
364 collapse of the model curves for arbitrary size distribution, although the solution can be computed for
365 any size distribution required.

366

367 **d. Iron oxidation state**

368 The bulk iron oxidation state was measured by positive analysis for FeO wt.% using potentiometric
369 titration based on the method described by Shapiro and Brannock (1956). Using potassium dichromate
370 (K₂Cr₂O₇) 0.01 N as titrant solution, a 665 Metrohm Titroprocessor, a 685 Dosimat automatic titrator
371 connected to an E 649 Magnetic Swing-out Stirrer with an electrode holder and a platinum reference
372 (silver-silver chloride) electrode. The Fe₂O₃ content was determined by difference between wet
373 chemically determined FeO and total Fe from electron microprobe analyses (see below). The initial
374 oxidation state of the iron in the sample was measured using a titration method to be 0.153 Fe³⁺/Fe_T
375 (Casas et al. 2019).

376

377 **e. Electron microprobe analysis of spatial gradients of calcium profiles**

378 For a subset of the experimental samples, we took the resultant post-experimental SO₂-treated glass
379 particles and mounted them in a layer 1-particle deep in resin before polishing them to a depth where
380 the exposed particle cross sections were approximately at a depth such that the particle centres were
381 visible (i.e. to a depth of 1 particle radius). The polished particle sections were analyzed using a
382 Cameca SX100 electron microprobe with a 15 kV accelerating voltage, 15 nA beam current, and a
383 defocused 3-10 μm spot size and ZAF correction procedures. Analyses were performed over 10 s with
384 5 s background measurements. To check for homogeneity and consistency, at least 10 measurements
385 were made at each spot. We measured transects across the particles to check for gradients in particular
386 elements.

387

388

4. Results and analysis

389 We interpret the leached concentrations of Ca^{2+} to be derived exclusively from the soluble salt
390 compounds on the particle surfaces, and not to represent any significant component of calcium derived
391 from the glass itself post-experiment. Here we describe and interpret these experimental results. In this
392 section we assume that $\text{Bi} \gg 1$, such that we use Eq. 8a, rather than Eq. 8b at the particle rim when
393 solving our diffusion model.

394 In this section, we present the experimental data from Figure 3 in dimensionless space. This has the
395 effect that for the monodisperse models (using Eq. 3), there exists a single universal model solution
396 (black curves in Fig. 4), and the data are normalized to compare with this model by fitting for the
397 diffusivity (see below). The same is true for the polydisperse models except that due to our specific
398 choice of normalization (Section 2), there is no single model solution, and subtle differences exist for
399 each model solution for each specific shape of particle size distribution (Fig. 4).

400

401 a. Analysis of data using a diffusivity dependent only on temperature

402 We can normalize the data assuming that the diffusivity \mathcal{D} is a function of temperature only and is not
403 dependent on concentration. To do this, we minimize for the best fit \mathcal{D} that describes a given time-
404 series of data at a given temperature using a least squares fitting algorithm. This results in the best fit
405 \mathcal{D} for each temperature and for each particle size distribution. In Fig. 4a we demonstrate that this
406 fitting results in a good collapse of the data and to the monodisperse solution given by Eq. 3 regardless
407 of polydispersivity, and that the global coefficient of determination is 0.90. For this approach, the
408 time-series data is normalized by $\bar{t} = t/\lambda_{\mathcal{D}} = \mathcal{D}t/R^2$ as described in §2b. In Fig. 4b we show that the
409 effect of polydispersivity can be extracted when compared with Eq. 3 solved with the weighting in Eq.
410 11, where the global coefficient of determination is 0.94. In this approach, the time-series data are
411 normalized by $\bar{t} = t/\lambda_{\mathcal{D}} = S^2\langle\mathcal{D}\rangle t/\langle R \rangle^2$ as described in §2e. The inset to Fig. 4b gives the individual
412 collapses for each particle size distribution compared with the weighted models. This fitting procedure
413 results in a given value of \mathcal{D} for each T given in Table 2.

414

415 b. Analysis of data using a concentration- and temperature-dependent diffusivity

416 Here we normalize the data assuming the diffusivity \mathcal{D} is a function of both temperature and the local
417 concentration of calcium and that this dependence has the form given in Eq. 7. A virtue of our choice
418 of non-dimensionalization is that we can globally fit the data for a given β and \mathcal{D}_{β} (or equivalently for
419 γ and \mathcal{D}_{γ}) in our definition of \mathcal{D}_{τ} . When we adjust these parameters, we find excellent agreement in all
420 cases.

421 First, we focus on the dependence of \mathcal{D} on concentration given in Eq. 7b, which is a linear
422 dependence. When we apply this global fitting procedure in the monodisperse model solving for Eq. 3
423 with Eq. 7b, we find a universal collapse of all data for all temperatures and particle size distributions
424 to the model $1 - \langle\bar{C}\rangle$ with a coefficient of determination of 0.95 (Fig. 4c). This approach uses the
425 normalization of time given by $\bar{t} = t/\lambda_{\mathcal{D}} = \langle\mathcal{D}\rangle t/R^2$ for all data where $\langle\mathcal{D}\rangle$ is computed from the
426 model. Second, when we apply this fitting procedure using the polydisperse model solving Eq. 3 with
427 Eq. 7b, weighted using Eq. 11, we also find a general collapse of each particle size distribution to the
428 respective model result for $1 - \langle\bar{C}\rangle$ with a coefficient of determination of 0.94 (Fig. 4d). This approach
429 uses the normalization of time given by $\bar{t} = t/\lambda_{\mathcal{D}} = S\langle\mathcal{D}\rangle t/\langle R \rangle^2$ for all data where, again, $\langle\mathcal{D}\rangle$ is
430 computed from the model. This provides a globally fitted β and \mathcal{D}_{β} for both monodisperse and
431 polydisperse conditions (Table 2).

432 When fitting using the exponential form of $\mathcal{D}(T, C)$ via Eq. 7c and minimizing for \mathcal{D}_γ and γ , we find a
433 comparable coefficient of determination as given above for the linear dependence of $\mathcal{D}(T, C)$.
434 However, the similarity in coefficient of determination imply that we do not have statistical grounds
435 on which to invoke an exponential form of $\mathcal{D}(T, C)$ and is therefore not shown here graphically.
436 Nevertheless, these results are given in Table 2.

437

438
439

5. Discussion

a. Diffusivity of calcium in rhyolite

440 Using different techniques to fit for the diffusivity of calcium as a function of (1) temperature or (2)
441 temperature and concentration, results in values or relationships that we can compare with previously
442 published data. We collate published values of $\mathcal{D}(T)$ computed from experimental data for diffusion of
443 calcium in rhyolitic materials (Jambon 1982; van der Laan et al. 1994; Mungall et al. 1999) as well as
444 an indirect estimation of calcium diffusivity from experiments performed in a similar manner as those
445 presented here (Ayrís et al. 2013).

446 In Fig. 5 we show how the fitted diffusivities arising from the approach described in §4 compare with
447 published diffusivities. When we use Eq. 7a defining a temperature dependence of \mathcal{D} only, we have a
448 single fit diffusivity for each temperature and each particle size distribution. In Fig. 5a & 5b we show
449 the fitted $\mathcal{D}(T)$ using the monodisperse solution (Fig. 5a) and the polydisperse solution (Fig. 5b). For a
450 given experimental temperature the diffusivities agree relatively well regardless of particle size or
451 particle size distribution. When we use Eq. 7b defining both temperature and a concentration
452 dependence of \mathcal{D} , we have a global Arrhenius fit for each particle size distribution used, rather than
453 individual results for each temperature (see Fig. 5c & 5d). In Fig. 5c, we make the arbitrary choice to
454 show the fit result for $C = C_r$, that is, for when the calcium concentration is at its initial value.
455 Whereas in Fig. 5d, we show the single, universal, polydisperse solution with a direct mapping for the
456 local calcium concentration. In all cases shown in Fig. 5, it is clear that the slope of our $\mathcal{D}(T)$ results
457 differ from previous work shown (Fig. 5), which would relate to an apparent difference in activation
458 energy of the calcium diffusion process. Our shallow temperature dependence relates to a lower
459 activation energy for diffusive mass transport in our case than in previous cases. Mungall et al. (1999)
460 finds that the activation energy is $E = 248 \text{ kJ.mol}^{-1}$. In contrast, we find a good description of our data
461 using a constant $E = 153 \text{ kJ.mol}^{-1}$, independent of the concentration of calcium (e.g. Fig. 5b). This
462 discrepancy warrants investigation.

463 In liquid systems, the relationship between \mathcal{D} and T has been proposed to relate simply to a
464 relationship between the liquid viscosity μ and T (Glasstone et al. 1941). The so-called Eyring
465 diffusivity \mathcal{D}_E is given by

466

$$\mathcal{D}_E = \frac{k_B T}{l \mu} \quad \text{Eq. 12}$$

467

468 where k_B is Boltmann's constant, and l is an effective jump distance of a given diffusing component.
469 The most appropriate diffusing component to associate with the viscosity is oxygen diffusion, with a
470 jump distance of the order of 0.1-1 nm. Specific estimates have been made of $l = 0.5 \text{ nm}$ (Dingwell
471 1990; Mungall 2002) or as low as $l = 0.14 \text{ nm}$ (Watkins et al. 2009).

472 We use a rhyolite viscosity model from Hess and Dingwell (1996) with an initial concentration of
473 dissolved water of 0.14 wt.% (Tuffen and Castro 2009) in order to compute μ for each experimental
474 temperature used here. We then use a subset of the published data for $\mathcal{D}(\mu)$ for calcium diffusion in
475 rhyolite, for which constraint of $\mu(T)$ was given (Mungall et al. 1999; Watkins et al. 2009; Ayrís et al.
476 2013). Taken together, this allows us to re-plot the $\mathcal{D}(T)$ results from Fig. 5 in terms of $\mathcal{D}(\mu)$, along
477 with Eq. 12 (Fig. 6).

478 In Fig. 6, we show that our model results for $\mathcal{D}(T, C)$ are broadly consistent with previous results for
479 calcium diffusivity in rhyolites when examined as $\mathcal{D}(\mu)$. Our model is constrained at a high range of μ
480 (low-temperature), compared with previous results (Fig. 6a). Extrapolation of our model (Fig. 6b)
481 encapsulates the previous data captured at higher relative μ . This leads us to conclude that both the
482 Watkins et al. (2009) and the Mungall et al. (1999) data straddle the transition from the so-called

483 'intrinsic diffusion' regime to the 'extrinsic diffusion' regime, as defined by Dingwell (1990). At high
484 temperatures, or low viscosities, calcium diffuses in rhyolite particles proportional to Eq. 12, while at
485 low temperatures, or high viscosities, calcium diffuses more quickly than would be predicted by Eq.
486 12.

487 A concentration dependence of \mathcal{D} for calcium in rhyolite reported here potentially further explains
488 some of the differences between the values of \mathcal{D} reported in the literature even for experimental work
489 performed in the same region of T . For example, Jambon (1982), Van der Laan et al. (1994), and
490 Mungall et al. (1999) all provide data for \mathcal{D} for calcium in rhyolitic liquids at similar temperatures (or
491 viscosities), and yet their values differ. While this may possibly be ascribed to analytical uncertainty in
492 their methods or to technical differences in experimental design, it is striking to note that the slightly
493 elevated $\mathcal{D}(T)$ values found by Jambon (1982) and Van der Laan et al. (1994) relative to those found
494 by Mungall et al. (1999) in the same range of temperatures, could be ascribed to the fact that the initial
495 concentration of calcium in the starting materials was higher in Jambon (1982) and Van der Laan et al.
496 (1994) compared with the trace quantities synthesized in Mungall et al. (1999). We note a positive
497 dependence of $\mathcal{D}(T)$ on C in our work and, although this dependence is spatially variable as diffusion
498 progresses, it is true that high bulk C_i would result in higher average measured $\langle \mathcal{D} \rangle(T)$.

499

500

b. Diffusion mechanisms and charge compensation by local iron oxidation

501 Casas et al. (2019) showed that Na^+ and K^+ are mobile in addition to Ca^{2+} , feeding the formation of
502 sodium- and potassium-sulfate at the particle surfaces. On the basis that the calcium sulfate is the
503 dominant salt formed, we have neglected the motion of these cations in our model development.
504 Nevertheless, their mobility, however slight, raises the question whether or not multicomponent
505 diffusion of cations affects the motion of calcium. The analysis given in Fig. 6 suggests that casting
506 diffusivities as a function of the material property viscosity effectively explains subtle differences in
507 the temperature-dependence of calcium diffusivity associated with small differences in base rhyolite
508 composition. This has the same effect as the approach taken by Renggli et al. (2019) who normalized
509 temperatures by the glass transition temperature for each composition considered. This approach also
510 suggests that small changes in viscosity associated with the removal of calcium or other cations from
511 the glass may influence the diffusivity of calcium; effectively rendering this a multi-component
512 diffusion problem. To test this, we use a multicomponent viscosity model (Giordano et al. 2008) to
513 confirm that the extraction of calcium and other cations have a negligible effect on viscosity (see sub-
514 vertical temperature-dependent curves in Fig. 6b), and therefore, we conclude that it is likely that
515 calcium is effectively moving independently in this rhyolitic composition. This is further consistent
516 with the initially high proportion of excess network-modifying cations (see Table 1), which are not
517 depleted by the SO_2 reaction.

518 The experiments analyzed here show that in the presence of SO_2 gas, rhyolite particles will form
519 calcium sulfate crystals on their surfaces. The analysis of the leachate data interrogated above then
520 confirms that this is rate-limited by the diffusion of calcium from the bulk of the rhyolite particle
521 toward the surfaces (e.g. Ayris et al. 2013). Conservation of charge neutrality in the melt implies the
522 movement of calcium out of the particles must be compensated. Following Cook and Cooper (2000),
523 we hypothesize that the observed extraction of Ca^{2+} is compensated by the local oxidation of iron
524 $\text{Fe}^{2+} \rightarrow \text{Fe}^{3+}$. The iron oxidation would be stabilized by monovalent cations Na^+ and K^+ if Fe^{3+} then
525 takes a tetrahedral network-forming role (Dickenson and Hess 1986).

526 The above suggests that the SO_2 -rhyolite reaction is accompanied by an inward-moving auto-oxidation
527 front in the particles that proceeds at the same rate as the diffusion of calcium outward, and limited by
528 the diffusivity of calcium out rather than the diffusivity of any oxidizing agent moving inward (Cook
529 and Cooper 2000). We test this hypothesis directly by confirming that while the initial $\text{Fe}^{3+}/\text{Fe}_T$ state
530 of the particles of rhyolite was 0.153, the oxidation state of the particles after 1 hour exposure to SO_2
531 at 800 °C, this value increased to 0.318. In Fig. 7 we show that the mol. fraction of calcium leached

532 from the surface deposits is linearly inversely proportional to the mol. fraction of Fe^{3+} , which is
533 consistent with this charge compensation mechanism.

534 The auto-oxidation of iron to accommodate the flux of calcium out of the particles allows us to place
535 the experimental results analyzed here in the wider context of rhyolites worldwide. For example, we
536 hypothesize this process is therefore limited by both the total Ca^{2+} and Fe^{2+} available initially in the
537 ash particles. Taking a global database of rhyolite glass compositions (Di Genova et al. 2017), we see
538 that the rhyolite used here has more initial total iron and calcium than the mean of natural rhyolite
539 glasses worldwide (Fig. 8). We suggest that it may be high-iron (specifically high reduced iron), high-
540 calcium rhyolites that will be most susceptible to SO_2 -ash reactions after fragmentation.

541

542 **c. The importance of surface reaction rates**

543 As a final test of our model, we apply it directly and without fitting to the spatial distribution of
544 calcium measured in an experimental run product, by using an electron microprobe method (see
545 Section 3). In this case the monodisperse model is sufficient because the particle radius is measured
546 directly.

547 In Fig. 9 we show a typical transect of calcium measured as CaO (wt.%). Running our model with the
548 constraints from the leachate data as inputs, and using the measured R for the particles in question, we
549 find reasonable agreement for $C(r)$ (Fig. 9). These data imply that the value of CaO close to the
550 boundary of the particles does indeed fall to low values consistent with $C_x \approx 0$, which implies $\text{Bi} \gg 1$.
551 Moreover, this suggests that the boundary reaction rate is not rate-limiting. If we relax this assumption
552 and fit for H in the solution to Eq. 3 applied to the data in Fig. 9, we find that $H = 1.96 \times 10^{-9} \text{ m.s}^{-1}$,
553 consistent with $\text{Bi} = 4.02$. This is in the condition $\text{Bi} > 1$, confirming that the reaction at the surface is
554 not rate-limiting. To a first order, this confirms that the kinetics of the particle surface reaction can be
555 neglected and that the process is dominantly rate limited by the motion of calcium from the bulk to the
556 surface by diffusion.

557 Casas et al. (2019) found no textural evidence that the surface of the particles became coated or
558 'armored' by sulfate deposits in these tumbling experiments. However, presumably longer duration
559 experiments without tumbling would result in complete surface coverage, which has been shown to
560 induce a change in dynamics (Renggli and King 2017). Surface coverage would presumably occur at
561 longer experimental timescales, and/or if more initial calcium were available to feed continued
562 reaction.

563

564

565

6. Implications

566 Our model solves the particle-scale kinetics of SO₂-particle reactions in volcanic environments, and is
567 motivated by evidence for sulfate salts on fresh volcanic ash (Rose 1977; Varekamp et al. 1984;
568 Gerlach and McGee 1994; De Hoog et al. 2001; de Moor et al. 2005), which could be evidence for
569 SO₂ scrubbing in volcanic conduits or plumes (Ayriss et al. 2013; Delmelle et al. 2018). Here, we
570 consider a few principal applications for this model.

571 Our kinetic model could be coupled with existing conduit (Degruyter et al. 2012) or volcanic plume
572 models (Costa et al. 2016), to predict the total mass of SO₂ scrubbed in a given eruption. Simple
573 calculations already suggest that the scrubbed quantity could be significant (Ayriss et al. 2013) and that
574 scrubbing may be similarly important for other gases such as HCl (Ayriss et al. 2014). However, these
575 predictions are based on a model that iterates a simplified diffusion calculation for either isothermal
576 conditions, or for average temperature-time pathways in volcanic plumes (Ayriss et al. 2013, 2014),
577 and all for monodisperse particles of a given representative size. In full 3D plume simulations, the
578 velocity and temperature field vary strongly in time and space (Costa et al. 2016), and in volcanic
579 conduits, particle-particle interactions may modify the particle size distribution dynamically (Dufek et
580 al. 2012). These complexities can be accounted for by the model presented herein, which can be
581 coupled to models that predict the evolution of the particle size distribution and temperature in time
582 and space, and therefore may be used to make improved predictions of the scrubbing potential of
583 eruptions of different magnitudes.

584 The model we present and validate herein is a particle-scale model. Solving a particle-scale diffusion
585 model for particles in an evolving plume or conduit flow model would be computationally expensive,
586 if not infeasible. Therefore, in our model, we specifically additionally validate a polydisperse model in
587 which the bulk response of an ensemble of reacting particles can be determined, if the distribution of
588 particle sizes is known. This strength of our polydisperse model would allow it to be coupled to plume
589 models without the need to solve individual particle-scale dynamics and to instead take a volume-
590 averaged approach for each spatial node, rendering the problem computationally tractable.

591 There is a well-documented mismatch between observed atmospheric loading of SO₂ from a given
592 eruption, and the petrological estimate of the sulfur budget from the same magma volume involved in
593 the eruption (Shinohara 2008; Edmonds et al. 2010; Wallace and Edmonds 2011). For silicic
594 eruptions, this mismatch has been resolved by invoking an additional source of sulfur from the crust –
595 e.g. degassed sulfur provided by deeper mafic magmas (Edmonds et al. 2010). Once a total scrubbing
596 potential of a given eruption was predicted using our model in conjunction with a 3D plume model,
597 that would have direct implications for these kinds of mass-balance calculations from which the
598 volumes of mafic magma at depth are estimated (Wallace 2001). Therefore, we envisage that
599 implications of a well-calibrated SO₂ scrubbing model are not only for atmospheric SO₂ loads, but also
600 for crustal sulfur budgets. The nature of these particle-SO₂ reactions is such that it may represent a key
601 missing reaction pathway in the volcanic sulfur cycle.

602 Future work could apply this model directly to estimate bulk SO₂ scrubbing efficiency in volcanic
603 conduits and in the hot core of volcanic plumes. This would require independent constraint of the
604 temperature and velocity field to which the particles are exposed, but once known, our model is
605 sufficiently flexible that the non-isothermal complex temperature-time pathway experienced by a
606 batch of particles can be accommodated. The desirable output is a prediction of the bulk mass of SO₂
607 reacted in any closed system, that goes beyond the simple calculations that have been performed
608 previously.

609

610 **Acknowledgments**

611 We are grateful to Elena Maters, Ed Llewelin, Antonio Costa, Mike James, and Madeleine
612 Humphreys for stimulating discussion, to Penny King and 3 anonymous reviewers for feedback, and to
613 Charles Lesher and Don Baker for editorial oversight and Daniel Neuville for editing the first
614 submission in the same journal. We acknowledge funding from the European Research Council, ERC
615 2018 ADV Grant 834225 (EAVESDROP) and support from the Centre for Advanced Study in
616 Munich, Germany.

617 **Author contributions**

618 F.B. Wadsworth, A.S. Casas, J. Vasseur, D.B. Dingwell and P.M. Ayris conceived and coordinated the
619 work, A.S. Casas led the experimental campaign published elsewhere (Casas et al. 2019), J. Vasseur,
620 K.-U. Hess, F.B. Wadsworth, and P. Delmelle analyzed the data. All authors contributed significantly
621 to the manuscript.

622

623

References cited

- 624 Ayris, P.M., Lee, A.F., Wilson, K., Kueppers, U., Dingwell, D.B., and Delmelle, P. (2013) SO₂ sequestration in large volcanic eruptions: High-temperature scavenging by tephra.
625
626 *Geochimica et cosmochimica acta*, 110, 58–69.
- 627 Ayris, P.M., Delmelle, P., Cimarelli, C., Maters, E.C., Suzuki, Y.J., and Dingwell, D.B. (2014) HCl
628 uptake by volcanic ash in the high temperature eruption plume: Mechanistic insights.
629 *Geochimica et Cosmochimica Acta*, 144, 188–201.
- 630 Ayris, P.M., Cimarelli, C., Delmelle, P., Wadsworth, F.B., Vasseur, J., Suzuki, Y.J., and Dingwell,
631 D.B. (2015) A novel apparatus for the simulation of eruptive gas-rock interactions. *Bulletin of*
632 *Volcanology*, 77, 1–5.
- 633 Baldini, J.U.L., Brown, R.J., and McElwaine, J.N. (2015) Was millennial scale climate change during
634 the Last Glacial triggered by explosive volcanism? *Scientific Reports*, 5.
- 635 Bluth, G.J.S., Rose, W.I., Sprod, I.E., and Krueger, A.J. (1997) Stratospheric loading of sulfur from
636 explosive volcanic eruptions. *Journal of Geology*, 105, 671–683.
- 637 Burnett, D.S., Goreva, J., Epstein, S., Haldemann, S.L., Johnson, M.L., and Rice, A. (1997) SO₂-rock
638 interaction on Io 2. Interaction with pure so₂. *Journal of Geophysical Research E: Planets*, 102,
639 19371–19382.
- 640 Carn, S.A., Fioletov, V.E., Mclinden, C.A., Li, C., and Krotkov, N.A. (2017) A decade of global
641 volcanic SO₂ emissions measured from space. *Scientific Reports*, 7, 1–12.
- 642 Casas, A.S., Wadsworth, F.B., Ayris, P.M., Delmelle, P., Vasseur, J., Cimarelli, C., and Dingwell,
643 D.B. (2019) SO₂ scrubbing during percolation through rhyolitic volcanic domes. *Geochimica et*
644 *Cosmochimica Acta*.
- 645 Cook, G.B., and Cooper, R.F. (2000) Iron concentration and the physical processes of dynamic
646 oxidation in an alkaline earth aluminosilicate glass. *American Mineralogist*, 85, 397–406.
- 647 Costa, A., Suzuki, Y., Cerminara, M., Devenish, B., Esposti Ongaro, T., Herzog, M., Van Eaton, A.,
648 Denby, L., Bursik, M., de' Michieli Vitturi, M., and others (2016) Results of the eruptive column
649 model inter-comparison study. *Journal of Volcanology and Geothermal Research*, 326, 2–25.
- 650 Crank, J. (1975) *The mathematics of diffusion*, 415 p.
- 651 De Hoog, J.C.M., Koetsier, G.W., Bronto, S., Sriwana, T., and Van Bergen, M.J. (2001) Sulfur and
652 chlorine degassing from primitive arc magmas: Temporal changes during the 1982-1983
653 eruptions of Galunggung (West Java, Indonesia). *Journal of Volcanology and Geothermal*
654 *Research*, 108, 55–83.
- 655 de Moor, J.M., Fischer, T.P., Hilton, D.R., Hauri, E., Jaffe, L.A., and Camacho, J.T. (2005) Degassing
656 at Anatahan volcano during the May 2003 eruption: Implications from petrology, ash leachates,
657 and SO₂ emissions. *Journal of Volcanology and Geothermal Research*, 146, 117–138.
- 658 Degruyter, W., Bachmann, O., Burgisser, A., and Manga, M. (2012) The effects of outgassing on the
659 transition between effusive and explosive silicic eruptions. *Earth and Planetary Science Letters*,
660 349, 161–170.
- 661 Delmelle, P., Wadsworth, F.B., Maters, E.C., and Ayris, P.M. (2018) 8. High Temperature Reactions
662 Between Gases and Ash Particles in Volcanic Eruption Plumes. In *High Temperature Gas-Solid*
663 *Reactions in Earth and Planetary Processes* pp. 285–308.

- 664 Di Genova, D., Kolzenburg, S., Wiesmaier, S., Dallanave, E., Neuville, D.R., Hess, K.U., and
665 Dingwell, D.B. (2017) A compositional tipping point governing the mobilization and eruption
666 style of rhyolitic magma. *Nature*, 552, 235–238.
- 667 Dickenson, M.P., and Hess, P.C. (1986) The structural role and homogeneous redox equilibria of iron
668 in peraluminous, metaluminous and peralkaline silicate melts. *Contributions to Mineralogy and
669 Petrology*, 92, 207–217.
- 670 Dingwell, D. (1990) Effects of structural relaxation on cationic tracer diffusion in silicate melts.
671 *Chemical Geology*.
- 672 Douglas, R.W., and Isard, J.O. (1949) The action of water and of sulphur dioxide on glass surfaces. *J.
673 Soc. Glass Technol*, 33, 289–335.
- 674 Dufek, J., Manga, M., and Patel, A. (2012) Granular disruption during explosive volcanic eruptions.
675 *Nature Geoscience*.
- 676 Edmonds, M., Aiuppa, A., Humphreys, M., Moretti, R., Giudice, G., Martin, R.S., Herd, R.A., and
677 Christopher, T. (2010) Excess volatiles supplied by mingling of mafic magma at an andesite arc
678 volcano. *Geochemistry, Geophysics, Geosystems*, 11, n/a-n/a.
- 679 Gerlach, T.M., and McGee, K.A. (1994) Total sulfur dioxide emissions and pre-eruption vapor-
680 saturated magma at Mount St. Helens, 1980–88. *Geophysical Research Letters*, 21, 2833–2836.
- 681 Giordano, D., Russell, J.K., and Dingwell, D.B. (2008) Viscosity of magmatic liquids: a model. *Earth
682 and Planetary Science Letters*, 271, 123–134.
- 683 Glasstone, S., Laidler, K., and Eyring, H. (1941) The theory of rate processes; the kinetics of chemical
684 reactions, viscosity, diffusion and electrochemical phenomena.
- 685 Henley, R.W., King, P.L., Wykes, J.L., Renggli, C.J., Brink, F.J., Clark, D.A., and Troitzsch, U.
686 (2015) Porphyry copper deposit formation by sub-volcanic sulphur dioxide flux and
687 chemisorption. *Nature Geoscience*, 8, 210–215.
- 688 Hess, K.-U.U., and Dingwell, D.B. (1996) Viscosities of hydrous leucogranitic melts: A non-
689 Arrhenian model. *American Mineralogist*, 81, 1297–1300.
- 690 Hsu, K.H. (1983) A Diffusion Model with a Concentration-Dependent Diffusion Coefficient for
691 Describing Water Movement in Legumes During Soaking. *Journal of Food Science*, 48, 618–
692 622.
- 693 Jambon, A. (1982) Tracer diffusion in granitic melts: experimental results for Na, K, Rb, Cs, Ca, Sr,
694 Ba, Ce, Eu to 1300 C and a model of calculation. *Journal of Geophysical Research: Solid Earth*.
- 695 Johnson, M.L., and Burnett, D.S. (1993) SO₂-rock interaction on Io: reaction under highly oxidizing
696 conditions. *Journal of Geophysical Research*, 98, 1223–1230.
- 697 King, P.L., Wheeler, V.M., Renggli, C.J., Palm, A.B., Wilson, S.A., Harrison, A.L., Morgan, B.,
698 Nekvasil, H., Troitzsch, U., Mernagh, T., and others (2018, November 1) Gas-solid reactions:
699 Theory, experiments and case studies relevant to earth and planetary processes. *Reviews in
700 Mineralogy and Geochemistry*. Mineralogical Society of America.
- 701 Kueppers, U., Scheu, B., Spieler, O., and Dingwell, D.B. (2006) Fragmentation efficiency of explosive
702 volcanic eruptions: A study of experimentally generated pyroclasts. *Journal of Volcanology and
703 Geothermal Research*, 153, 125–135.
- 704 Mavrogenes, J., and Blundy, J. (2017a) Crustal sequestration of magmatic sulfur dioxide. *Geology*, 45,

- 705 211–214.
- 706 ——— (2017b) Crustal sequestration of magmatic sulphur dioxide as an ore-forming process. *Applied*
707 *Earth Science*, 126, 76–77.
- 708 Morioka, M. (1981) Cation diffusion in olivine-II. Ni-Mg, Mn-Mg, Mg and Ca. *Geochimica et*
709 *Cosmochimica Acta*, 45, 1573–1580.
- 710 Mungall, J. (2002) Empirical models relating viscosity and tracer diffusion in magmatic silicate melts.
711 *Geochimica et Cosmochimica Acta*.
- 712 Mungall, J., Dingwell, D., and Chaussidon, M. (1999) Chemical diffusivities of 18 trace elements in
713 granitoid melts. *Geochimica et Cosmochimica*.
- 714 Mysen, B., and Richet, P. (2005) *Silicate Glasses and Melts*, 544 p. *Silicate Glasses and Melts*.
- 715 Palm, A.B., King, P.L., Renggli, C.J., Hervig, R.L., Dalby, K.N., Herring, A., Mernagh, T.P., Eggins,
716 S.M., Troitzsch, U., Beeching, L., and others (2018) Unravelling the consequences of SO₂-basalt
717 reactions for geochemical fractionation and mineral formation. *Reviews in Mineralogy and*
718 *Geochemistry*, 84, 257–283.
- 719 Renggli, C., and King, P. (2017) An experimental investigation of reactions between SO₂ and
720 Anorthite-Diopside glasses.
- 721 Renggli, C.J., and King, P.L. (2018) SO₂ Gas Reactions with Silicate Glasses. *Reviews in Mineralogy*
722 *and Geochemistry*, 84, 229–255.
- 723 Renggli, C.J., Palm, A.B., King, P.L., and Guagliardo, P. (2019) Implications of Reactions Between
724 SO₂ and Basaltic Glasses for the Mineralogy of Planetary Crusts. *Journal of Geophysical*
725 *Research: Planets*, 124, 2563–2582.
- 726 Robock, A. (2000) Volcanic eruptions and climate. *Reviews of Geophysics*, 38, 191–219.
- 727 Rose, W.I. (1977) Scavenging of volcanic aerosol by ash: Atmospheric and volcanologic implications.
728 *Geology*, 5, 621–624.
- 729 Saadatfar, M., Brink, F., Latham, S., King, P., Middleton, J., Troitzsch, U., Turner, M., and Henley,
730 R.W. (2020) High resolution 3D mapping of grain kinematics during high temperature
731 sequestration of SO₂ from flue gas by carbonate aggregates. *Scientific Reports*, 10, 2201.
- 732 Shapiro, L., and Brannock, W. (1956) Rapid analysis of silicate rocks.
- 733 Shinohara, H. (2008) Excess degassing from volcanoes and its role on eruptive and intrusive activity.
734 *Reviews of Geophysics*, 46.
- 735 Textor, C., Graf, H.F., Herzog, M., and Oberhuber, J.M. (2003) Injection of gases into the stratosphere
736 by explosive volcanic eruptions. *Journal of Geophysical Research D: Atmospheres*, 108.
- 737 Torquato, S. (2013) *Random heterogeneous materials: microstructure and macroscopic properties*, 701
738 p. Vol. 16. Springer Science & Business Media.
- 739 Tuffen, H., and Castro, J.M. (2009) The emplacement of an obsidian dyke through thin ice:
740 Hrafnatinnuhryggur, Krafla Iceland. *Journal of Volcanology and Geothermal Research*, 185, 352–
741 366.
- 742 van der Laan, S., Zhang, Y., Kennedy, A.K., and Wyllie, P.J. (1994) Comparison of element and
743 isotope diffusion of K and Ca in multicomponent silicate melts. *Earth and Planetary Science*

- 744 Letters, 123, 155–166.
- 745 Varekamp, J.C., Luhr, J.F., and Prestegard, K.L. (1984) The 1982 eruptions of El Chichón Volcano
746 (Chiapas, Mexico): Character of the eruptions, ash-fall deposits, and gasphase. *Journal of*
747 *Volcanology and Geothermal Research*, 23, 39–68.
- 748 Vasseur, J., Wadsworth, F.B., and Dingwell, D.B. (2020) Permeability of polydisperse magma foam.
749 *Geology*.
- 750 Wadsworth, F.B., Vasseur, J., Llewellyn, E.W., and Dingwell, D.B. (2017a) Sintering of polydisperse
751 viscous droplets. *Physical Review E*, 95, 033114.
- 752 Wadsworth, F.B., Vasseur, J., Llewellyn, E.W., Genareau, K., Cimarelli, C., and Dingwell, D.B.
753 (2017b) Size limits for rounding of volcanic ash particles heated by lightning. *Journal of*
754 *Geophysical Research: Solid Earth*, 122, 1977–1989.
- 755 Wallace, P.J. (2001) Volcanic SO₂ emissions and the abundance and distribution of exsolved gas in
756 magma bodies. *Journal of Volcanology and Geothermal Research*, 108, 85–106.
- 757 Wallace, P.J., and Edmonds, M. (2011) The sulfur budget in magmas: Evidence from melt inclusions,
758 submarine glasses, and volcanic gas emissions. *Reviews in Mineralogy and Geochemistry*, 73,
759 215–246.
- 760 Watkins, J., DePaolo, D., and Huber, C. (2009) Liquid composition-dependence of calcium isotope
761 fractionation during diffusion in molten silicates. *et Cosmochimica Acta*.
- 762 Zhang, Y., and Cherniak, D.J. (2010) Diffusion in minerals and melts: Introduction. *Reviews in*
763 *Mineralogy and Geochemistry*, 72, 1–4.
- 764 Zhang, Y., and Ni, H. (2010) Diffusion of H, C, and O components in silicate melts. *Reviews in*
765 *Mineralogy and Geochemistry*, 72, 171–225.
- 766
- 767
- 768

769

Table 1 The composition of the experimental material

Oxide	From Tuffen & Castro (2009) n=100 [in wt.%]	This study using electron microprobe n=9 [in wt.%]	This study using electron microprobe n=9 [in re- normalized mol.%]
SiO ₂	75.17 ± 0.41	75.20 ± 0.33	80.89 ± 0.39
TiO ₂	0.22 ± 0.02	0.21 ± 0.04	0.17 ± 0.04
Al ₂ O ₃	12.02 ± 0.16	12.06 ± 0.12	7.64 ± 0.08
FeO _T	3.13 ± 0.18	3.11 ± 0.11	2.37 ± 0.09
Fe ₂ O ₃	-	0.88 ± 0.03	0.36 ± 0.01
FeO	-	2.23 ± 0.08	2.01 ± 0.08
MnO	0.11 ± 0.04	0.11 ± 0.04	0.10 ± 0.04
MgO	0.09 ± 0.02	0.08 ± 0.03	0.13 ± 0.05
CaO*	1.66 ± 0.10	1.66 ± 0.11	1.91 ± 0.14
Na ₂ O	4.58 ± 0.09	4.61 ± 0.08	4.81 ± 0.09
K ₂ O	2.88 ± 0.11	2.90 ± 0.09	1.99 ± 0.07
Total	99.86 ± 0.41	99.94 ± 0.50	100.0
Mol. fraction excess oxides**			0.032
NBO/T**			0.070

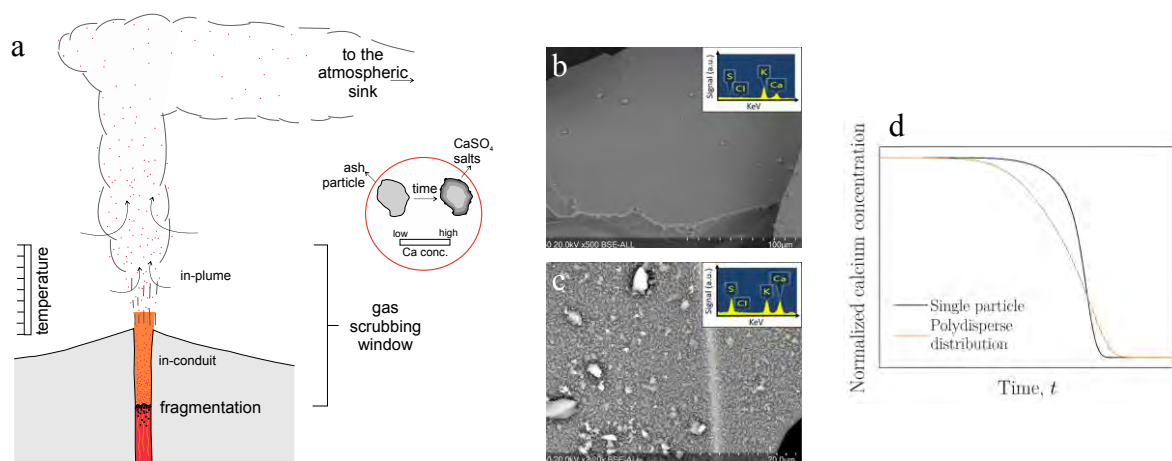
*The wt.% value of CaO from this study is taken as C_r .

**Calculated using Mysen and Richet (2005)

770

771

772



773

774 **Figure 1.**

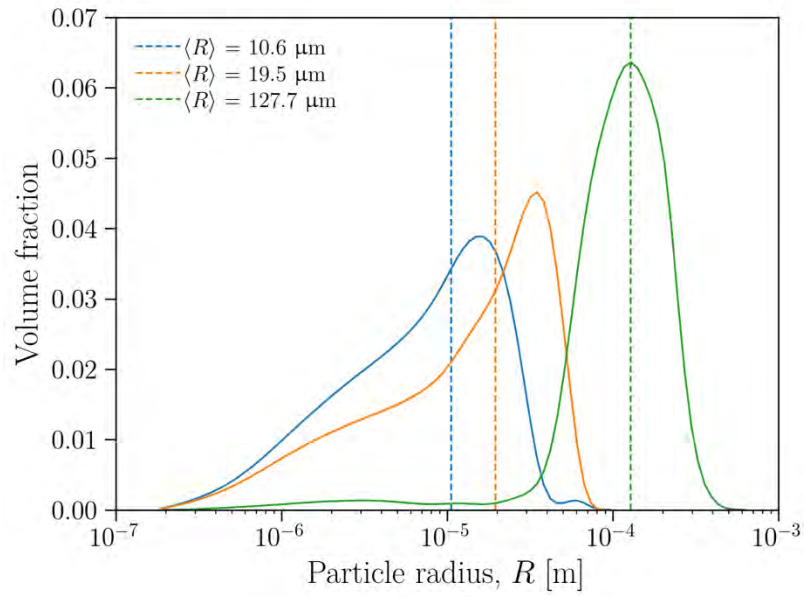
775 The gas-particle reaction window in natural volcanic systems. (a) A cartoon of an eruption column
776 cross section, showing the fragmentation level above which the gas, ash, and pyroclasts decouple and
777 reactions can take place. *Inset:* A schematic of the particle-scale reaction described in Ayriss et al.
778 (2013) in which a particle exposed to SO₂ for time t at temperature T will form surface salts fed by
779 cation diffusion toward the particle surface. (b-c) SEM images of the experimental products analysed
780 here; (b) the starting material prior to treatment with SO₂, and (c) a surface of the experimental particle
781 after exposure to SO₂ at 800 C for 1 hour. The images in panels (b) and (c) are reproduced from Casas
782 et al. (2019) with permission. (d) A schematic view of the important difference of a polydisperse
783 compared with a monodisperse (or single particle) solution to Fickian diffusion.

784

785

786

787

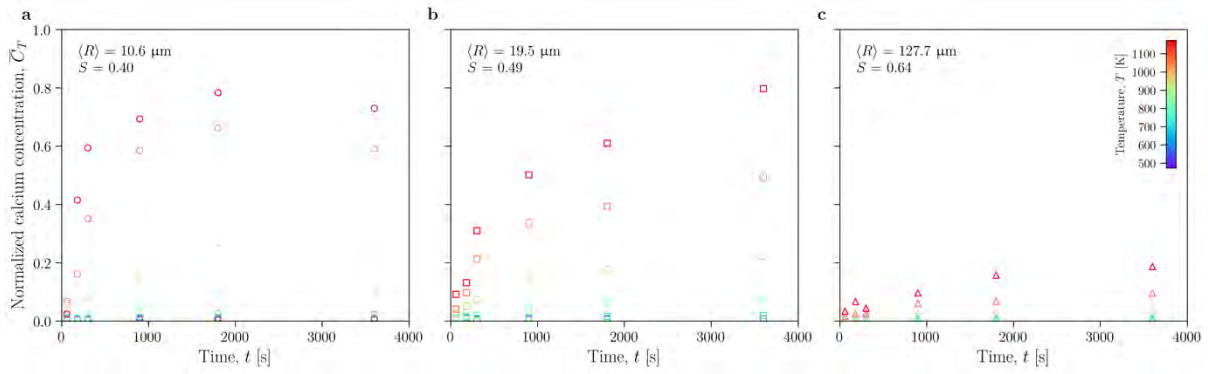


788

789 **Figure 2.**

790 The particle size distribution of the three populations of rhyolitic particles used in this study. Vertical
791 dashed lines mark the arithmetic mean radius $\langle R \rangle$, which is also given on the figure as a numeric
792 value.

793



794

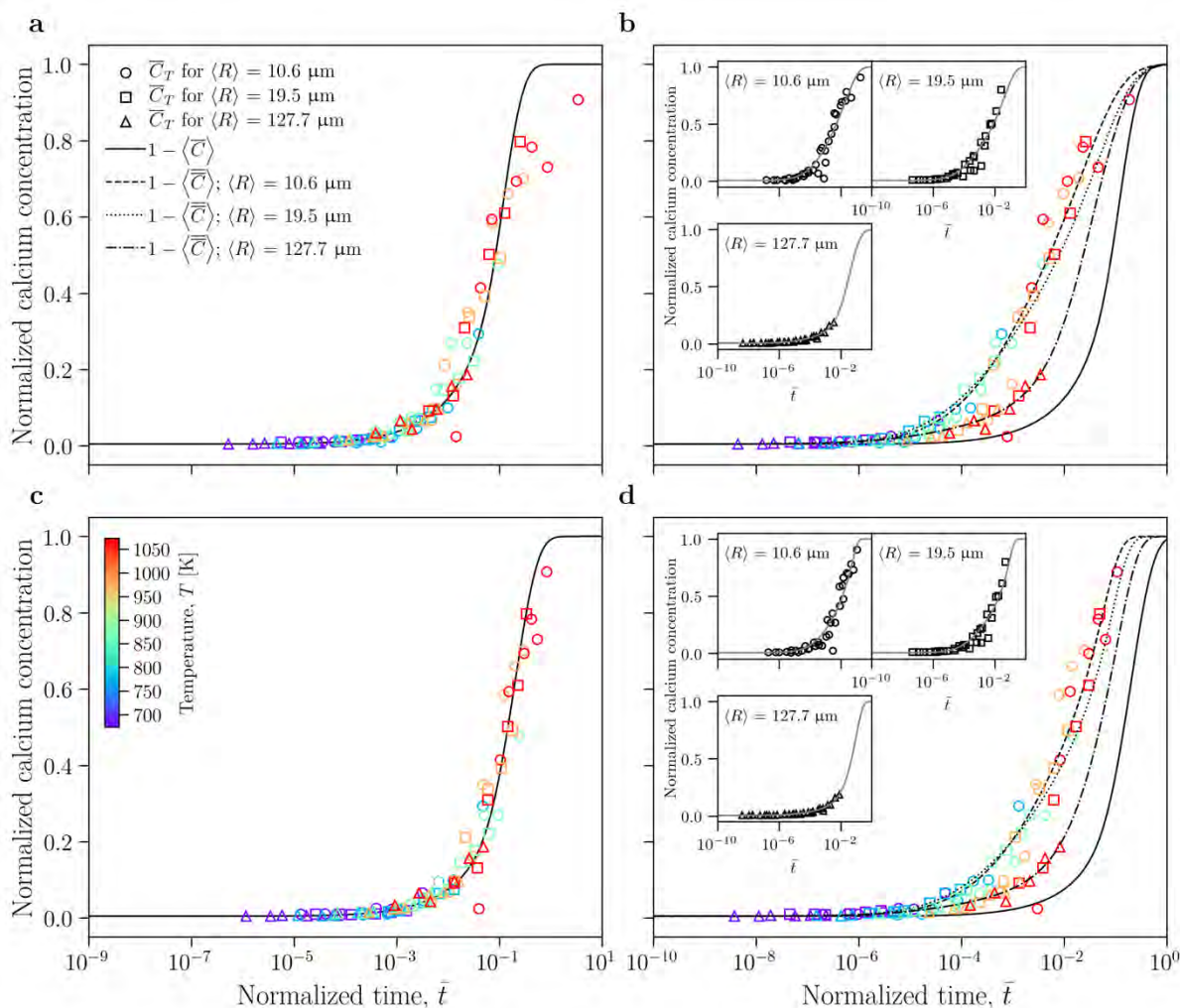
795 **Figure 3.**

796 Raw experimental results for calcium concentration leached from samples post-experiment normalized
797 by the total calcium in the initial material, giving dimensionless $1 - \bar{C}_T$, as a function of the time of
798 exposure to bypassing high temperature SO_2 -Ar gas mixtures for each particle size distribution (a)
799 $\langle R \rangle = 10.6 \mu\text{m}$, (b) $\langle R \rangle = 19.5 \mu\text{m}$, and (c) $\langle R \rangle = 127.7 \mu\text{m}$. Colors refer to temperature of exposure
800 and labelled are the polydispersity values S for each distribution.

801

802

803



804

805 **Figure 4.**

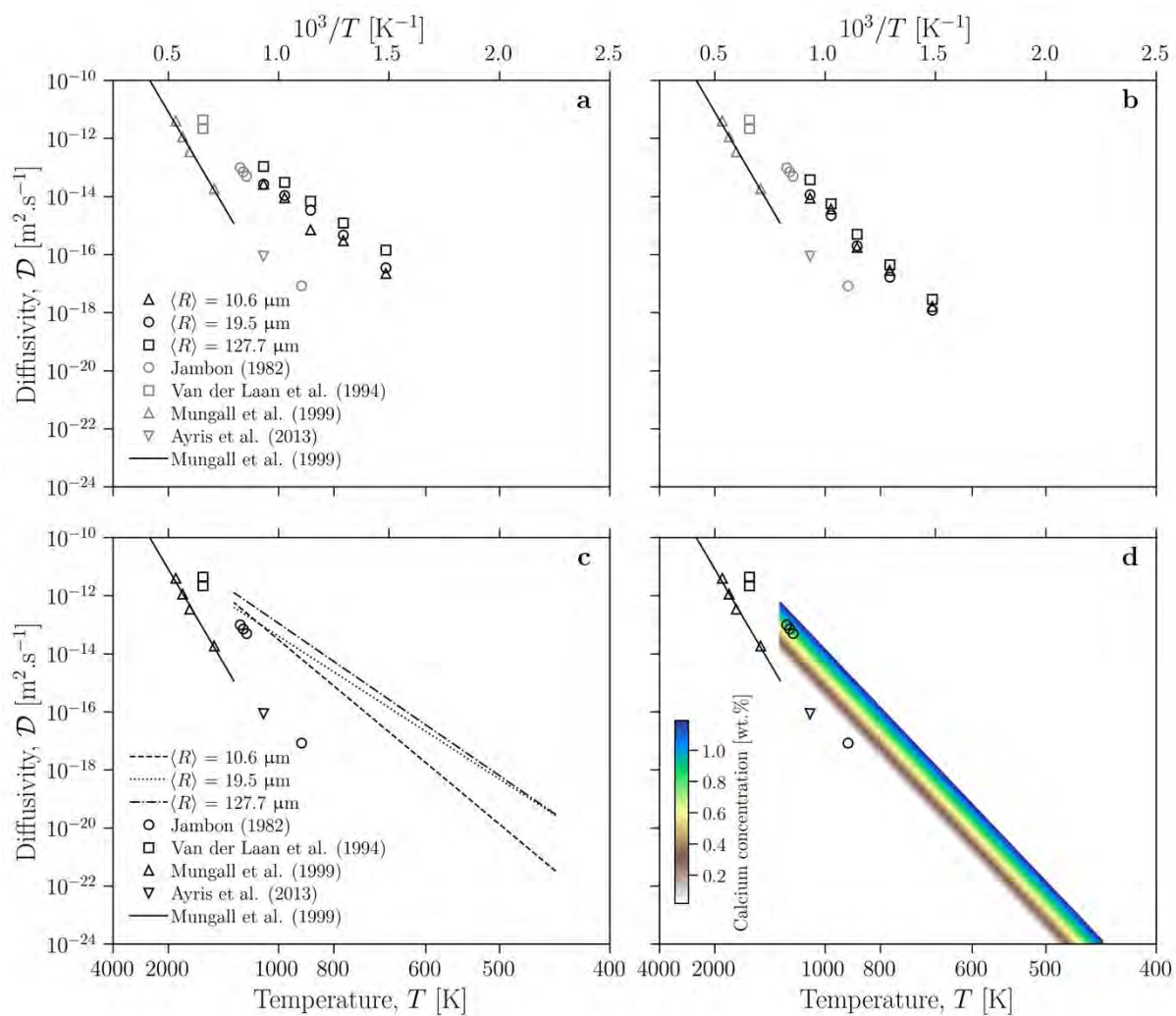
806 The data presented in Fig. 3 where time t is transformed to dimensionless time \bar{t} by $\bar{t} = t/\lambda_D$ (see
 807 text). In panels (a) and (b) we show the comparison between the experimental data and our model for
 808 the case where the calcium diffusivity in λ_D depends only on temperature via Eq. 4 (or equivalently,
 809 via Eq. 7a). In this case the fit parameters are \mathcal{D}_α and α (see Table 2). In panels (c) and (d) we show
 810 the same comparison between the experimental data and our model, but for the case where the calcium
 811 diffusivity additionally depends on the local concentration of calcium via Eq. 5a (or equivalently, via
 812 Eq. 7b). In this case the fit parameters are \mathcal{D}_γ and γ (see Table 3). In panels (a) and (c), we assume the
 813 particles are monodisperse with a size described by the mean particle size, whereas in panels (b) and
 814 (d), we use Eq. 11 to account for the particle size distribution polydispersity (*insets*: the individual
 815 results for each particle size distribution).

816

817

818

819



820

821 **Figure 5.**

822 The fitted calcium diffusivity as a function of temperature for the case where the diffusivity is
 823 temperature-dependent only – shown in (a) and (b) – and for the case where the diffusivity is
 824 dependent on both temperature and the local concentration of calcium – shown in (c) and (d). In panels
 825 (a) and (c), we assume the particles are monodisperse with a size described by the mean particle size,
 826 whereas in panels (b) and (d), we account for the particle size distribution polydispersity.

827

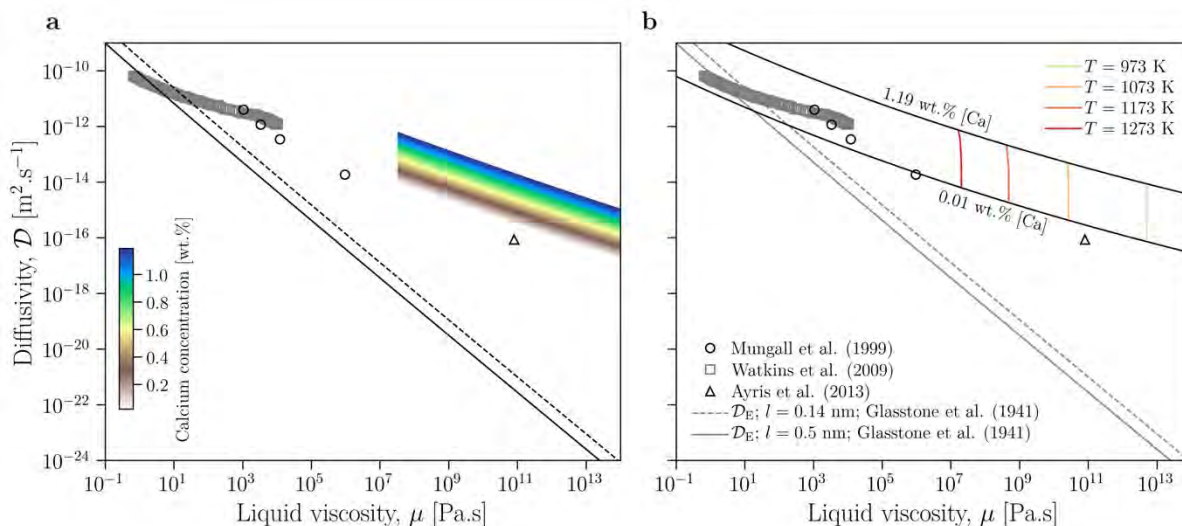
828

829

830

831

832



833

834 **Figure 6.**

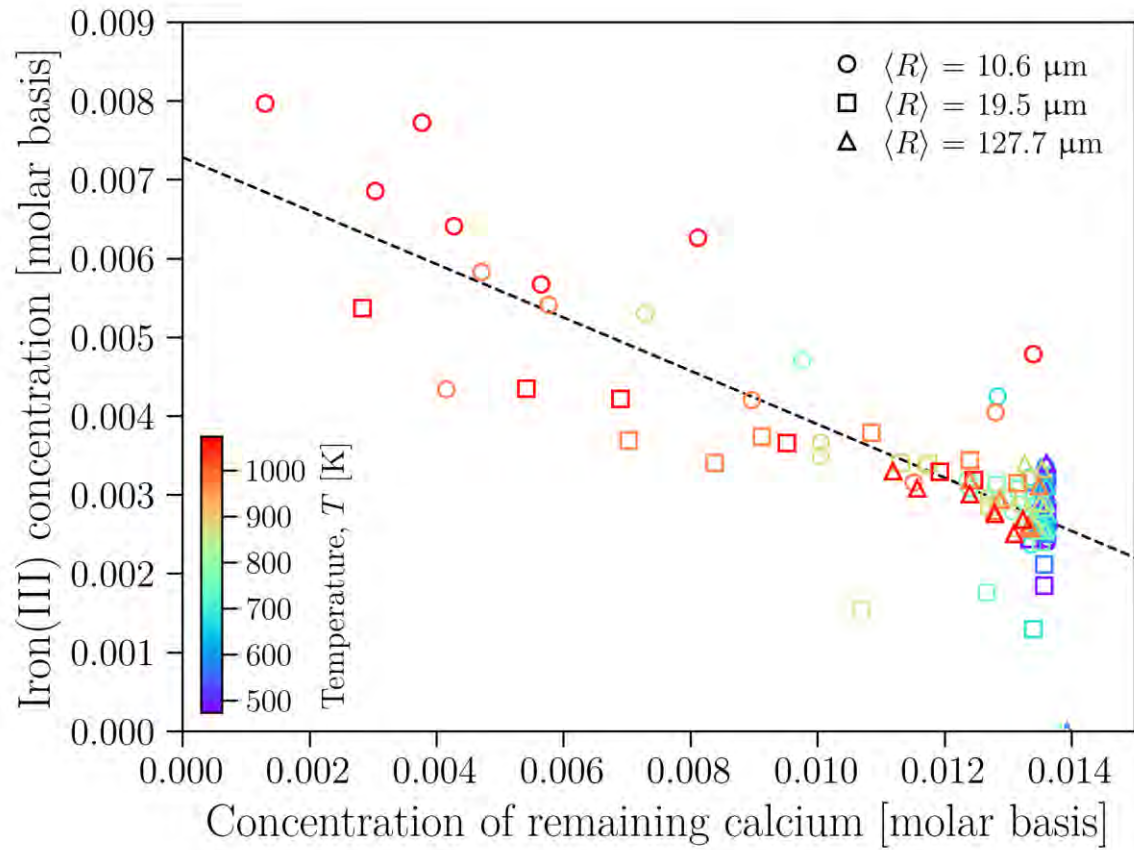
835 The results shown in Fig. 5, but where T has been converted to μ , using either measured μ values for
 836 the materials used (i.e. in the case of Mungall et al. 1999), or calculated $\mu(T)$ values given by the
 837 model by Hess and Dingwell (1996), valid for rhyolites assuming nominally dry values of 0.06 wt.%
 838 total H_2O dissolved in the liquid (applied to the results of this study and to Ayris et al. 2013). In the
 839 case of Watkins et al. (2009), the values of μ are given directly. Additionally shown is the $\mathcal{D}_E(\mu)$
 840 relationship Eq. 12, calculated for oxygen diffusivity using a jump-distance of either $l = 0.14$ or
 841 $l = 0.5$ nm. (a) Our model is given for a continuous range of calcium concentrations. (b) Our model is
 842 given for two different calcium concentrations and extrapolated to low- μ . The curves in (b) that are
 843 marked for temperature are the combined \mathcal{D} and μ variation as particles lose calcium, showing that μ
 844 is not strongly affected by diffusive loss of calcium in this case (calculated using Giordano et al.
 845 2008).

846

847

848

849

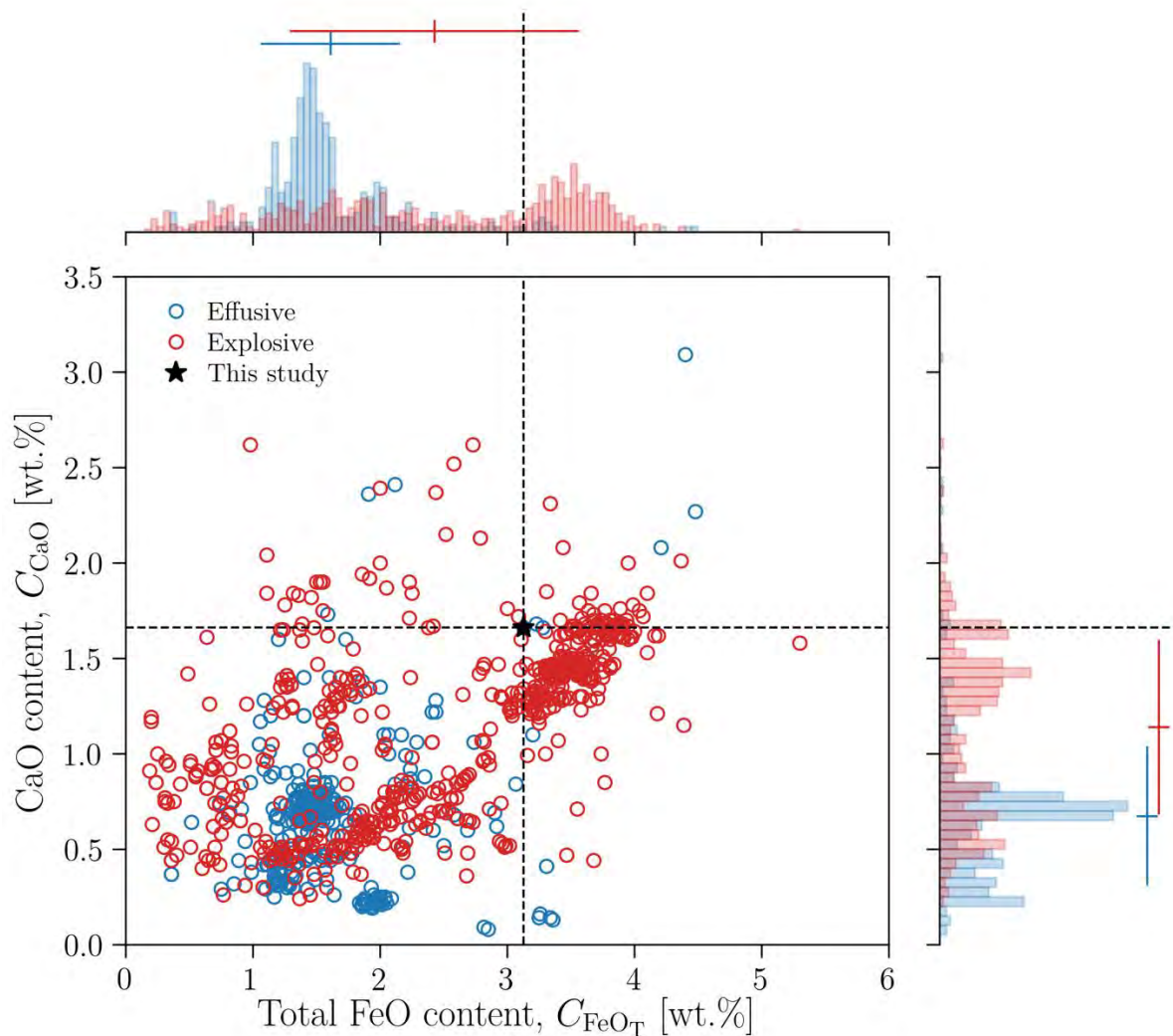


850

851 **Figure 7.**

852 The relationship between Fe^{3+} and C_T (both in mol. fraction and calculated on a single oxygen basis),
853 showing a linear relationship for which the slope is approximately $-1/3$.

854



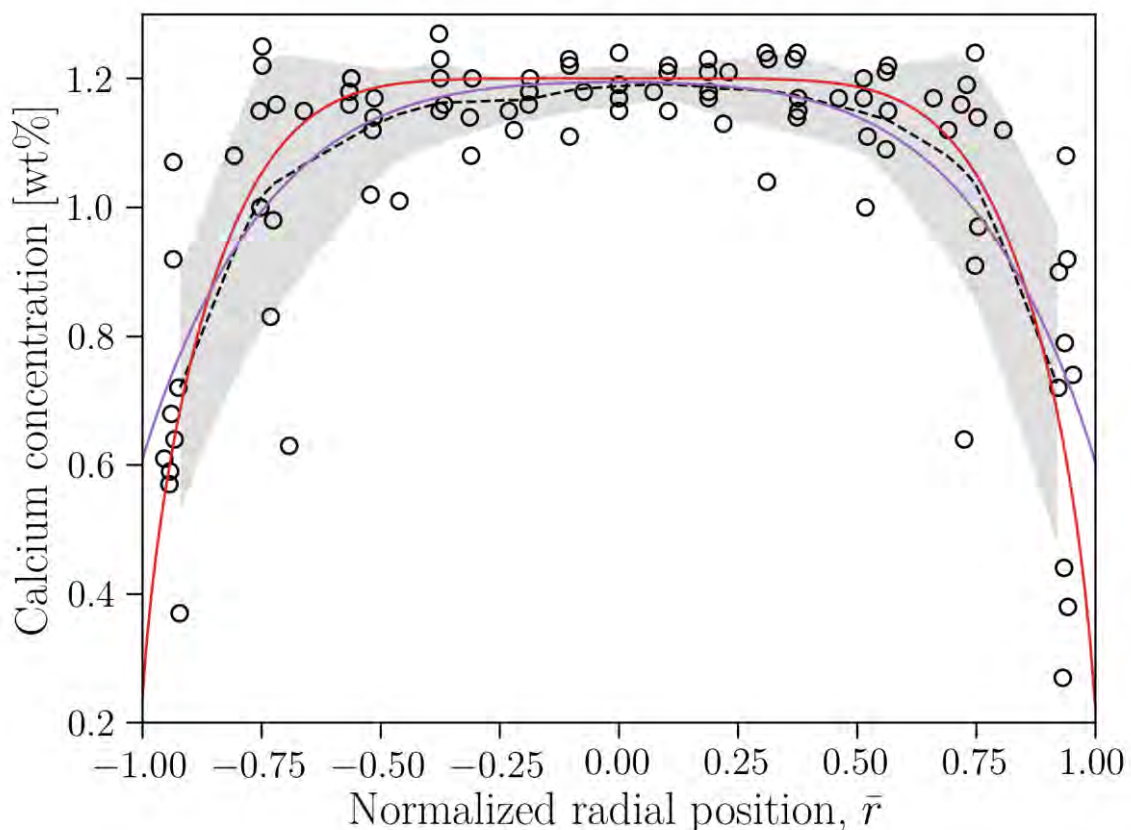
855

856 **Figure 8.**

857 CaO and FeO_T compositions of natural rhyolite glass worldwide (Using the composition database
858 from Di Genova et al. 2017), divided into glass deposited from explosive or effusive eruptions. Here
859 we note that it is the iron-rich, calcium-rich eruption products that are most susceptible to SO_2 -ash
860 reactions.

861

862



863

864 **Figure 9.**

865 Independent model verification using spatial gradients of calcium within rhyolite particles that have
866 been exposed to SO_2 for 1 hour at 800 °C. Here $\langle R \rangle = 19.5 \mu\text{m}$ and $\bar{r} = r/R$ is the spatial position in
867 the particle relative to the centre where $\bar{r} = 0$. Plotted are single transects across individual particles
868 (black circles). The grey shaded area represents the standard error about the mean on repeat
869 measurements. The interpolated mean of the data is the dashed curve. The red curve represents our
870 model where $\text{Bi} \gg 1$ and the reaction rate is neglected (such that $C \rightarrow 0$ at $\bar{r} = 1$ and $\bar{r} = -1$ for all
871 $\bar{t} > 0$), and the purple curve represents our model where $\text{Bi} = 4.02$.

872

873

874

Table 2

T [K]
673
773
873
973
1073

T [K]
673
773
873
973
1073

\bar{C}_r
 \bar{C}_r

Results of analysis for temperature-dependent diffusivity

Monodisperse

$\langle R \rangle = 10.6 \mu\text{m}$	$\langle R \rangle = 19.5 \mu\text{m}$	$\langle R \rangle = 127.7 \mu\text{m}$	Average
D	D	D	
$[\text{m}^2.\text{s}^{-1}]$	$[\text{m}^2.\text{s}^{-1}]$	$[\text{m}^2.\text{s}^{-1}]$	$[\text{m}^2.\text{s}^{-1}]$
$2.25 \pm 0.25 \times 10^{-17}$	$3.49 \pm 0.75 \times 10^{-17}$	$1.43 \pm 0.65 \times 10^{-16}$	6.68×10^{-17}
$3.03 \pm 0.21 \times 10^{-16}$	$4.76 \pm 0.39 \times 10^{-16}$	$1.23 \pm 0.25 \times 10^{-15}$	6.70×10^{-16}
$7.12 \pm 1.23 \times 10^{-16}$	$3.35 \pm 0.36 \times 10^{-15}$	$6.86 \pm 1.11 \times 10^{-15}$	3.64×10^{-15}
$8.94 \pm 2.13 \times 10^{-15}$	$1.09 \pm 0.18 \times 10^{-14}$	$3.10 \pm 0.27 \times 10^{-14}$	1.70×10^{-14}
$2.64 \pm 0.94 \times 10^{-14}$	$2.64 \pm 0.39 \times 10^{-14}$	$1.06 \pm 0.12 \times 10^{-13}$	5.30×10^{-14}

Polydisperse

$\langle R \rangle = 10.6 \mu\text{m}$	$\langle R \rangle = 19.5 \mu\text{m}$	$\langle R \rangle = 127.7 \mu\text{m}$	Average
D	D	D	
$[\text{m}^2.\text{s}^{-1}]$	$[\text{m}^2.\text{s}^{-1}]$	$[\text{m}^2.\text{s}^{-1}]$	$[\text{m}^2.\text{s}^{-1}]$
$1.60 \pm 0.16 \times 10^{-18}$	$1.20 \pm 0.23 \times 10^{-18}$	$2.89 \pm 1.41 \times 10^{-18}$	1.90×10^{-18}
$2.96 \pm 0.58 \times 10^{-17}$	$1.65 \pm 0.13 \times 10^{-17}$	$4.49 \pm 0.84 \times 10^{-17}$	3.03×10^{-17}
$1.78 \pm 0.22 \times 10^{-16}$	$1.97 \pm 0.14 \times 10^{-16}$	$5.08 \pm 0.39 \times 10^{-16}$	2.94×10^{-16}
$3.67 \pm 1.19 \times 10^{-15}$	$2.21 \pm 0.62 \times 10^{-15}$	$5.57 \pm 1.16 \times 10^{-15}$	3.82×10^{-15}
$8.77 \pm 3.46 \times 10^{-15}$	$1.14 \pm 0.49 \times 10^{-14}$	$3.79 \pm 0.53 \times 10^{-14}$	1.94×10^{-14}

Table 3

\beta
D_\beta
E

\beta
D_\beta
E

\gamma
D_\gamma
E

\gamma
D_\gamma
E

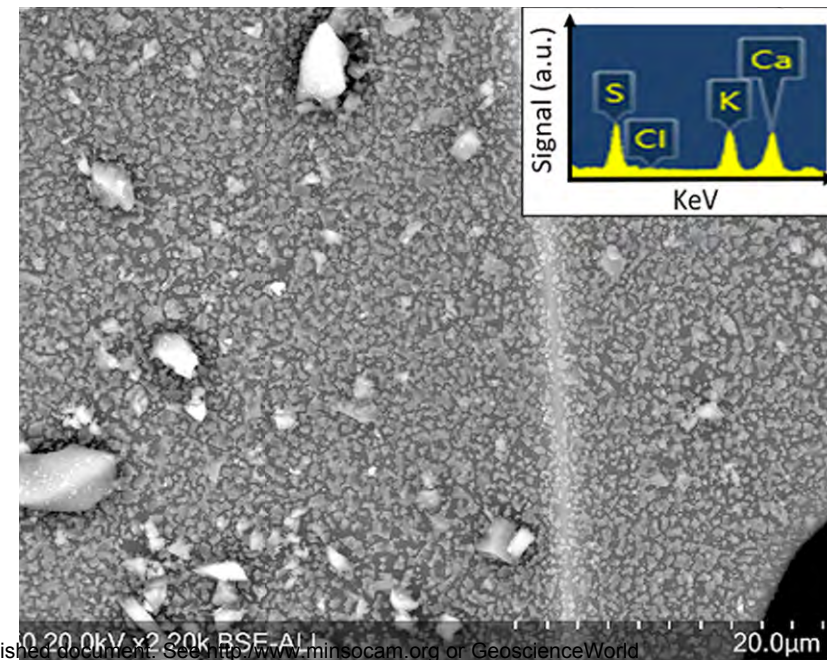
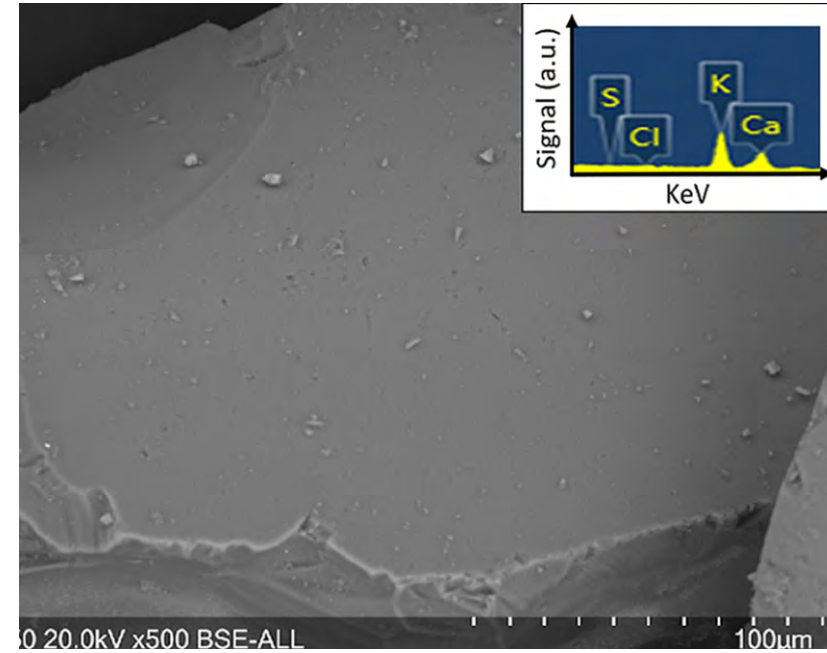
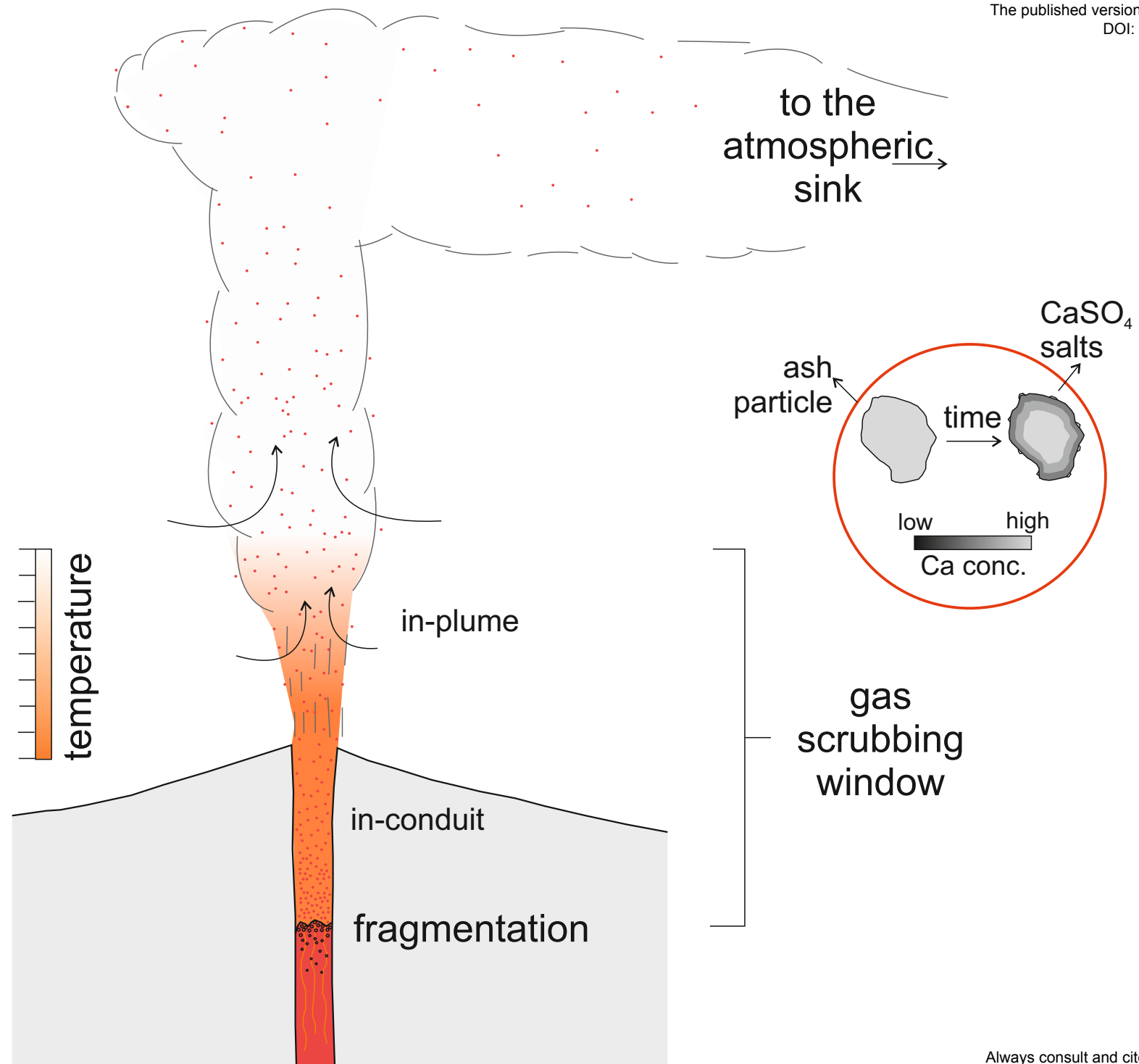
Results of analysis for temperature- and concentration-dependent diffusivity

	Monodisperse			
	$\langle R \rangle = 10.6 \mu\text{m}$	$\langle R \rangle = 19.5 \mu\text{m}$	$\langle R \rangle = 127.7 \mu\text{m}$	Average
[K]	14613.9 ± 1092.3	11324.2 ± 736.6	12075.5 ± 953.7	12671.2
$[\text{m}^2 \cdot \text{s}^{-1}]$	$5.64 \pm 3.83 \times 10^{-8}$	$2.80 \pm 1.46 \times 10^{-9}$	$1.64 \pm 0.98 \times 10^{-8}$	2.52×10^{-8}
$[\text{kJ} \cdot \text{mol}^{-1}]$	121.51 ± 9.08	94.15 ± 6.12	158.70 ± 9.79	133.79

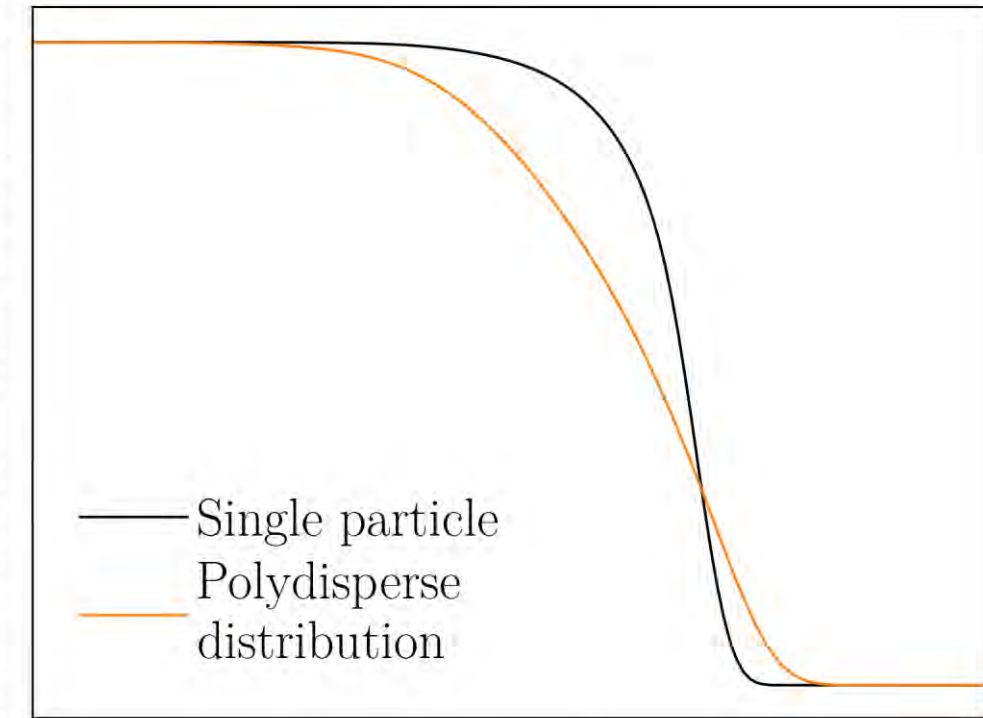
	Polydisperse			
	$\langle R \rangle = 10.6 \mu\text{m}$	$\langle R \rangle = 19.5 \mu\text{m}$	$\langle R \rangle = 127.7 \mu\text{m}$	Average
[K]	17489.5 ± 1498.1	18625.5 ± 1855.3	19087.2 ± 1178.3	18400.7
$[\text{m}^2 \cdot \text{s}^{-1}]$	$3.64 \pm 2.88 \times 10^{-7}$	$1.03 \pm 0.87 \times 10^{-6}$	$4.33 \pm 2.91 \times 10^{-6}$	1.91×10^{-6}
$[\text{kJ} \cdot \text{mol}^{-1}]$	145.42 ± 12.46	154.86 ± 15.43	158.70 ± 9.80	152.99

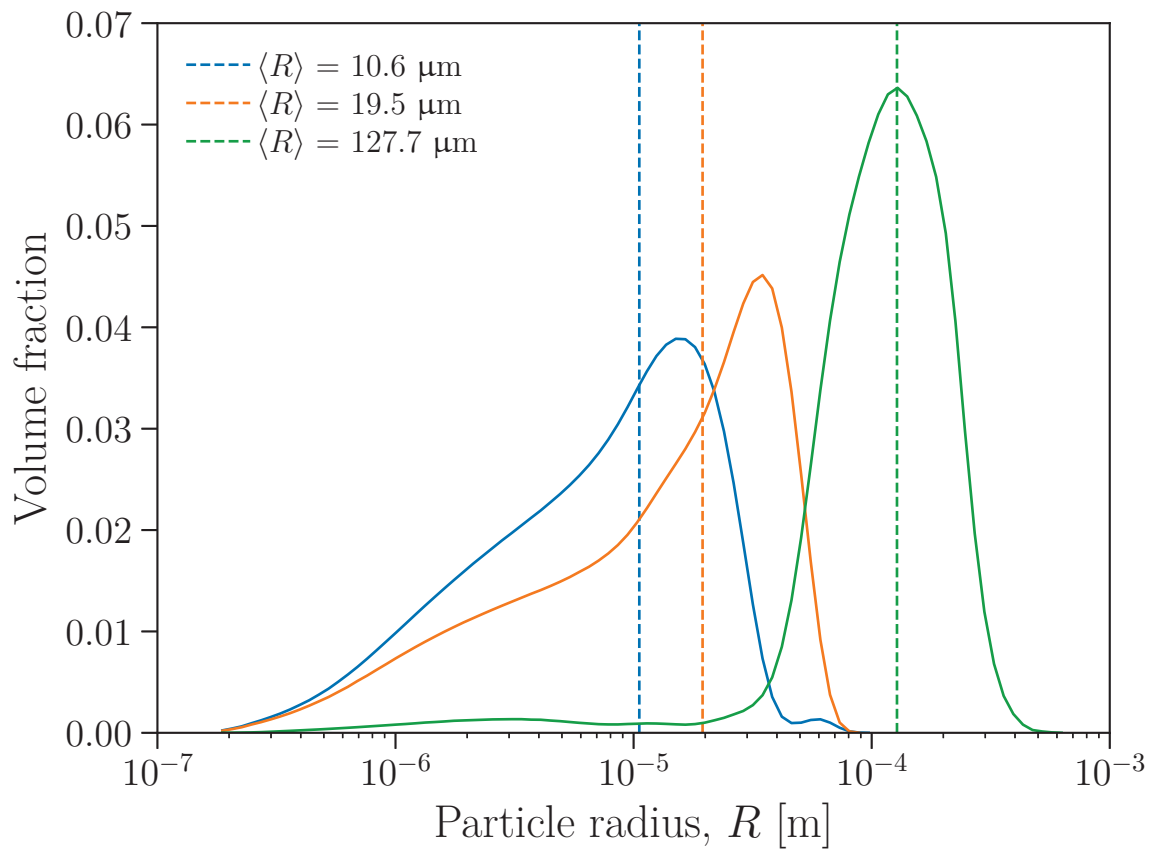
	Monodisperse			
	$\langle R \rangle = 10.6 \mu\text{m}$	$\langle R \rangle = 19.5 \mu\text{m}$	$\langle R \rangle = 127.7 \mu\text{m}$	Average
[K]	15180.2 ± 1353.4	10222.2 ± 1019.2	12415.3 ± 917.4	12605.9
$[\text{m}^2 \cdot \text{s}^{-1}]$	$6.70 \pm 5.04 \times 10^{-8}$	$1.57 \pm 4.23 \times 10^{-10}$	$1.76 \pm 1.03 \times 10^{-8}$	2.84×10^{-8}
$[\text{kJ} \cdot \text{mol}^{-1}]$	126.22 ± 11.25	84.99 ± 8.47	103.23 ± 7.63	104.81

	Polydisperse			
	$\langle R \rangle = 10.6 \mu\text{m}$	$\langle R \rangle = 19.5 \mu\text{m}$	$\langle R \rangle = 127.7 \mu\text{m}$	Average
[K]	17046.9 ± 1444.1	17421.0 ± 1603.5	19590.7 ± 1238.9	18139.5
$[\text{m}^2 \cdot \text{s}^{-1}]$	$1.48 \pm 1.15 \times 10^{-7}$	$2.26 \pm 1.82 \times 10^{-7}$	$5.16 \pm 3.57 \times 10^{-6}$	1.84×10^{-6}
$[\text{kJ} \cdot \text{mol}^{-1}]$	141.74 ± 12.01	144.85 ± 13.33	162.89 ± 10.30	149.83

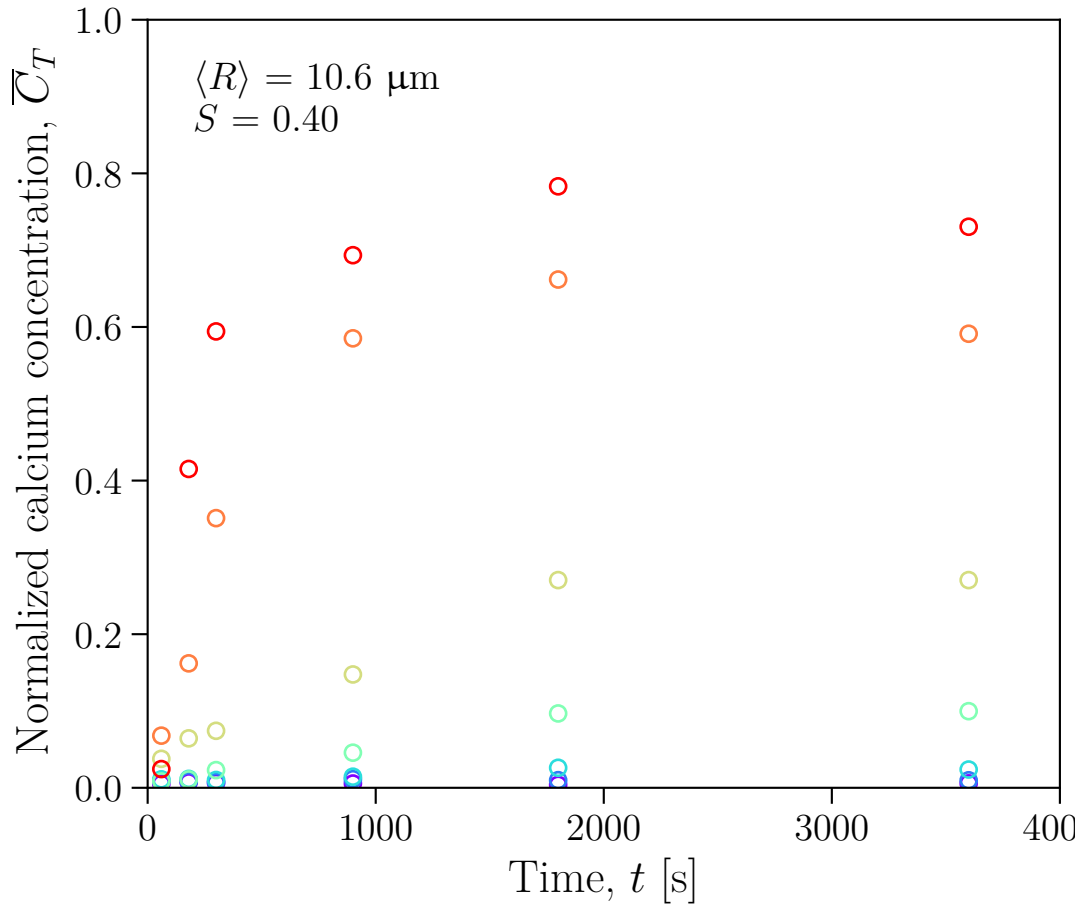


Normalized calcium concentration

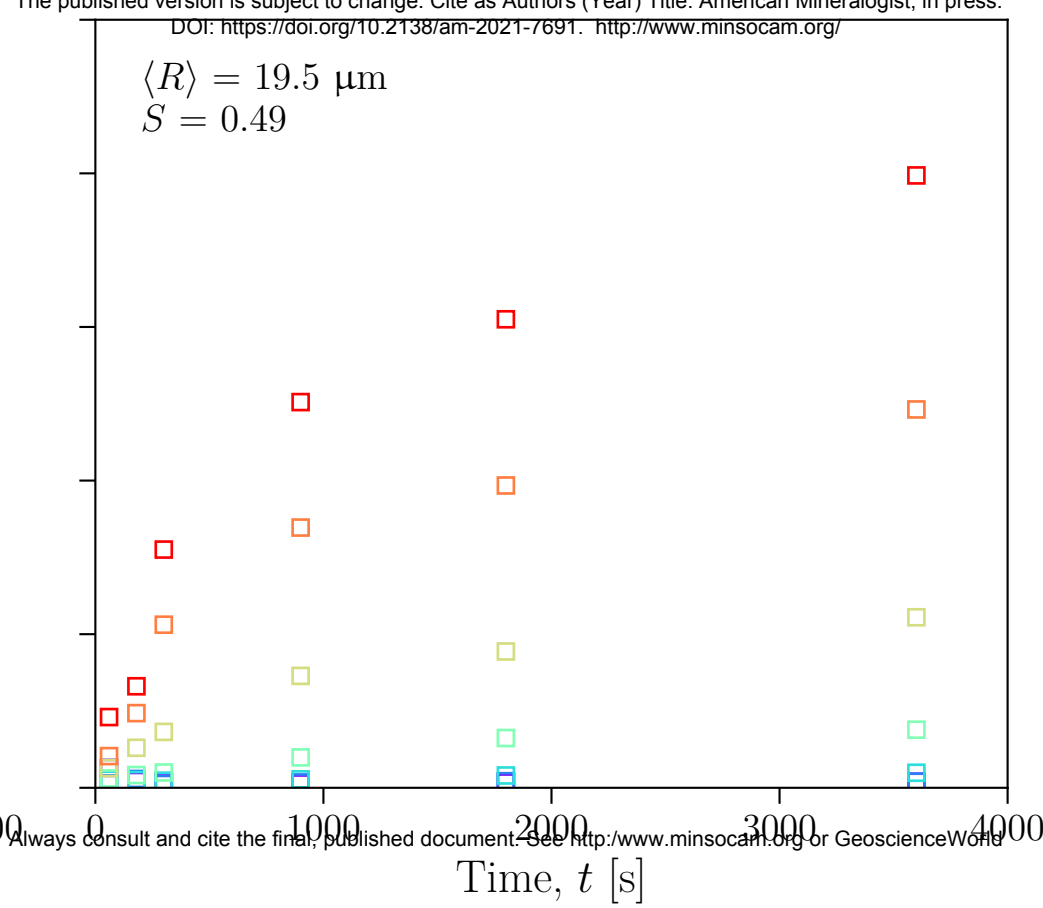




a



b



c

