Liquid properties in the Fe-FeS system under moderate pressure: tool box to model small planetary cores

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Abstract

Physical properties of liquid Fe-S alloys (from 10 to 50 at\%S) under high pressure were investigated by in situ X-ray diffraction (up to 5GPa and 1900 K) and by ab initio calculations. The local structure of Fe-S liquid alloys clearly show how S modifies the local arrangement of the Fe atoms. Density has been extracted from the diffuse scattering by minimization of the oscillation in the short distance of the radial distribution function g(r). Two different formalisms for the P-V-T-X equation of state are presented in order to model density and sound velocity as a function of pressure, temperature and sulfur content. Based on these results, Moon’s core composition is discussed. This coherent dataset will serve as a thermodynamically consistent ground for modeling the core of small telluric planets and large icy satellites.
Introduction

Elements lighter than iron are expected to be present in planetary cores, due to their abundance in iron meteorites and their affinity with the metallic phase. Among the potential light elements (Poirier 1994), sulphur is likely the most abundant in the cores of iron meteorites parent bodies (Chabot 2004). From tungsten isotopes data (Kleine et al. 2009) and experimental studies of percolation processes (Yoshino et al. 2003), core formation is expected to occur within a few million years of the solar system formation, even in small planetesimals (Greenwood et al. 2006). It is therefore important to constrain physical properties of Fe-S liquid alloys in order to better understand core formation in the early stage of the solar system.

Melting properties in the Fe-FeS system under high pressure have been intensively investigated by in situ (Campbell et al. 2007; Morard et al. 2007b; Chen et al. 2008a; G. Morard et al. 2008; Andrault et al. 2009) or ex situ methods (Chudinovskikh and Boehler 2007; Stewart et al. 2007; Chen et al. 2008b; Kamada et al. 2012). However, while the Fe-rich side of the phase diagram begins to be relatively well described, we still lack precise knowledge of important physical properties, especially in the liquid state, such as density or viscosity (LeBlanc and Secco 1996; Sanloup et al. 2000; Vočadlo et al. 2000; Balog et al. 2003; Nishida et al. 2008).

As today, the inner structures and in particular the cores of planetary bodies in the Solar system are not well known, mostly because of the absence of seismological data, with the clear exception for the Earth and, to some extent, for the Moon. Inferences about planetary cores can be obtained from geodesy, electrical and magnetic data, and size, density, and composition have been estimated (e.g. Yoder et al. 2003; Margot et al. 2007; Rivoldini et al. 2009; Zhang and Pommier 2017). However, those inferences require detailed modeling of the interior structure, and the precision of the estimates, in particular core composition, calls for extensive knowledge about core material properties. In this study we investigated the density of liquid Fe-S alloys up to 5 GPa and 1900 K and assessed how it affects the Lunar interior.

Simultaneous measurements of density and liquid structure have been performed to emphasize the strong link between evolution of the local atomic arrangement and changes in the liquid density. This experimental study has been complemented by ab initio calculations of the liquid structure in similar P-T conditions. We present two models describing thermoelastic properties of liquid Fe-S as a function of pressure, temperature, and S content.
based on our data and results published in the literature. These models are finally applied to estimate S content in Moon’s core.

**Experimental procedure**

**Paris-Edinburgh Press experiments**

X-ray diffraction experiments were carried out at the High Pressure beamline ID27 at ESRF in Grenoble, France (Mezouar et al. 2005) using a large volume VX5 type Paris-Edinburgh press (PEP) (Besson et al. 1992; Klotz et al. 2005). This press allows large opening angle in the equatorial plane. The very high brilliance X-ray beam delivered by two in-vacuum undulators was collimated down to 50*50 microns (typical values). The X-ray wavelength was fixed to λ=0.24678 Å (Gadolinium K-edge) using a Si(111) channel cut monochromator. A multichannel collimator (Mezouar et al. 2002; Morard et al. 2011) was used to minimize the X-ray background coming from sample environment materials. The data were collected using a MAR345 imaging plate system (X-Ray research company GmbH, Nodersted, Germany). The sample-detector distance was calibrated with a LaB₆ standard powder and the diffraction images were treated and integrated using the Fit2D software (Hammersley et al. 1996). Typical exposure time to obtain the hereby presented data is 300s.

The high-pressure chamber consists of two opposed tungsten carbide anvils which have quasi-conical hollows. We used 7 mm boron epoxy gaskets with a classical cell assembly (graphite cylinder furnace, MgO capsule used as pressure medium and electrical insulator (Mezouar et al. 1999)). MgO polycrystalline capsule is softer than single crystal alumina capsule, used for conventional X-ray radiography technique, so to minimize pressure gradients in the cell (Nishida et al. 2016). Pressure was calibrated using equation of state (EoS) of MgO (Utsumi et al. 1998) close to the sample, in order to reduce P-T uncertainties. Temperature was fitted using a previously established calibration curve (Morard et al. 2007a). By combined use of this calibration and MgO EoS, pressure was found to be stable up to 1570 K and to decrease at higher temperatures (up to 0.7 GPa for the highest temperature). With increasing temperature we could track the structural transitions in Fe (Klotz et al., 2008) and FeS (Urakawa et al., 2004), confirming the validity of the temperature calibration. As addressed in details in a previous paper (Morard et al., 2007b), metrological uncertainties are the following : 170 K in temperature and 0.6 GPa in pressure.
The samples were a mixture of pure Fe (99.99 %, Alfa Aesar) and FeS powders (99.9%, Sigma Aldrich). Electron microprobe (EMP) analyses were performed on quenched samples (Centre Camparis, UPMC, Paris) using a Cameca SX100 wavelength dispersive spectrometer (WDS) to measure the three main elements in this study (Fe, S and O, as potential pollutant). Operating conditions in both cases were 15 kV and 40 nA for a counting time of 20 s on peak and 10 s on background. Standards for the three elements were Fe, FeS$_2$ and Fe$_2$O$_3$, respectively. Our samples show fine dendritic textures; therefore we used a defocused beam of ~20 microns to average the compositions of quenched texture (Figure 1).

Density and liquid structure of liquid Fe-S alloys were extracted from the diffuse scattering signal recorded under high pressure and high temperature. Diffuse signal with high quality were recorded on a Q range up to almost 100 nm$^{-1}$, allowing measurements up to the third oscillation of the structure factor S(Q) (Figure 2). Analysis of angle dispersive liquid diffuse signal recorded using PEP is detailed in a previous paper (Morard et al., 2013). Uncertainties on experimentally measured density values of ±3 atoms/nm$^3$ (equivalent to ~250 kg/m$^3$) were estimated from Q range, self-absorption and minimal distance on the radial distribution function g(r) (Table 1) (Morard et al, 2013).

Molecular Dynamics Calculations

Simulations were performed using the ABINIT package (Gonze et al. 2016). The electrons are treated quantum mechanically using density functional theory (DFT) while the ions movement is treated classically using the resulting forces. The calculations used the projected augmented wave (PAW) (Blochl 1994; Torrent et al. 2008) method for the computation of the electronic structure and the generalized gradient approximation (GGA) following the parametrization of Perdew, Burke and Ernzerhof (PBE) for the exchange-correlation energy and potential (Perdew et al. 1996). For iron we generated a pseudo-potential with 3s, 3p, 3d, and 4s states as valence electrons (see (Dewaele et al. 2008; Bouchet et al. 2013) for a detailed description of the generation of the pseudopotential and a comparison with recent experimental data). For sulfur the pseudo-potential includes 3s and 3p states as valence electrons. The simulations were performed using a cutoff energy, $E_c$, for the plane wave basis chosen equal to 350 eV. The radius of the augmentation regions for the PAW pseudo-potential were chosen small enough to avoid an overlapping of the spheres surrounding each atom at the highest densities. An efficient scheme for the parallelization
present in ABINIT was used to perform the simulations involving a large number of atoms and time steps (Bottin et al. 2008).

We used supercells of liquid Fe of 128 atoms and we replace 13, 17, 29, 38, 50 and 64 atoms of Fe by atoms of S to obtain composition of 10%atS, 23%atS, 30%atS, 39%atS, and 50%atS respectively. The simulations were first performed in the NPT (constant number of atoms, pressure and temperature) ensemble to reach the desired pressure (5 GPa) and then in the NVT (constant number of atoms, volume and temperature) ensemble in order to avoid any fluctuations of cell parameters. The temperature was fixed to 2500 K and we systematically used an electronic temperature equal to the ionic temperature in our simulations. This relatively high temperature is necessary to ensure the complete melting of the supercells.

Once the box of simulated atoms is equilibrated (after at least 5 ps), partial radial distributions functions, g(r), for Fe–Fe, Fe–S and S–S atoms were calculated from 2.5 ps time averages taken different starting times to ensure the convergence. It is well known that at room pressure the experimental equilibrium volume of iron is underestimated in DFT-GGA-PBE calculations due to an incorrect modelling of magnetic effects (Dewaele et al. 2008) by around 7% and therefore the density is overestimated by the same percentage. The bulk modulus is also strongly overestimated (Dewaele et al. 2008), and therefore the density discrepancies between DFT_GGA and experiments hold up to high pressure (around 150 GPa). Note that it is possible to correct the GGA results by using the dynamical mean-field theory (Pourovskii et al. 2013) and to recover a good agreement with the experimental equation of state, but this approximation cannot be used in molecular dynamics calculations due to its enormous computational cost. For solid Fe-S in the B2 structure the density overestimation is similar (Sherman 1995). We reproduce these discrepancies in our calculations for liquid Fe-S with a constant overestimation of the experimental density by about 1000 kg/m$^3$, due to the GGA-PBE, but also to a larger pressure in the calculations. Noteworthy, the overall evolution of the density as a function of the sulfur composition is comparable to that experimentally measured.

Results and discussion

Liquid structure
Analysis by Fourier transform of the diffuse X-ray scattering of liquid metals gives access to the local structure g(r) of the liquid (Figure 3). In addition to the experimental results, ab initio calculations have been performed on this system in a similar P-T range. As already mentioned, inherent problems of calculations involving iron, without modeling magnetic properties correctly, leads to a known overestimation of the density. However, main features of experimental and theoretical liquid structures are relatively similar. It should be noticed that oscillations present in the experimental data of the structure of pure FeS liquid are not real but are related to spurious signal coming from limited Q range on the corresponding structure factor.

Irrespectively of the density shift, simulations bring additional information impossible to obtain with the present XRD experiments: partial g(r) have been be extracted from the calculations (Figure 4), showing no S-S polymerization on the Fe-FeS side of the phase diagram, in agreement with previous published calculations using smaller supercells (64 atoms compared to 128 in our MD simulations) (Alfè and Gillan 1998; Vočadlo et al. 2000). The S-S bond remains quite large, emphasizing S-S repulsion, and no formation of S-S clusters for all studied compositions. This definitely rules out the idea that S-S interactions were at the origin of the anomalous values of the viscosity (LeBlanc and Secco 1996).

Partial g(r) do not show any drastic change in the bond lengths with increasing S content (Figure 4). The Fe-S bond is shorter than the Fe-Fe bond, this is related to the covalent status of this bond (Flory et al. 2005; Morard et al. 2008). The position of the first peak in the g(r), directly related to the first coordination sphere (CS), was observed in the experiments to remain relatively unchanged up to ~30at%S and to drastically decrease for larger sulfur concentrations (Figure 5). As the contribution of each X-X atomic pair to the total g(r) probed by X-ray is related to the weight of the atoms, the first peak position is essentially associated with the Fe-Fe network. S-S bond is relatively longer (3.4 Å) but with lower intensity and larger dispersion, whereas Fe-S bonds are shorter (2.2 Å) than Fe-Fe bonds (2.5 Å) and have a higher intensity. It is therefore the addition of Fe-S bonds that influence the total g(r) observed in our experiments, and explained the shortening of the observed first CS position (Figure 5). The hereby presented peak position are in relative good agreement with previous results from (Kono et al. 2015).
Comparison of our dataset with the literature

The Fe-FeS binary system has been studied extensively over a large pressure range. Density of liquid Fe-S alloys has been determined by ex-situ sink-float method (Balog et al. 2003; Nishida et al. 2008), by in situ X-ray radiography (J. Chen et al. 2014), and by X-ray absorption (Sanloup et al. 2000; Nishida et al. 2011). These datasets do not provide a consistent picture, and the derived densities show large discrepancies, even when measurements have been performed with the same method. Sound velocities of liquid Fe-S alloys have been probed in situ under high pressure by conventional ultrasonic method but, as for the density measurements, the results by different research groups are in disagreement (Nishida et al. 2013, 2016; Jing et al. 2014). A thorough comparison of our new measurements (Table 1) with literature results will sheds light on the properties of the Fe-S binary system at high pressure and high temperature.

Regarding pure liquid FeS, previous studies have found contradictory results (Nishida et al. 2011; B. Chen et al. 2014) in spite of using the same X-ray absorption technique. Chen et al. argue that the reported density difference can be explained by the temperature difference. Specifically, they claim that temperature difference of 150K can account for a density difference of almost 400 kg/m$^3$. However such a large thermal expansion is incompatible with ambient pressure measurements (for pure FeS, the liquid density only changes from ~3900 kg/m$^3$ to ~3820 kg/m$^3$ between 1473 K and 1623 K (Kaiura and Toguri 1979)). Furthermore, a combined fit of either of the two high-pressure sets of density for pure liquid FeS with ambient pressure results is impossible using conventional EoS values, and requires an unrealistically small K$_{T0}$ (Nishida et al. 2011). Conversely, the here-measured density for pure liquid FeS at high pressure are in good agreement with ambient pressure measurements (Nagamori 1969; Kaiura and Toguri 1979), while differ from the previous high-pressure measurements (the difference between the present dataset at 4 GPa and 1900 K and previous determination is 6% (Nishida et al. 2008), 10% (Nishida et al. 2011), and 20 % (J. Chen et al. 2014) (Figure 6).

Unlike liquid density measurements at high pressure, literature concerning measurements on liquid Fe-S alloys at ambient pressure is rather limited and relatively old (Nagamori 1969; Kaiura and Toguri 1979). It should also be noted that there are no density measurements over a wide compositional range (0-40 at% S) at ambient pressure. Regarding pure liquid Fe, measurements have been recently performed over a wide temperature range, allowing a very
good estimation of the density/temperature relation (Assael et al. 2006), which is used to extract density of liquid Fe-S alloys at 1900 K as a function of S content (Figure 7).

The density of the liquid Fe-S alloys at ambient pressure (\(\rho_0\)) is a sensitive parameter in our study. Drastically different models have been used in previous studies (Figure 7), with a deviation of \(~1000\) kg/m\(^3\) for close compositions in the 15-20 at\% S range. We anchor our study on ambient pressure measurements, fixing the density for the 40-50 at\% S alloys on those by Nagamori (1969) extrapolated at 1900 K using thermal expansion from Kaiura and Togori (1979). A quadratic polynomial fit is then performed to interpolate between density of S-rich alloys and that of pure liquid Fe, fixed to the value of Assael et al. (2006). This trend is in relative good agreement with densities calculated by ab initio techniques (Kuskov and Belashchenko 2016) for low S concentration (Figure 7). The so-derived second order polynomial gives:

\[
\rho = -3108X_S^2 - 5176X_S + 6950
\]  

where \(X_S\) is the S concentration in at\%. Looking at this relation, it comes clearly out that values of density from Sanloup et al., (2000) and the related model from Jing et al. (2014) are too low, in particular for low S content, and in disagreement with our model (Figure 7).

Regarding sound velocity, most recent measurements (Nishida et al. 2016) and calculations (Kuskov and Belashchenko 2016) are incompatible with ambient pressure measurements (Nasch et al. 1997; Nasch and Manghnani 1998) (Figure 8 and 9), while previous measurements on Fe-16at\%S are (Jing et al. 2014) (Figure 9). Our model is therefore adjusted to agree with Jing et al, (2014).

To conclude, a survey of the literature on Fe-S liquid alloys under high pressure show relatively scattered results. Strong disagreement exists for density, sound velocity and thermal expansion. Our approach is to anchor our model on ambient pressure measurements, with only few assumptions, as for example the interpolation of liquid density between 0 and 40 at\% S.

**Parametrized equation of state**

In order to fit our set of density data, we use a 3\(^{rd}\) order Birch Murnaghan equation of state relating pressure \(P\) and density \(\rho\) at the fixed temperature of 1900 K:
where $K_{T0}$ is the isothermal bulk modulus at ambient pressure and 1900 K, $K'$ is the pressure derivative of the bulk modulus and $\rho_0$ is the density at ambient pressure and 1900 K. Each parameter is a function of molar S content in the liquid ($X_S$). The isothermal bulk modulus at ambient pressure and its derivative can be expressed as follows (J. Chen et al. 2014):

$$K_{T0} = K_{T,Fe}^{1-X_S} * K_{T,S}^{X_S}$$  \hspace{1cm} (3)$$

A linear mixing law for bulk moduli could be used, but for a shorter range of composition (Morard et al. 2013). In the present study, we found more suitable to use such type of mixing law. More details regarding different mixing law for bulk moduli can be found in Chen et al, 2014.

For pure Fe we assume $K_{T,Fe}=76$ GPa. This value is deduced from sound velocity measurements at 1bar (Nasch and Manghnani 1998), assuming a value for the specific heat at constant volume $C_V$ of 4R and the value of specific heat at constant pressure $C_P$ as in (Nasch and Steinemann 1995). Regarding sulfur, we take $K_{T,S}=1.6$ GPa (Tsuchiya 1994) as in Chen et al. (2014), which gives an overall good agreement with compressibility data from Nishida et al. (2011). However, it should be noticed that this value is measured at 673 K and not at 1900 K. Accordingly, this parameter should be rather considered a fitting parameter, restraining the validity of our model only between Fe and FeS compounds.

As measurements in the 0-5 GPa range do not cover a large enough pressure range to adequately constrain $K'$ values, this is adjusted to fit the velocity measurements, following the relation $K'_{Fe}=X_S*3$, with $K'_{Fe}=6.5$.

As our measurements were performed at different temperatures, we also need to estimate thermal expansion as a function of pressure, temperature, and S content. We assume a linear dependence of density on temperature (experimentally proven for liquid iron(Assael et al. 2006)). We calculate the thermal expansion at 1900 K, assuming linearity between Fe and FeS:
\[ \alpha_0 = -\frac{1}{\rho_0} \left( \frac{\partial \rho}{\partial T} \right)_0 \] (4)

\[ \left( \frac{\partial \rho}{\partial T} \right)_{Fe,0} = 0.926 \text{ from (Assael et al. 2006)} \]

\[ \left( \frac{\partial \rho}{\partial T} \right)_{FeS,0} = 0.533 \text{ from (Kaiura and Toguri, 1979)} \]

\[ \left( \frac{\partial \rho}{\partial T} \right)_0 = 2 \left( \frac{\partial \rho}{\partial T} \right)_{Fe,0} - \left( \frac{\partial \rho}{\partial T} \right)_{FeS,0} \right) X_S \] (5)

Due to the lack of data on thermal expansion for intermediate compositions, we chose a linear mixing for the thermal expansion mixing model. More detailed investigation is required in the future to better constrain this linear model.

Then, thermal expansion at a given pressure is calculated under the assumption:

\[ \alpha_0 K_{T,0} = \alpha K_T \] (6)

With

\[ K_T = \frac{K_{T0}}{2} \left[ 7 \left( \frac{\rho}{\rho_0} \right)^{\gamma_3} - 5 \left( \frac{\rho}{\rho_0} \right)^{\gamma_5} \right] \left[ 1 + \frac{3}{4} \left( K'_0 - 4 \right) \left( \frac{\rho}{\rho_0} \right)^{\gamma_3} - 1 \right] + \frac{3K_{T0}}{4} \left( K'_0 - 4 \right) \left( \frac{\rho}{\rho_0} \right)^{3} - \left( \frac{\rho}{\rho_0} \right)^{\gamma_3} \right] \] (7)

A parameter that could be directly derived from our Eos is the compressional sound velocity in the liquid:

\[ V_p = \sqrt{\frac{K_T (1 + \gamma T)}{\rho}} \] (8)

The value of the Grüneisen parameter of the liquid Fe-S alloy is calculated from the \( V_p \) of pure Fe and Fe-16\%at. S (Nasch et al. 1997; Nasch and Manghnani 1998) (Figure 8, inset) assuming that the Grüneisen parameter evolves linearly with the sulfur concentration. The values obtained are in agreement with ab initio calculations in (Kuskov and Belashchenko 2016) (Figure 8, inset). However, calculations from (Kuskov and Belashchenko...
2016) argue for an increase of the sound velocity as a function of S content, whereas experimental data show the opposite behavior (Figure 8). In addition, Grüneisen parameter is assumed to be constant in the pressure range 0-10 GPa. This seems quite reasonable over a relatively small pressure range (Kuskov and Belashchenko 2016). The Grüneisen parameter varies here as $\gamma = 1.52 + 2.5X_S$.

**Thermodynamic model**

In addition to the empirical parametrization introduced above, we also provide a description of the thermoelastic properties of liquid Fe-S alloys based on thermodynamic solution modeling. This approach is complementary to the previous one and is based on the mixing of the end-members: pure liquid Fe and FeS. For liquid Fe we use the EoS of Komabayashi (2014) and for liquid FeS, an EoS deduced from our density data and from sound velocity data of Nishida et al. (2016). For both end-members we use a Vinet equation for isothermal compression and an Anderson-Grüneisen parametrization for isobaric heating (see Komabayashi 2014). The parameters for liquid FeS EoS are given in Table 2. To assess the solution model that best describes our density data and the sound velocity data of Nishida et al (2016) over the entire Fe-FeS binary, we test three different mixing models that affect the volume of the solution, assuming (a) ideality; (b) concentration dependent non-ideality; and (c) concentration and pressure dependent non-ideality.

To model the excess contribution we use the Margules parameterization, introduced by Buono and Walker (2011), to describe the non-ideal behavior of Fe-FeS liquidus on the iron-rich side of the Fe-FeS phase diagram. In this parametrization, the excess contribution to the Gibbs energy is linear in temperature. As a consequence, the thermal expansivity and isobaric heat capacity of the solution depends only on the end-members.

The ideal and excess thermoelastic properties of the solution are obtained from its Gibbs energy (e.g. Poirier et al. 2000), and the Grüneisen parameter required for calculating the sound velocity (Eq 8) is computed from

$$\gamma = \left(\frac{C_p}{\alpha K T V} - \alpha T\right)^{-1}, \quad (9)$$

where $\alpha$, $K_T$, $V$, and $C_p$ are the thermal expansivity, isothermal bulk-modulus, volume, and isobaric heat capacity of the solution.
Ideal mixing

The ideal mixing volume between the two end-members is described by the following equation:

\[ V = (1 - x)V_{Fe} + xV_{FeS}, \quad (10) \]

where \( V \) is the volume of the considered liquid alloy, \( x \) the mole fraction of FeS, and \( V_{Fe} \) the volume of liquid Fe and \( V_{FeS} \) the volume of liquid FeS.

Such a simple model cannot reproduce our density dataset (Figure 10). The density calculated from Eq.10 decreases too rapidly with increasing S concentration and does not reproduce the positive curvature of density as function of S content seen in the data (Figure 10).

Pressure independent volume Margules parameters

In this non-ideal mixing model, the volume is:

\[ V = (1 - x)V_{Fe} + xV_{FeS} + (1 - x)x(W_{V,Fe} (1 - x) + W_{V,FeS} x), \quad (11) \]

where \( W_{V,Fe} \) and \( W_{V,FeS} \) are the volume Margules parameters. The values of those parameters have been estimated by fitting Eq. 11 on the density data (case A) and on the sound velocity data (case B) separately (Table 3).

The density data is well described in case A, but the predicted sound velocities do not agree with the measurements. Conversely, a fit to the sound velocity data provides a good representation for the sound velocity (case B), but fails to describe the experimental density data. This simple Margules mixing model is well suited for modeling the Fe-rich side of the Fe-S phase diagram and the density of Fe-S solutions however, as shown in case B, a different non-ideal behavior is required to describe the sound velocity data. With Eq 11, the sound velocity is only affected by the non-ideality through the volume but not through the bulk modulus. A non-ideal contribution to the bulk modulus requires that the volume Margules coefficients depend on pressure, as discussed in the next section.

Pressure dependent volume Margules parameters

By trial and error, we have found that the density and sound velocity data can well be described if we adopt the following parametrization for the volume Margules parameters:
\[ W_{V,Fe} = W_{11} + W_{12} \ln \left( \frac{3}{2} + P \right) \]  \hspace{0.5cm} (11)

\[ W_{V,FeS} = W_{21} + W_{22} \cdot P. \]  \hspace{0.5cm} (12)

The estimated values for the parameters \( W \) are given in Table 3. They have been obtained by fitting the equations describing the volume and sound velocity on the density and sound velocity data at the same time. As can be seen from Fig. 10, this parameterization for the liquid solution gives an adequate description for the experimental data in the 0-8 GPa pressure range. Note that this parametrization cannot be used to model thermoelastic properties beyond that pressure range since an already moderate extrapolation in pressure would lead to unphysical properties.

**Implications**

To infer the size and sulfur content of planetary cores from seismic or geodesy data requires an accurate knowledge of density and velocity properties of solid and liquid iron alloys as a function pressure, temperature and sulfur content. Until now, no consensus on experimental measurements on Fe-S liquid alloys has been reached, with strong contradictions between the different high-pressure studies, and also large disagreement with ambient pressure data. In the present work, we presented new experimental density data of liquid Fe-S alloys at high pressure and high temperature, together with ab initio liquid structure calculations. Combining this new data set and literature data, we were able to establish an empirical and a thermodynamic model for liquid Fe-S solutions that allow to compute thermoelastic properties of the liquid alloy as a function of pressure, temperature, and sulfur concentration.

To illustrate the effect of the new density data on the internal structure of small planets, we compare the composition of the Moon's core predicted by our empirical and thermodynamic Fe-S mixing model with a core model assuming ideal mixing between liquid iron and liquid Fe-10wt%S (Figure 11). The thermoelastic properties of the core in the ideal mixing model are calculated following (Dumberry and Rivoldini 2015) and are based on the EoS of liquid Fe (Komabayashi 2014) and Fe-10wt%S (Balog et al. 2003). The aim of this model is to emphasize that liquid Fe-S alloys EoS directly affect the Moon’s core composition deduced from its radius.
For our global Moon model, we use the density profile of the silicate part of the Moon structure deduced by Weber et al. 2011 from seismic data, the core models listed in the previous paragraph, and assume that the entire core is in the liquid state, convecting and composed of iron and sulfur. In order to agree with the most recent moment of inertia estimate (Williams et al. 2014), we had to reduce the density of the upper part of the mantle by about 0.1%. The pressure in the planet is calculated by solving the hydrostatic pressure equation and Poisson equation. The temperature profile in the core is obtained by integrating the adiabatic gradient equation in the core (e.g. (Dumberry and Rivoldini 2015)) assuming a core-mantle boundary (CMB) temperature of 1750K. All thermoelastic properties of the core are calculated from the mixing models as a function of pressure, temperature, and composition.

Our results show that for a given core radius, models based on the new elastic data require approximately twice the amount of sulfur than models based on an equation of state deduced from previous elastic data (Figure 11). Noteworthy, the sulfur concentrations required by the empirical and the thermodynamic model are very similar, even though when the two models are based on a different sets of acoustic velocity data. Models that agree at 1 sigma with the moment of inertia have a core radius below about 360 km but, the specifics of the core model have only a very negligible effect on the moment inertia of the planet. However, the larger amount of sulfur required by the new elastic data significantly affects the thermal evolution of the core (i.e. the liquidus of the core decreases with increasing S content) and with it the dynamo action, by delaying the onset of crystallization of iron-rich compounds in the core. The present model assumes a specific CMB temperature (1750 K) which is expected in the present day core of the Moon (Laneuville et al. 2014). Lowering the CMB temperature will somewhat increase the sulfur content required for the core to match the mass constraint and initiate the crystallization of iron-rich compounds close to the CMB, since the core isentrope is likely steeper than the liquidus (Williams 2009; Breuer et al. 2015). Irrespectively of the assumptions, the present model is a clear illustration of the impact of our two models on Moon’s core composition.

The present study, anchored on experimental measurements and theoretical calculations, allows to present a coherent model for planetary cores, with applications ranging from the direct simulations of small planetary cores dynamics to the inversion of seismic data (Garcia et al. 2011; Weber et al. 2011; Antonangeli et al. 2015).
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Figure Captions

Figure 1: Scanning Electron Microscope (SEM) images of polished cross sections of recovered samples. Here are presented two different compositions, with typical quench textures of S-poor and S-rich liquids. Microprobe measurements were performed on large area, of ~20 microns, in order to average the composition over these textures.

Figure 2: Structure factors S(Q) of liquid Fe-S alloys as a function of the composition for similar P-T conditions.

Figure 3: Comparison between pair distribution function g(r) obtained from ab initio calculations (5 GPa and 2500 K) and experimental results (similar P-T conditions and composition as in Figure 2).

Figure 4: Partial pair distribution function for different composition extracted from the ab initio calculations.

Figure 5: First peak position as a function of the S content for the different g(r), compared with theoretical work, and previous results from (Kono et al. 2015).
Figure 6: Density of liquid iron alloys from this study and the corresponding thermodynamic model compared with available measurements. The color of the different symbol is related to the composition, rounded to the closest nominal value.

Figure 7: Density of liquid Fe-S alloys at ambient pressure from different studies. Our thermodynamic model uses an interpolation between the density of pure Fe from (Assael et al. 2006) and densities of S-rich liquids (Nagamori 1969) corrected for temperature difference following (Kaiura and Toguri 1979).

Figure 8: Sound velocity fit for this model as a function of S content based on measurements at ambient pressure from (Nasch et al. 1997; Nasch and Manghnani 1998), compared with theoretical calculations from Kuskov and Belashchenko (2016). Inset: Gruneisen parameter used in this model compared with ab initio calculations from Kuskov and Belashchenko (2016).

Figure 9: Sound velocity extracted from our thermodynamic model compared to the available literature (Nasch et al. 1997; Nasch and Manghnani 1998; Jing et al. 2014; Nishida et al. 2016). Different colors correspond to different compositions, rounded to the closest nominal value. Representative error bars for the two high pressure studies have been added in the figure legend.

Figure 10: Thermodynamic models of density and sound velocity of liquid Fe-S alloys at 1900 K. Four different models have been tested: ideal solution, pressure independent (Model A and B) and pressure dependent Margules parameters. It is clear that the pressure dependence of the Margules parameters is required to represent adequately the experimental values.
Figure 11: Radius of Moon’s core as a function of S content of an entirely liquid core, using models presented in this study (parametrized EoS and thermodynamic model) compared with an ideal solution based on previous density measurements (Balog et al. 2003). Calculations were performed for a temperature at the Moon CMB of 1750 K.

Tables

<table>
<thead>
<tr>
<th>Run number</th>
<th>Pressure (GPa)</th>
<th>Temperature (K)</th>
<th>xS (at%)</th>
<th>density (atoms/nm³)</th>
<th>Density at 1900 K (kg/m³)</th>
<th>Density at 1900 K (kg/m³)</th>
<th>CN1</th>
<th>CN2</th>
<th>r1 (nm)</th>
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Table 1: Pressure-temperature conditions, chemical composition of quenched samples, densities measured and calculated at 1900 K.

Reference temperature T0 (K) 1650
Reference volume (after Nagamori corrected for T=1650 K) (cm³/mol) 11.8
Reference thermal expansion (after Kaiura) (K-1) 1.29E-04
KT0 (Gpa) 16.5 (0.5)
K'T0 6.21
Y 1
Q 0

Table 2: Parameters for liquid FeS Equation of state, following Vinet and Anderson-Gruneisen parameterization (see (Komabayashi 2014) more details).
\[ W_{V,Fe} \quad -6.74 \pm 0.63 \quad 4.93 \pm 0.92 \]
\[ W_{V,FeS} \quad -0.47 \pm 0.51 \quad 5.70 \pm 0.87 \]
\[ P \text{ dependent model} \]
\[ W11 \quad -7.78 \pm 0.95 \]
\[ W12 \quad 2.31 \pm 0.14 \]
\[ W21 \quad -2.32 \pm 0.88 \]
\[ W22 \quad 0.057 \pm 0.024 \]

**Table 3**: Margules parameters for pressure independent and pressure dependent parameterization. These parameters are given cm\(^3\)/mol.

**Bibliography**


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Figure 1

Fe + 9.9 at%S

Fe + 38 at%S
Figure 2

![Graph showing S(Q) vs. Wave vector (nm⁻¹) with different lines for various conditions: 65-66: 2.1 GPa; 1860 K; 9.9 at% S; 35-36: 3.2 GPa; 1860 K; 17.4 at% S; 53-54: 2.9 GPa; 1860 K; 22.9 at% S; 63: 2.7 GPa; 1690 K; 29.4 at% S; 17: 3.5 GPa; 1600 K; 38 at% S; 48: 2.6 GPa; 1770 K; 50 at% S.](image-url)
Figure 3

Molecular dynamics calculations vs. Experimental measurements

5 GPa and 2500 K
Figure 4

- Fe-S bonds
- Fe-Fe bonds
- S-S bonds

- 10 at% S
- 23 at% S
- 30 at% S
- 39 at% S
- 50 at% S
Figure 5

Experimental results

Ab initio calculations

Kono et al., 2015
Figure 6

$T = 1900$ K

- **This study**
- **Nishida et al., 2008**
- **Nishida et al., 2011**
- **Chen et al., 2014**
- **Kuskov and Belashchenko, 2016**
- **Sanloup et al., 2000**
- **Kaiura and Toguri, 1979**
Figure 7

Ambient pressure

Density (kg/m³)

S content (at%)

- Assael et al, 2002 (1900 K)
- Nagamori et al, 1969
- Nagamori et al, 1969 (recalculated at 1900 K)
- Ab initio calculations
- This study (1900 K)
- Nishida et al, 2016
- Jing et al, 2014 (after Sanloup et al, 2000)
- Kuskov and Belashchenko, 2016

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Figure 8

![Graph showing the relationship between Sulfur content (at%) and Vp (km/s). The graph includes data points from different studies and pressures.](image-url)

- **This study**
- Nasch et al., 1997, 1998
- Kuskov and Belashchenko, 2016

The inset graph shows the variation of $\gamma$ with Sulfur content at different pressures (5 GPa, 10 GPa).
Figure 9

T = 1900 K

Sound velocity (m/s) vs. Pressure (GPa) for various sulfur concentrations. The data points are from:
- Nasch et al., 1997, 1998
- Nishida et al., 2016
- Jing et al., 2014

At 0 at% S, the sound velocity increases with pressure. The curves for 10 at%, 20 at%, 30 at%, 40 at%, and 50 at% S show similar trends but with different slopes and intercepts.
Figure 10

Ideal solution | Pressure independent Margules model A | Pressure independent Margules model B | Pressure dependent Margules model

Density (kg/m³)

Sound velocity (m/s)
Figure 11

CMB temperature : 1750 K

- Ideal mixing
- This study
- Parametrized EoS
- Thermodynamic model

Sulfur content in Moon's core (wt%)

Radius of Moon's core (km)