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2 **Quantitative determination of Al/Si order in sillimanite by high angular resolution**
3 **electron channeling X-ray spectroscopy**

4 **Yohei IGAMI^{1,*}, Takahiro KURIBAYASHI², and Akira MIYAKE¹**

5 ¹Department of Geology and Mineralogy, Graduate School of Science, Kyoto University,

6 Kyoto 606-8502, Japan

7 ²Department of Earth Science, Graduate School of Science, Tohoku University, Sendai

8 980-8578, Japan

9 *Corresponding author. E-mail: y-igami@kueps.kyoto-u.ac.jp

10
11 **Abstract**

12 Sillimanite is a polymorph of Al₂SiO₅ that is widely used as an indicator of pressures and
13 temperatures reached during metamorphism. The degree of disorder in the double chains of
14 SiO₄ and AlO₄ tetrahedra in sillimanite, particularly at high temperatures, is of interest as a
15 factor in the phase relations of the Al₂SiO₅ polymorphs. We determined the Al/Si order
16 parameter (*Q*) of sillimanite from Rundvågshetta, Antarctica, by the High Angular
17 Resolution Electron Channeling X-ray Spectroscopy (HARECXS) method using
18 transmission electron microscopy with energy dispersive X-ray spectrometry. HARECXS
19 profiles were successfully obtained from regions ~1 μm in diameter by automated control

20 of beam tilting and X-ray detection. The obtained Q value was close to that previously
21 estimated by single-crystal X-ray diffraction. Moreover, the Q values of annealed samples
22 were obtained while avoiding interference from mullite or SiO₂-rich glass domains formed
23 by annealing. For quantitative determination of Q , we also performed theoretical
24 calculations of HARECXS profiles and evaluated sample thicknesses by convergent-beam
25 electron diffraction. The experimentally obtained profiles were successfully fitted by a
26 linear combination of simulated profiles of completely ordered and completely disordered
27 sillimanite, which yielded Q values. The Q values obtained from 18 measurements showed
28 no effect from differing sample thicknesses. Moreover, the results from annealed samples
29 showed that Q decreases continuously with increasing annealing temperature. The
30 temperature dependence of Q values, formulated by least-squares fitting on the basis of the
31 Bragg-Williams approximation, yielded a transition temperature from order to disorder at
32 1727 °C. The obtained curve is more accurate at high temperatures than previous estimates.
33 It indicates that the sample material reached peak temperatures greater than ~1000 °C,
34 which is close to previous estimates of the peak metamorphic temperature of
35 Rundvågshetta sillimanite. This study also implies that the HARECXS method is suitable
36 for accurate analyses of other natural samples with complicated microtextures.

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38 **Keywords:** ALCHEMI, HARECXS, transmission electron microscope, sillimanite,
39 Al/Si-disordering

40

41 **Introduction**

42 Sillimanite is one of Al_2SiO_5 polymorphs that are valuable as indicators of the
43 pressure and temperature experienced by metamorphic rocks. The crystal structure of
44 sillimanite consists of AlO_6 octahedral chains that are linked with double chains of
45 $\text{SiO}_4/\text{AlO}_4$ tetrahedra oriented parallel to the *c*-axis. The tetrahedral Si/Al chains are usually
46 ordered, but it has been suggested that they become disordered at high temperatures (e.g.,
47 Zen 1969; Holdaway 1971; Greenwood 1972). Disordering of Al and Si tetrahedra in
48 sillimanite has been studied for its possible effects on phase relations of the Al_2SiO_5
49 polymorphs. Therefore, annealing experiments since the 1970s have sought to determine
50 the Al/Si order parameter (*Q*) at high temperatures (e.g., Navrotsky et al. 1973), but
51 quantitative determinations have had poor success. The main problem is the difficulty of
52 separating sillimanite from microscopic precipitates of mullite and SiO_2 -rich glass, which
53 appear in sillimanite at temperatures greater than 1200 °C (e.g., Tomba et al. 1999; Igami et
54 al. 2017). Transmission electron microscope (TEM) observations of sillimanite heated to
55 high temperatures have shown that the mineral is partly transformed to mullite
56 ($\text{Al}_2[\text{Al}_{2+2x}\text{Si}_{2-2x}]\text{O}_{10-x}$, $x \approx 0.17\text{--}0.59$) with glass inclusions (e.g., Holland and Carpenter
57 1986; Raterron et al. 1999, 2000; Rahman et al. 2001). These textures are too fine for the
58 spatial resolution of a scanning electron microscope (SEM) combined with energy
59 dispersive X-ray spectrometry (EDS). X-ray diffraction (XRD) and neutron diffraction
60 experiments cannot distinguish these coexisting phases from sillimanite because glasses do
61 not show clear diffraction peaks and mullite is crystallographically very similar to
62 sillimanite. Moreover, the similarity of Al and Si in their X-ray scattering factors makes

63 them difficult to determine Q by XRD.

64 Spence and Taftø (1983) developed the ALCHEMI (Atom Location by
65 CHanneling-Enhanced Microanalysis) analytical method in which TEM-EDS is used to
66 determine the crystallographic sites of impurity elements in crystals on the basis of
67 channeling-enhanced X-ray emissions. This method uses TEM to analyze micrometer-sized
68 regions and can distinguish elements with similar atomic number by EDS. However,
69 ALCHEMI is specialized for site determination of impurity elements in the crystals that
70 consist of the atomic planes on which two host elements arranged separately, in general.
71 Therefore, it is not applicable to the determination of order degree between mixed two
72 cations, including Al/Si order in sillimanite. However, improvements in ALCHEMI have
73 subsequently led to the HARECXS (High Angular Resolution Electron Channeling X-ray
74 Spectroscopy) method, which is suitable for wider application (e.g., Soeda et al. 2000;
75 Yasuda et al. 2006, 2007; reviewed by Muto and Ohtsuka 2017). HARECXS acquires
76 many EDS measurements as the direction of the incident electron beam is continuously
77 varied to yield X-ray intensity profiles of the elements against beam tilting. Because such
78 profiles can be precisely simulated, if the crystallographic structure is fully known, by
79 using the program ICSC (Oxley and Allen 2003) based on dynamic electron
80 elastic/inelastic scattering theory, HARECXS can be applied to crystals with complex
81 structure. Although HARECXS is not yet a fully quantitative procedure, we expected it to
82 provide improved quantitative determinations of Q in sillimanite.

83 In this study, we used HARECXS for quantitative determination of Q in a
84 sillimanite sample. The same sample was also examined by single-crystal XRD to evaluate

85 the suitability of this HARECXS application. Annealed samples of the same sillimanite
86 were then examined by HARECXS to investigate the effect of heating on Q in sillimanite
87 suggested by theoretical studies (e.g., Zen 1969; Holdaway 1971; Greenwood 1972).

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Experimental methods

91 Samples

92 Our starting material was sillimanite crystals approximately 2 mm across separated
93 from cordierite-bearing rock collected in Rundvågshetta, East Antarctica (sample
94 RVH92011102A, Kawasaki et al. 1993, 2011). The crystals were optically homogeneous
95 and prismatic, with a chemical composition of $Al_{1.99}Fe_{0.01}Si_{1.00}O_5$ as determined with a
96 SEM-EDS system (JEOL JSM-7001F SEM, Oxford Inca EDS). Sillimanite from the same
97 locality, measured with a JEOL JCSA-8800 SEM at Ehime University (Kawasaki et al.
98 2011), had a similar composition. Because the amount of impurity is sufficiently small, we
99 assume that the impurity does not affect in our analysis and the composition of this sample
100 is Al_2SiO_5 .

101

102

103 Single-crystal XRD measurement of the starting material

104 We investigated a single sillimanite crystal using an automated four-circle X-ray
105 diffractometer (Rigaku, AFC-7S, Tohoku University) with monochromatized $MoK\alpha$
106 radiation ($\lambda = 0.71069 \text{ \AA}$, 50 kV, 20 mA). See Table 1 for a summary of the measurement

107 and its results. The cell parameters were determined from 22 centered reflections in the 2θ
108 range between 21° and 29° . The structure was refined by using reflection data from $F_o >$
109 $4\sigma(F_o)$ and diffraction intensities measured over the range $2\theta = 3\text{--}60^\circ$ in the quadrant of
110 reciprocal space ($h = -10\text{--}10$, $k = 0\text{--}10$, $l = 0\text{--}8$) by the $2\theta - \omega$ scan method. After
111 making Lorentz polarization corrections, we averaged the intensities of symmetrically
112 equivalent reflections in the Laue group $2/m2/m2/m$ to produce a set of 413 unique data. No
113 crystal absorption correction was applied given the crystal's small size and low absorption
114 coefficient. The structure was calculated using the SHELXL-97 program (Sheldrick 1997).
115 The initial parameters of sillimanite with space group $Pbnm$ were from Winter and Ghose
116 (1979), and neutral atom scattering factors were used for all atoms. In addition to fractional
117 atom coordinates, the site occupancies of tetrahedral sites (Al2, Si) were refined to evaluate
118 Q . However, the site occupancies refined by peak intensities alone may not be accurate
119 because X-ray scattering factors of Al and Si are so similar. Instead, it is common practice
120 to determine the Al content in the O-tetrahedra of aluminum silicates from the linear
121 relation between average T–O distance and Al content in tetrahedra (e.g., Smith and Bailey
122 1963). Therefore, in this study we estimated the Al content in tetrahedral sites also by
123 comparison of T–O distance with previous studies. Bish and Burnham (1992) also
124 attempted to estimate the Al content in tetrahedral sites from existing sillimanite data,
125 obtaining results consistent with Al occupancies refined from neutron diffraction patterns.

126

127 **Annealing experiment**

128 Sillimanite crystals were placed in a platinum crucible and annealed in a muffle

129 furnace at five different combinations of temperature and time (1090 °C for 1255 h, 1140
130 °C for 1512 h, 1373 °C for 300 h, 1476 °C for 90 h, and 1530 °C for 5 h). The temperature
131 was measured with a Pt₇₀Rh₃₀–Pt₉₄Rh₆ (B-type) thermocouple placed at the top of the
132 sample space in the furnace. The run temperature, controlled by the thermocouple, was kept
133 constant within ±1 °C of the nominal value. After heating, samples were allowed to cool to
134 room temperature.

135

136 **Preparation for TEM analysis**

137 Sections for TEM examination were prepared from the starting material and
138 annealed samples using a focused ion beam apparatus (FEI Quanta 200 3DS or Helios
139 NanoLab G3 CX). Predefined areas measuring 15 μm × 2 μm were coated with Pt, the
140 surrounding areas were removed to a depth of ~10 μm using a Ga⁺ ion gun, and then the
141 sections were cut off and mounted on a TEM grid. The extracted sections were thinned to a
142 thickness of 70–350 nm using a Ga⁺ ion beam at 30 kV with beam currents of 0.1–3 nA.
143 The final processing used a Ga⁺ ion beam at 5 kV with a beam current of 48 pA to remove
144 amorphous layers on the surface of the sections.

145

146 **TEM observations**

147 Before conducting the HARECXs observations, we observed the sillimanite
148 sections using a TEM (JEOL, JEM-2100F) operated at 200 kV, paying particular attention
149 to the presence of mullite. Images were recorded by a CCD camera (Gatan, Orius SC200D).
150 Crystallographically, mullite (space group: *Pbam* (No. 55), cell parameters: $a = 7.54336\text{\AA}$,

151 $b = 7.69176\text{\AA}$, $c = 2.88402\text{\AA}$; Balzar and Ledbetter 1993) is very similar to sillimanite
152 (space group: *Pbnm* (No.62), cell parameters: $a = 7.4883\text{\AA}$, $b = 7.6808\text{\AA}$, $c = 5.7774\text{\AA}$;
153 Winter and Ghose 1979), but mullite formation in sillimanite can be identified by the
154 presence of glass phases rich in SiO_2 that should be formed with mullite (e.g., Holland and
155 Carpenter 1986; Raterron et al. 1999, 2000; Rahman et al. 2001). The mullite formation is
156 also distinguished by its selected area electron diffraction (SAED) pattern, because there is
157 a periodic difference between the two phases: Al/Si configuration in the tetrahedral sites is
158 ordered (alternated along *c*-axis) in sillimanite whereas it is disordered in mullite. This
159 Al/Si alternation in the tetrahedral sites along *c*-axis can be identified by the reflections
160 with $l = \text{odd}$ ($2n+1$) of sillimanite. These reflections are invisible in mullite because of
161 reduction of the Al/Si alternation, although these reflections are no longer “ $l = \text{odd}$ ”
162 reflections in mullite because the *c*-axial length of mullite is also no longer half the size as
163 that of sillimanite. For simplification, we indexed all the SAED patterns based on the
164 standard cell of sillimanite. In this case, the “ $l = \text{odd}$ reflections” are typical in sillimanite
165 but are absent in mullite. Therefore, we also observed the sections by dark field imaging
166 taken from the $l = \text{odd}$ reflections of sillimanite.

167

168 **HARECXs measurements**

169 HARECXs profiles (X-ray intensities of Al-K, Si-K and O-K vs. beam tilting
170 angle) were obtained for each sillimanite section by using a TEM-EDS system (JEOL
171 JEM-2100F, JED-2300T) operated in the beam-rocking mode with a 200 keV electron
172 beam (Fig. 1), following Muto and Ohtsuka (2017). The change in EDS spectra in response

173 to beam tilting was observed by using electron channeling along the (202) plane, on which
174 Al and Si tetrahedra are arranged separately in ordered sillimanite but are mixed in
175 disordered sillimanite (Fig. 2a). In such condition, the HARECXS profiles are expected to
176 change the most significantly with Al/Si disordering. Firstly, the samples were tilted to the
177 condition where only the 101 systematic row reflections were excited and other reflections
178 were possibly not excited (Fig. 2b). Then, the measurement was performed by automated
179 control of beam tilting and X-ray detection. The beam tilting was controlled with a small
180 step in the direction of the 101 systematic row reflections including from $\bar{8}0\bar{8}$ to 808
181 Bragg conditions. The convergent angle of the incident beam was set to 0° (parallel beam)
182 by checking diffraction spots to emphasize the channeling phenomenon. In this condition,
183 our TEM illuminated a region $\sim 1 \mu\text{m}$ in diameter, allowing us to analyze sillimanite regions
184 without mullite or glass.

185 Convergent-beam electron diffraction (CBED) patterns were also obtained from the
186 measurement region to estimate the thickness of the sample, which is known to affect
187 HARECXS profiles. The sample thickness was determined by comparing obtained and
188 simulated CBED patterns, calculated with the program MBFIT (Tsuda and Tanaka 1999).

189

190 **Simulation of HARECXS profiles**

191 We evaluated the observations by producing simulated HARECXS profiles with the
192 program ICSC (Oxley and Allen 2003). We used structural parameters of sillimanite from
193 Winter and Ghose (1979), and the degree of Al/Si order was varied by manipulating the site
194 occupancies of tetrahedral sites. In the calculation of inelastic scattering process, the sample

195 thickness determined from CBED patterns was used as an input parameter.

196

197

Results and discussion

198 Single-crystal XRD of the starting material

199 The results of the structural refinement of the starting material from XRD data,
200 which converged to a final $R1$ factor of 3.19%, are listed in Tables 1 and 2. The refined site
201 occupancy was 96.8% Al with 3.2% vacancy in the Al2 site, and 96.5% Si with 3.5%
202 vacancy in the Si site. This is not consistent with chemical composition obtained by
203 SEM-EDS. This indicates that the site occupancies cannot be determined accurately from
204 peak intensities alone, as expected. Instead, we estimated the site occupancies by
205 comparison of T–O distance with various other sillimanite, following previous studies
206 (Bish and Burnham, 1992). Table 3 shows that the average distances of the Al2–O and Si–O
207 bonds in tetrahedra, calculated from atomic positions, are close to those reported by
208 Peterson and McMullan (1986). Among previous studies, Burnham (1963) reported the
209 most ordered trend of average T–O distances; therefore, we used those results for our
210 estimates under the assumption that the sillimanite of Burnham (1963) was a perfectly
211 Al/Si-ordered sillimanite (i.e., $Q = 1$). Figure 3 shows the linear relationship from Burnham
212 (1963) between Al content in tetrahedral sites and bond distance of sillimanite, with Al
213 content in the Si site plotted as zero and Al content in the Al2 site plotted as one. The line
214 through these two points is expressed by the relation $y = 6.4516x - 10.418$. This relation
215 compares favorably with the work of Jones (1968), who showed that data from feldspar and
216 other framework silicates fit the relations $y = 6.3481x - 10.178$ and $y = 6.4116x - 10.282$,
217 respectively. Applying the relation in Figure 3 to our sillimanite data yields a proportion of
218 5.8(19)% Al in the Si site and 94.3(19)% Al in the Al site, for a Q value of 0.88(4). This

219 result also agrees with the bulk composition of tetrahedral sites, in which Si:Al = 1:1.

220

221 **TEM observations and SAED patterns of untreated and annealed samples**

222 TEM images of the starting material showed no characteristic textures such as
223 antiphase boundaries, lamellae, or inclusions, and the SAED pattern displayed diffraction
224 intensities from sillimanite alone (Fig. 4). However, in a sample heated at 1530 °C for 5 h, a
225 dark-field TEM image taken from 111 reflection of sillimanite showed dark-contrast
226 regions containing small inclusions elongated in the direction of the *c*-axis of the host
227 sillimanite (Fig. 5). This texture resembles that resulting from mullite formation, as
228 reported by previous TEM studies (e.g., Raterron et al. 1999, 2000; Rahman et al. 2001),
229 and indicates that the dark region consists of mullite + SiO₂-rich glass. These mullite
230 regions were also observed in a sample heated at 1476 °C for 90 h and in a sample heated at
231 1373 °C for 300 h, but they were not observed in samples heated at 1140 °C for 1512 h and
232 at 1090 °C for 1255 h. This result is consistent with the 1200 °C formation temperature of
233 mullite in sillimanite, as previously determined from high-resolution XRD analysis of
234 annealed sillimanite (Igami et al. 2017). However, all samples retained regions of
235 sillimanite that displayed *l* = odd reflections in their SAED patterns, which indicates that
236 their Al/Si distribution was not completely disordered.

237

238 **HARECXS profiles of the starting material**

239 HARECXS profiles of the starting material were obtained at 18 points in two
240 sections of differing thicknesses. Although these profiles displayed significant differences

241 (Fig. 6), the difference is thought to reflect the differences in sample thickness rather than
242 differences in Q , as suggested by the differences in CBED patterns (Fig. 6). This result
243 clearly shows the importance of accounting for sample thickness in quantitative
244 determinations of the order parameter by HARECXS. In this study, thicknesses determined
245 by the CBED method corresponded to the calculated HARECXS profiles.

246 For traditional ALCHEMI, 1/4–3/4 of the extinction distance (ξ) is acknowledged as
247 suitable sample thickness (Buseck and Self, 1992). The extinction distance of the 101
248 reflection of sillimanite can be calculated using the structural parameters from Winter and
249 Ghose (1979), to be $\xi_{101} = 887\text{nm}$. Therefore, the suitable sample thickness of this study is
250 roughly expected as 222–665nm. Our HARECXS results agree with this value; for example,
251 gradients of the profiles at the thickness of $\sim 380\text{nm}$ (Fig. 7, right) is larger than those of the
252 thickness of $\sim 80\text{nm}$ (Fig. 7, left). This means that the measurement at the region of the
253 thickness of $\sim 380\text{nm}$ is expected to provide more precise result because of larger
254 channeling effect than that of the thickness of $\sim 80\text{nm}$.

255 The order parameter Q was determined by fitting with linear combinations of two
256 simulated profiles, f_i calculated assuming complete order ($Q = 1$) and g_i calculated
257 assuming complete disorder ($Q = 0$), where $i = \text{O, Al, or Si}$. The linear-combination profiles
258 F_i are expressed as

$$259 \quad F_i = a_i(Qf_i + (1 - Q)g_i) + b_i,$$

260 where a_i is a scale factor and b_i is the background value. The Q parameter in this expression,
261 corresponding to the contribution ratio of f_i and g_i , was determined by least-squares
262 fitting of observed profiles of O, Al, and Si to their respective F_i . The scale and offset of the

263 horizontal axis of the HARECXs profiles were also optimized at the time of this fitting.

264 The HARECXs profile with optimized Q shown in Figure 6 along with linear
265 combination profiles with $Q = 1, 0.9, 0.8, \dots, 0$, indicates that profiles were successfully
266 reproduced by the linear combination model. The Q values determined from each of our 18
267 measurement points (Table 4 and Fig. 7a) were consistent, averaging 0.80(1), despite the
268 widely varying thickness of the starting material.

269

270 **Comparison of results between HARECXs and XRD**

271 The average Q value of 0.80(1) for sillimanite determined by HARECXs is roughly
272 consistent with the result of 0.88(4) determined by XRD, but there is a slight discrepancy.
273 Oxley and Allen (2003) noted that a term for absorptive scattering of the incident electron
274 beam other than thermal diffuse scattering (additional mean absorption λ_{ABS}) must be added
275 to calculations to make experimental and observed HARECXs profiles agree. They
276 reported that $\lambda_{ABS} = 1300 \text{ \AA}$ (mean free path equivalent) was required to ensure agreement
277 between the calculated and experimental HARECXs profile of Matsumura et al. (2001),
278 obtained from a specimen of spinel (MgAl_2O_4) 1800 \AA thick by using a 120 keV incident
279 electron beam. They mentioned that λ_{ABS} may also simulate the effect of experimental
280 factors, such as uncertainty in sample thickness, specimen drift, or beam convergence.
281 Therefore, we tested three different values of the parameter λ_{ABS} in this study (Table 4 and
282 Fig. 7) and found that $\lambda_{ABS} = 2500 \text{ \AA}$ yielded the best match with the XRD result of $Q =$
283 0.88(4). The additional term $\lambda_{ABS} = 2500 \text{ \AA}$ for correction of systematic errors may be
284 suitable for quantitative analyses of HARECXs profiles obtained by our method.

285

286

287 **HARECXS measurements of annealed sillimanite**

288 HARECXS profiles were also obtained from annealed samples, avoiding
289 precipitation of mullite or glass inclusions, and Q values were determined by fitting
290 simulated profiles using $\lambda_{ABS} = 2500 \text{ \AA}$. The linear combination model successfully
291 reproduced the HARECXS profiles (Table 5 and Fig. 8). We found that annealing
292 temperatures as high as 1530 °C did not completely disorder sillimanite in the HARECXS
293 profiles (Fig. 9). This result is consistent with our SAED patterns, which displayed $l = \text{odd}$
294 reflections in all samples. The HARECXS results improved upon SAED patterns by
295 showing more quantitatively that Q decreased continuously from ~ 0.9 to ~ 0.5 with
296 increasing temperature.

297

298

299 **Temperature dependence of Q in sillimanite**

300 We used our experimental results to determine the order parameter Q as a function
301 of temperature by least-squares fitting with the Bragg-Williams model (solid line in Fig. 9).
302 This well-known model can predict how the equilibrium order parameter Q varies with
303 temperature, on the assumption that the enthalpy of cation mixing arise only from the
304 interactions between the nearest neighbor cations. We used the lowest Q value at each
305 heating temperature for the fitting, considering it to be the result closest to the equilibrium
306 disordered state. This fitting yielded a transition temperature of 1727 °C, although the real

307 transition temperature might be slightly lower if the samples did not reach an equilibrium
308 disordered state. This result implies that sillimanite becomes completely disordered above
309 approximately 1700 °C. However, at low pressures, the transformation to mullite with
310 partial melting begins immediately at these temperatures (e.g., Tomba et al. 1999; Igami et
311 al. 2017). Greenwood (1972) also studied Al/Si order in sillimanite and predicted values of
312 Q as a function of temperature with the Bragg-Williams model, but that prediction was
313 loosely constrained (between the dotted lines in Fig. 9) because experimental evidence at
314 high temperatures was lacking. The curve established by this study is well constrained at
315 high temperatures.

316

317

318

Implications

319 In this study, we successfully quantified the Al/Si order parameter Q in annealed
320 sillimanite, and the results shows that Q decreases with increasing temperature. According
321 to the Bragg-Williams Q - T curve (Fig. 9), our Q value of 0.88 ± 0.01 for the sillimanite
322 sample from Rundvågshetta corresponds to an experienced temperature of 1000 ± 30 °C.
323 This result is comparable to independent estimates that put the peak metamorphic
324 temperature at Rundvågshetta higher than at least 900–1000 °C (e.g., Kawasaki et al. 1993,
325 2011; Harley 1998; Fraser et al. 2000). The real peak temperature may have been higher,
326 because Q may have increased during slow cooling from the peak metamorphic stage. More
327 exact temperature will be estimated by further experiments taking cooling rate into account.
328 Subsequent precise experiments about temperature dependence of Q in sillimanite would

329 also shed more light on the effect of Al/Si disordering to the phase equilibria of the Al_2SiO_5
330 polymorphs, which in turn should increase the accuracy of the Al_2SiO_5 phase diagram.

331 The HARECXS method enabled us to obtain crystallographic information from a
332 $\sim 1 \mu\text{m}$ diameter region, avoiding even the smallest interfering phases. Our results suggest
333 that HARECXS will be useful for determining site occupancies of other common
334 rock-forming minerals, such as Mg/Fe in olivine or pyroxene, and possibly for obtaining
335 new information about the formation process of minerals with complicated microtextures.

336

337

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343

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431 **Figure captions**

432

433 **Tables**

434 **Table 1.** Details and results of the single X-ray diffraction measurement of the starting
435 material

436

437 **Table 2.** Atomic coordinates of the starting material from the single X-ray diffraction
438 measurement

439

440 **Table 3.** Average distances of the four T–O bonds in Al/SiO₄ tetrahedra

441

442 **Table 4.** Order parameter Q from individual measurements of the starting material

443

444 **Table 5.** Order parameter Q in annealed sillimanite

445

446

447 **Figures**

448 **Figure 1.** Schematic illustrations of HARECXS experiment. Electron beam is tilted to one
449 direction (along the systematic row reflections of 101), keeping the pivot point on the
450 measurement region. The characteristic X-ray intensities are obtained with respect to the
451 tilting angle.

452

453 **Figure 2.** Channeling plane and diffraction condition for the HARECXS experiments of
454 this study. (a) The channeling plane (202) in crystal structure of ordered sillimanite. Only
455 AlO_4 and SiO_4 tetrahedra are drawn in this figure. On the (202) planes, AlO_4 or SiO_4
456 tetrahedra are arranged separately in ordered sillimanite, but are mixed in disordered
457 sillimanite. (b) An example of diffraction patterns at suitable diffraction conditions.

458

459 **Figure 3.** Average T–O distances in tetrahedra and estimated Al content in tetrahedral sites.
460 The data of Burnham (1963) are plotted as perfectly ordered sillimanite in which Al content
461 is zero in the Si site and 1 in the Al2 site. Solid line represents the linear relationship
462 between Al contents and bond distances ($y = 6.4516x - 10.418$).

463

464 **Figure 4.** TEM image and SAED pattern of starting material. (b) is enlargement of the
465 dashed outline in (a). Characteristic microtextures are absent in the TEM image, and the
466 SAED pattern (inset of b) shows $l = \text{odd}$ reflections.

467

468 **Figure 5.** TEM images and SAED pattern of a sample heated at 1530 °C for 5 h. Dark field
469 TEM image taken from 111 reflection of sillimanite (a) shows dark contrast regions. Bright
470 field image of the dashed outline (b) shows small inclusions elongated with the c -axis of
471 the host sillimanite, which are similar to textures of mullite formation observed by previous
472 studies (e.g., Raterron et al. 1999, 2000; Rahman et al. 2001). SAED pattern (inset of a)
473 shows $l = \text{odd}$ reflections.

474

475 **Figure 6.** Examples of HARECXS profiles and CBED patterns obtained from the starting
476 material. Dotted lines show the linear combination profiles with $Q = 1, 0.9, 0.8, \dots, 0$. Solid
477 lines show the profiles with optimized Q .

478

479 **Figure 7.** Histograms of the Q values determined for the starting material with (a) no λ_{ABS}
480 addition, (b) $\lambda_{ABS} = 3000 \text{ \AA}$, (c) $\lambda_{ABS} = 2500 \text{ \AA}$, and (d) $\lambda_{ABS} = 2000 \text{ \AA}$.

481

482 **Figure 8.** HARECXS profiles and CBED patterns obtained from sillimanite annealed at
483 $1476 \text{ }^\circ\text{C}$ for 90 h (left) and at $1530 \text{ }^\circ\text{C}$ for 5 h (right). Dotted lines show the linear
484 combination profiles with $Q = 1, 0.9, 0.8, \dots, 0$. Solid lines show the profiles with
485 optimized Q .

486

487 **Figure 9.** Values of Q determined by the HARECXS method plotted against annealing
488 temperature (circles). The solid line shows predicted Q values determined by least-squares
489 fitting with the Bragg-Williams model, and the region between the dotted lines is the range
490 of Q values predicted by Greenwood (1972). The cross indicates the starting material,
491 plotted by applying the average Q of 0.88 to the determined Bragg-Williams Q - T curve and
492 corresponding to a temperature of $\sim 1000 \text{ }^\circ\text{C}$ for the starting material.

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sample	Sillimanite
Crystal data	
Chemical formula	Al ₂ SiO ₅
Cell weight	648.20
Crystal system, space group	Orthorhombic, <i>Pbnm</i>
Temperature (K)	300 (Room temperature)
<i>a</i> , <i>b</i> , <i>c</i> (Å)	7.4867(6), 7.6750(5), 5.7720(5)
<i>α</i> , <i>β</i> , <i>γ</i> (°)	90, 90, 90
<i>V</i> (Å ³)	331.66
<i>Z</i>	4
<i>F</i> (000)	320.0
<i>D_x</i> (Mg m ⁻³)	3.245
<i>μ</i> (mm ⁻¹)	1.12
Crystal shape	Block
Color	Colorless
Crystal size (μm)	80 × 80 × 60
Data collection	
Diffractometer	Rigaku, AFC-7S
Radiation source	Sealed Tube
Total reflections measured	1206
Independent reflections	527
Observed reflections	413
<i>R_{int}</i>	0.049
<i>θ</i> values (°)	<i>θ</i> _{max} = 60.0, <i>θ</i> _{min} = 3.0
(<i>sin θ</i> / <i>λ</i>) _{max} (Å ⁻¹)	0.7035
Range of <i>h</i> , <i>k</i> , <i>l</i>	<i>h</i> = -10→10, <i>k</i> = 0→10, <i>l</i> = 0→8
Refinement	
Refinement on	<i>F</i>
<i>R</i> 1	0.0319
<i>R</i> (all)	0.0429
w <i>R</i> 2	0.0749
GooF(obs)	1.082
GooF(all)	1.082
No. of reflections(<i>F_o</i> > 4σ(<i>F_o</i>))	413
No. of parameters	49
Weighting scheme	$w = 1/[(\sigma^2(F_o^2) + (0.0144P)^2 + 0.00 P]$, where $P = [\text{Max}(F_o^2, 0) + 2(F_c^2)]/3$
(Δ/σ) _{max}	0.00
Δρ _{max} , Δρ _{min} (e Å ⁻³)	0.41, -1.05

Site	Occ	<i>x</i>	<i>y</i>	<i>z</i>	<i>U₁₁</i>	<i>U₂₂</i>
All	1	0	0	0	0.0064(3)	0.0053(3)
Al2	0.968(4)	0.14181(8)	0.34503(7)	0.25	0.0056(4)	0.0047(4)
Si	0.965(4)	0.15336(8)	0.34035(7)	0.75	0.0048(3)	0.0038(3)
O1	1	0.36044(18)	0.40921(19)	0.75	0.0069(6)	0.0099(7)
O2	1	0.35646(19)	0.43384(19)	0.25	0.0080(7)	0.0076(6)
O3	1	0.4766(2)	0.00129(16)	0.75	0.0123(8)	0.0092(7)
O4	1	0.12569(16)	0.22326(13)	0.51458(17)	0.0093(5)	0.0066(5)

U_{33}	U_{23}	U_{13}	U_{12}
0.0064(4)	-0.0002(2)	0.0002(2)	-0.0001(2)
0.0042(4)	0	0	-0.0007(3)
0.0046(4)	0	0	-0.0002(3)
0.0045(8)	0	0	-0.0009(6)
0.0046(8)	0	0	0.0006(6)
0.0151(10)	0	0	-0.0038(6)
0.0070(5)	-0.0005(4)	-0.0003(4)	-0.0016(5)

	Average bond distance (Å)		Remarks
	Al2 – O	Si – O	
Burnham (1963)	1.770(2)	1.615(3)	single crystal XRD
Winter & Ghose (1976)	1.764(5)	1.626(5)	single crystal XRD
Peterson & McMullan (1986)	1.759(3)	1.623(2)	single crystal ND
Bish & Burnham (1992)	1.753(4)	1.634(3)	powder ND fibrolitic sillimanite
This study	1.761(3)	1.624(3)	single crystal XRD

Measurement No.	Thickness (nm)	Degree of order Q			
		No λ_{ABS} addition	λ_{ABS} = 3000 Å	λ_{ABS} = 2500 Å	λ_{ABS} = 2000 Å
1	252	0.79(3)	0.87(3)	0.88(3)	0.90(3)
2	292	0.75(2)	0.82(2)	0.84(2)	0.86(2)
3	222	0.75(2)	0.82(2)	0.84(2)	0.86(2)
4	272	0.80(3)	0.88(3)	0.89(3)	0.92(3)
5	249	0.76(2)	0.83(2)	0.85(2)	0.87(2)
6	289	0.73(2)	0.80(2)	0.82(2)	0.84(2)
7	199	0.77(2)	0.83(2)	0.85(2)	0.87(2)
8	269	0.73(2)	0.80(2)	0.82(2)	0.84(2)
9	178	0.80(2)	0.86(2)	0.88(2)	0.90(2)
10	75	0.93(4)	0.96(4)	0.96(4)	0.97(4)
11	175	0.85(2)	0.92(2)	0.93(2)	0.95(2)
12	350	0.78(2)	0.86(2)	0.88(2)	0.90(2)
13	166	0.87(2)	0.94(2)	0.95(2)	0.97(2)
14	74	0.90(3)	0.93(3)	0.94(3)	0.94(3)
15	355	0.79(2)	0.87(2)	0.89(2)	0.91(2)
16	348	0.76(2)	0.84(3)	0.85(3)	0.88(3)
17	174	0.77(2)	0.83(3)	0.85(3)	0.87(3)
18	73	0.86(4)	0.91(5)	0.92(5)	0.94(5)
Mean		0.80	0.87	0.88	0.90
Standard error		0.01	0.01	0.01	0.01

Annealing condition				
T (°C)	Time (h)	Measurement no.	Thickness (nm)	Order parameter Q
1530	5	1	147	0.55(2)
		2	113	0.61(3)
1476	90	1	314	0.74(3)
		2	314	0.68(2)
		3	304	0.68(2)
1373	300	1	234	0.79(2)
1140	1512	1	261	0.79(2)
		2	261	0.82(3)
1090	1255	1	304	0.84(2)
		2	237	0.86(2)
		3	345	0.85(2)
Starting Material		1 – 18	73 – 355	0.88(1)*

* Average and standard error of 18 measurements.

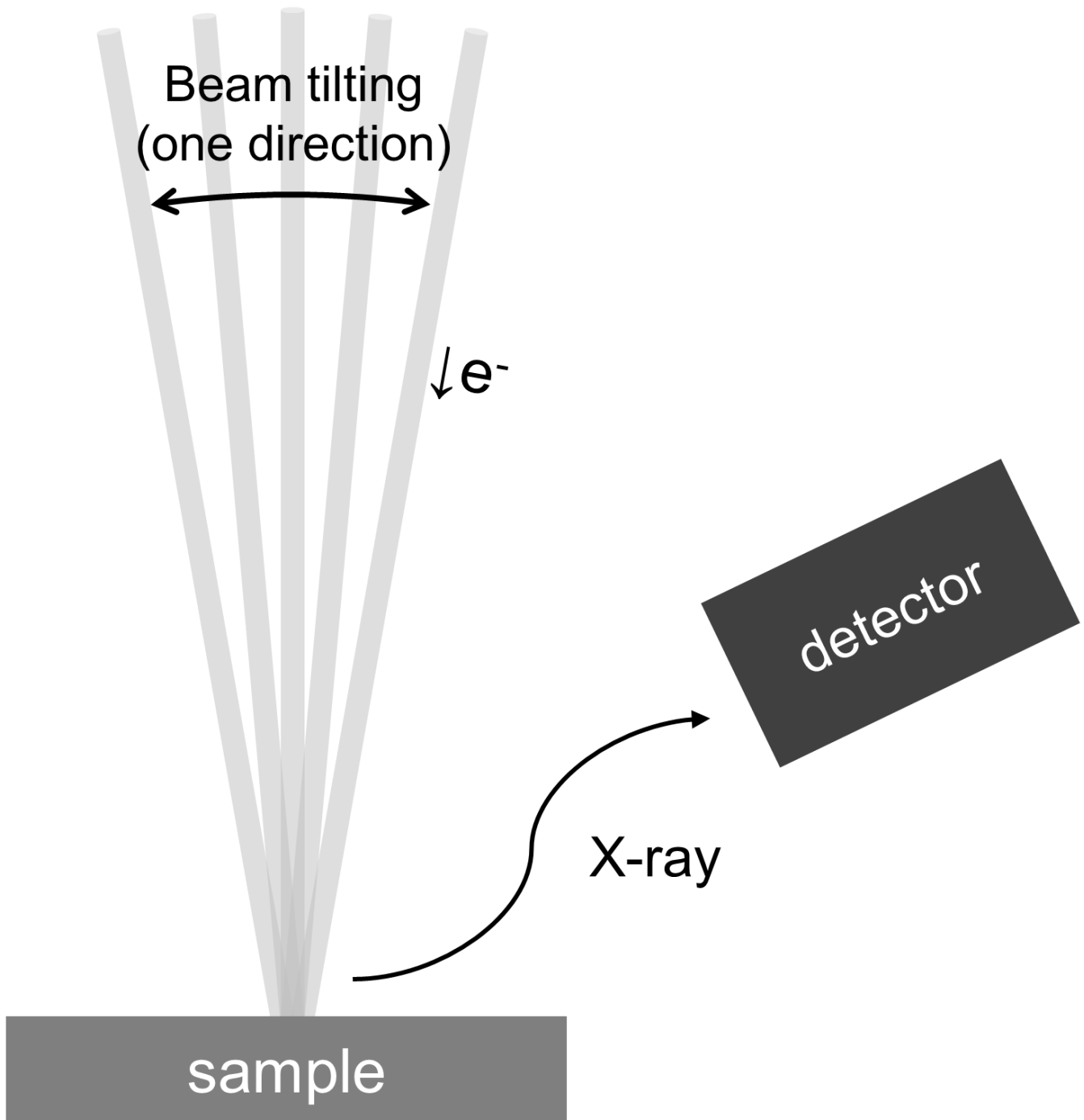


Figure 1

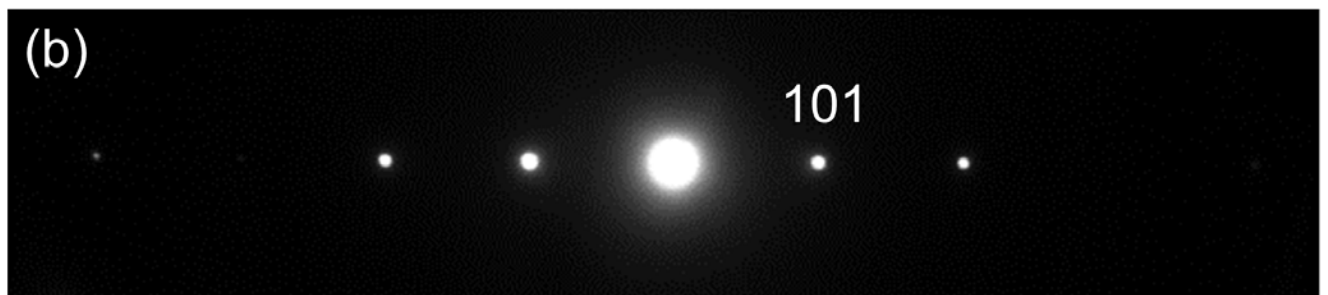
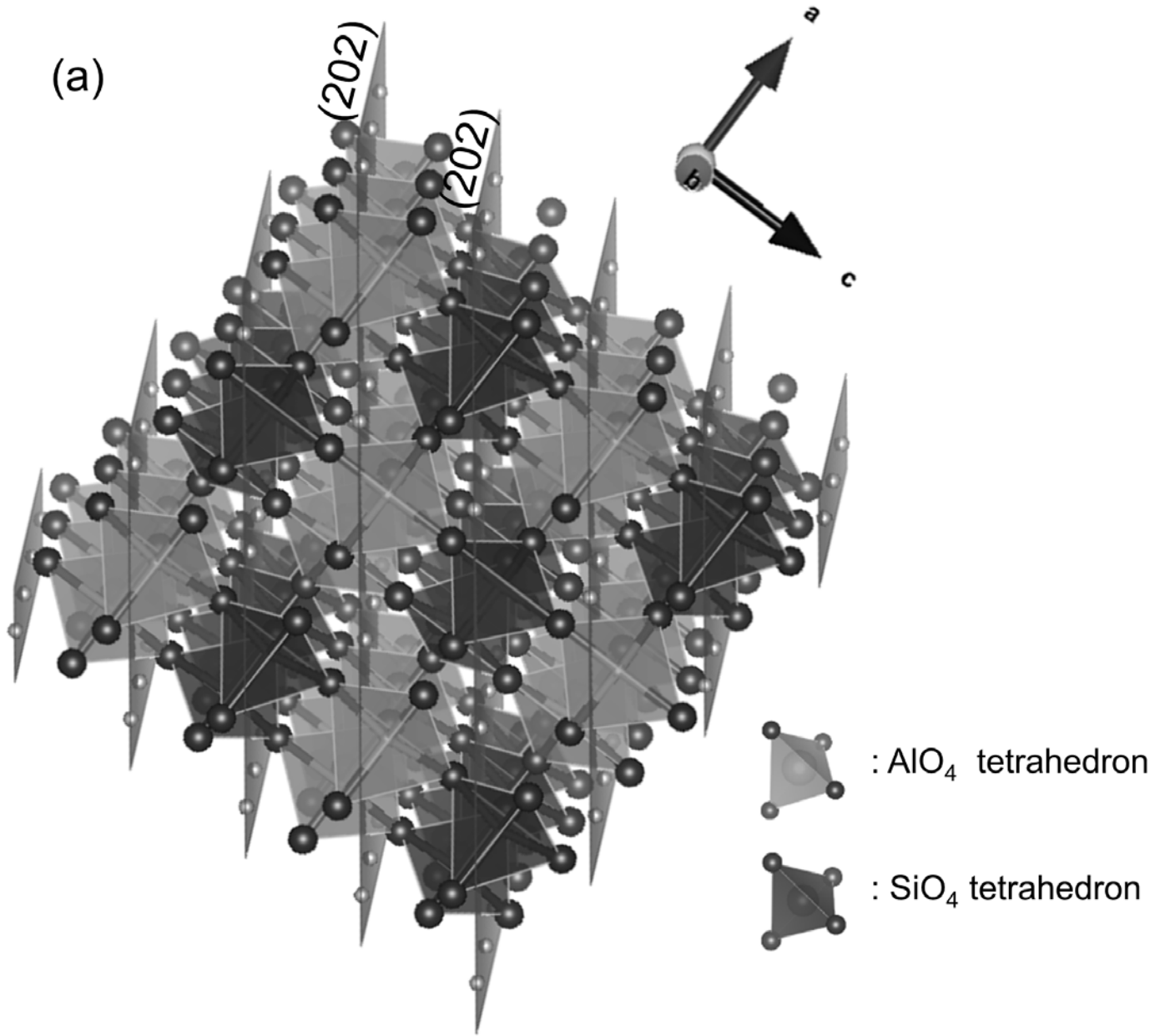


Figure 2

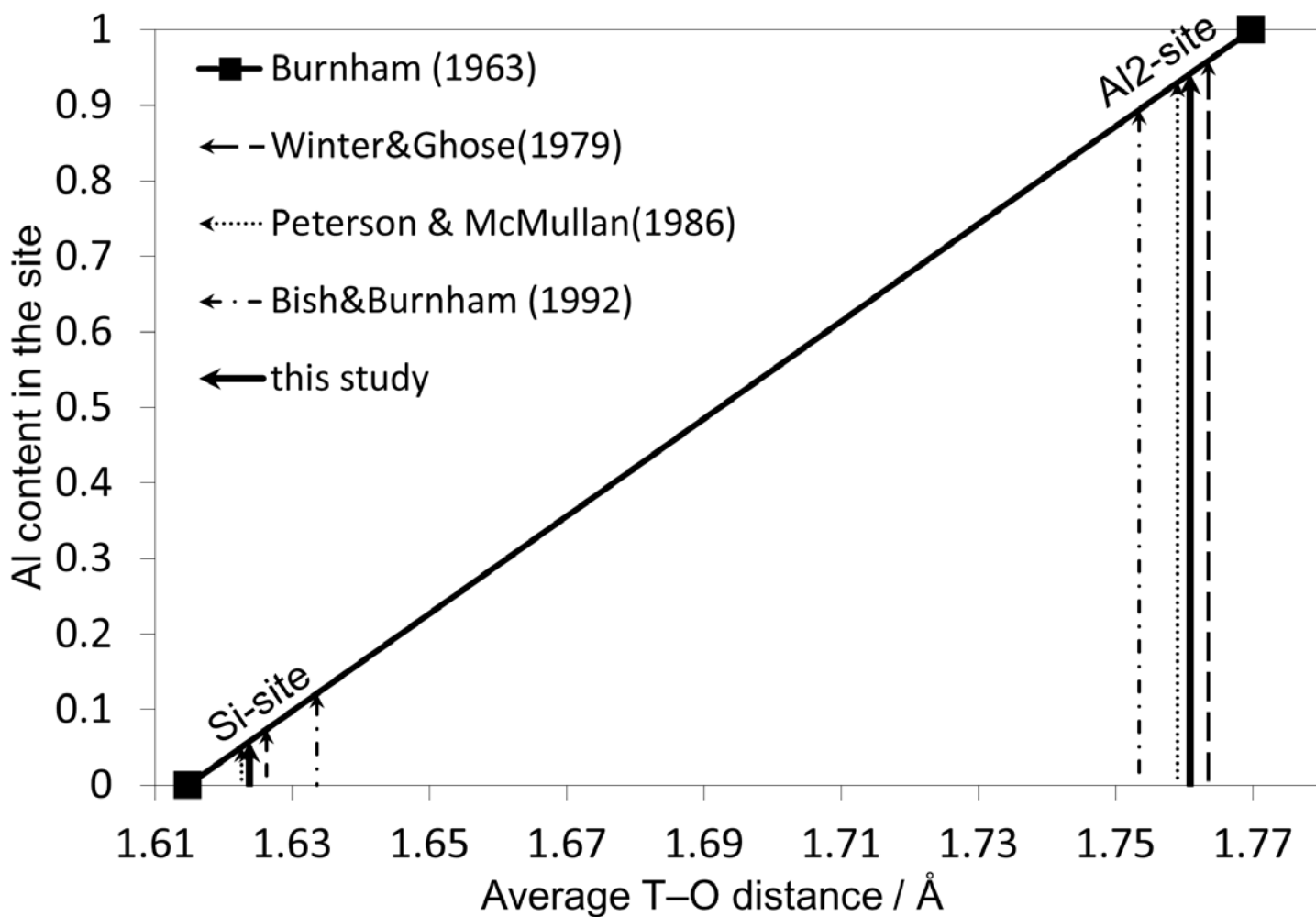


Figure 3

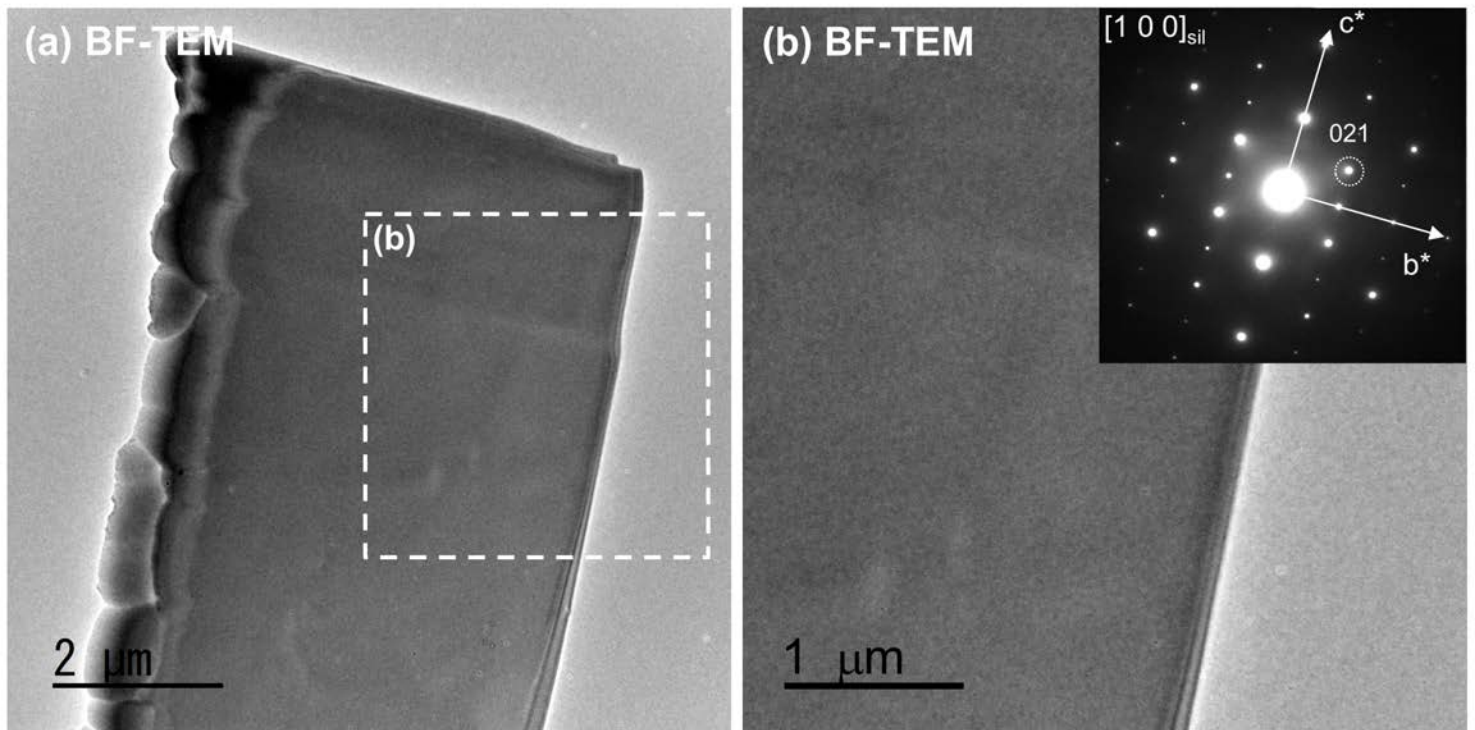


Figure 4

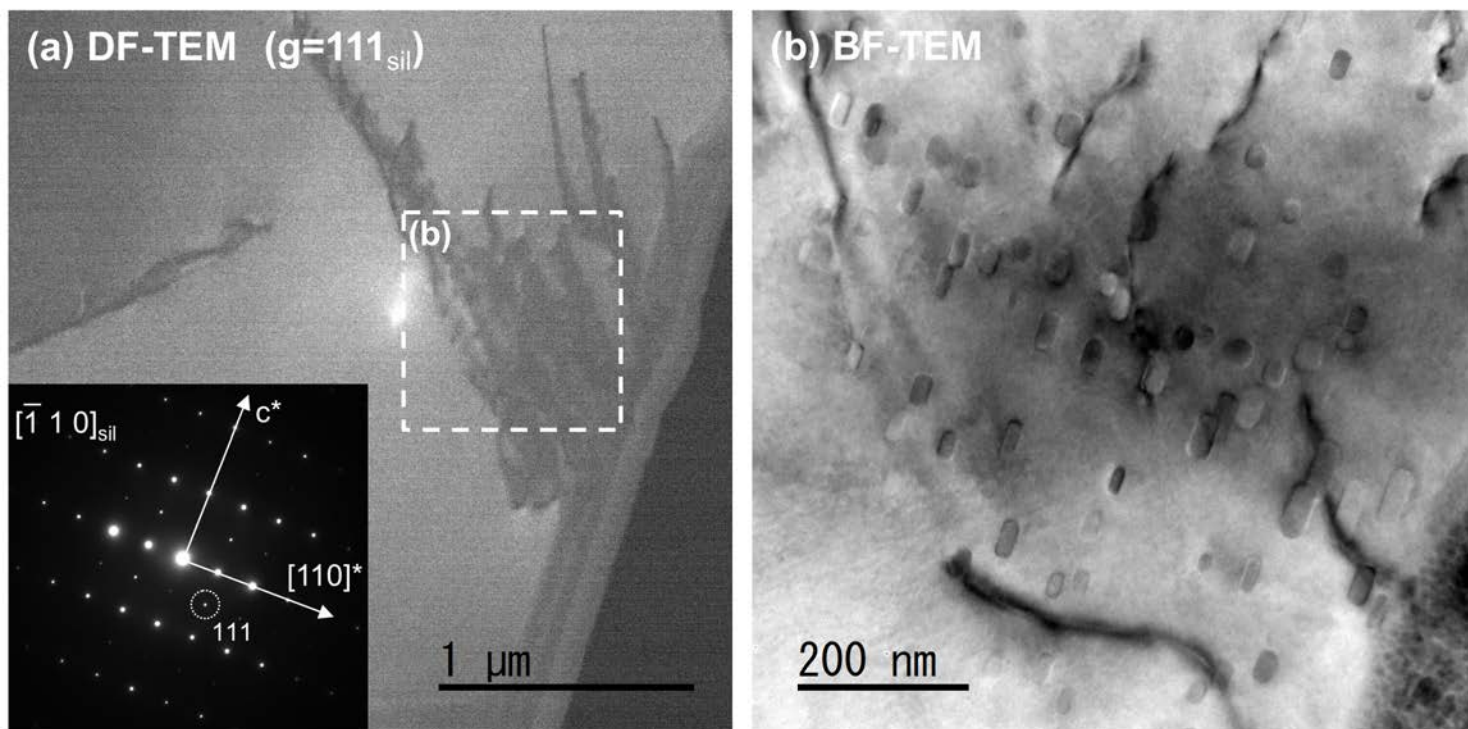


Figure 5

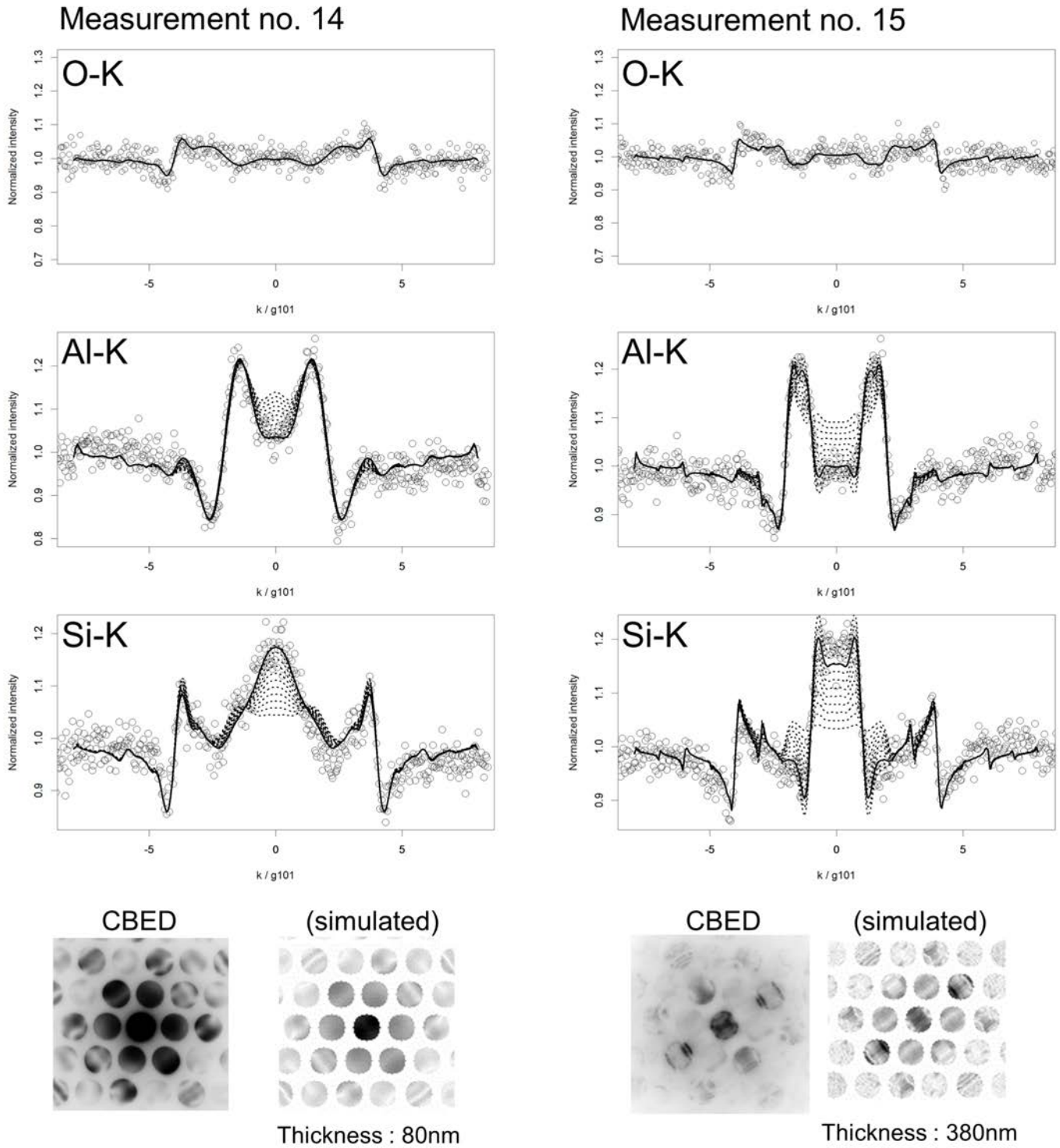


Figure 6

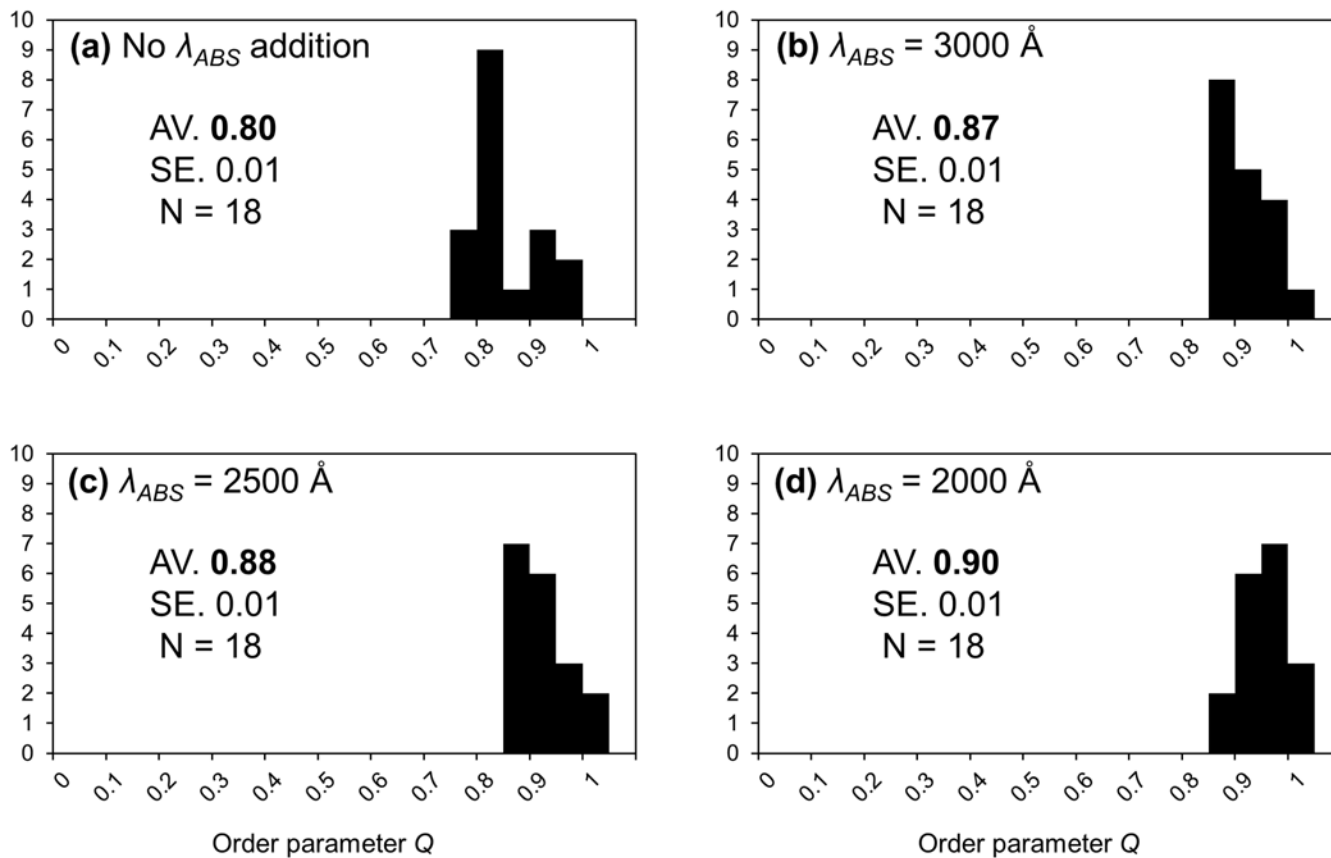


Figure 7

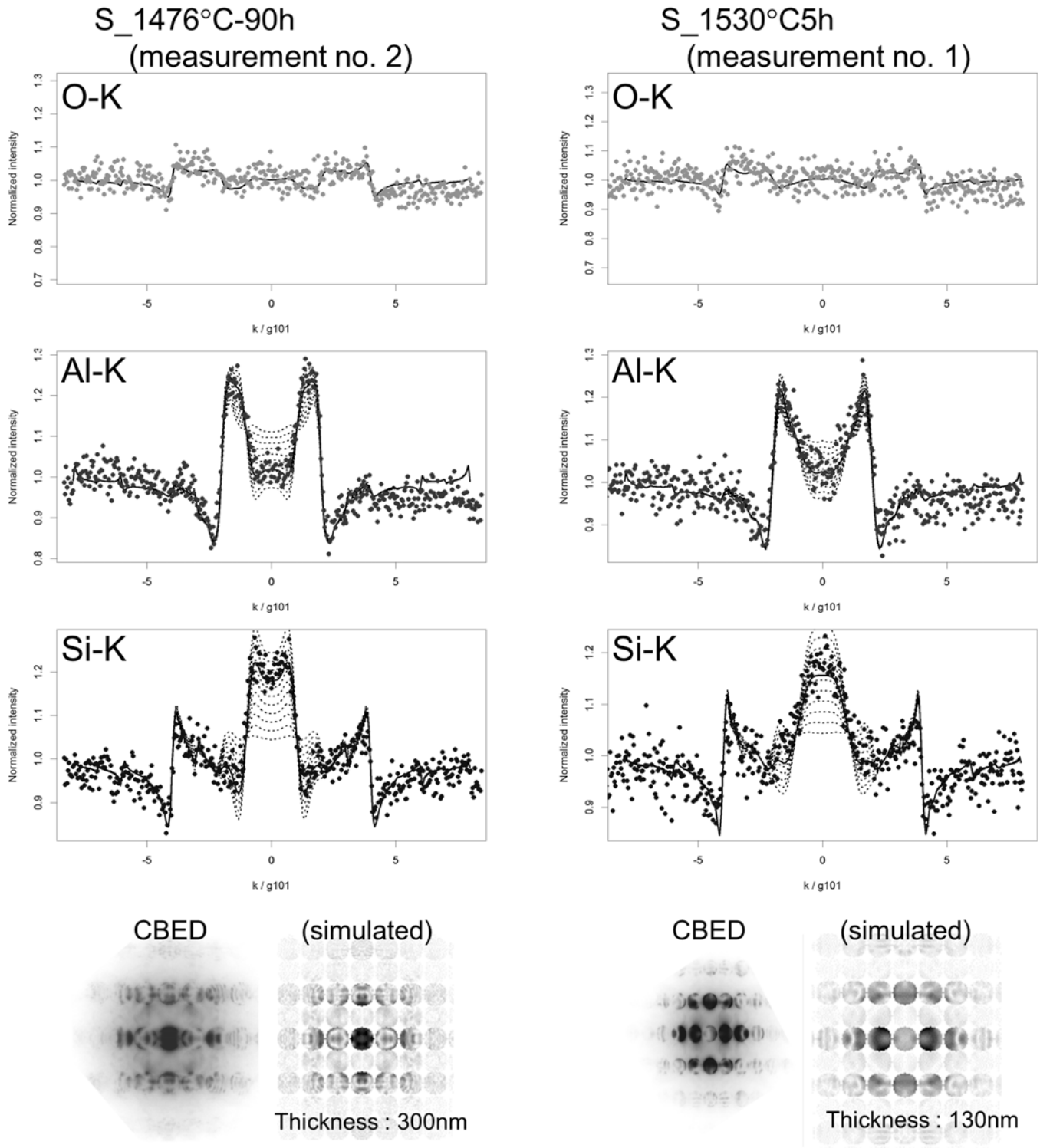


Figure 8

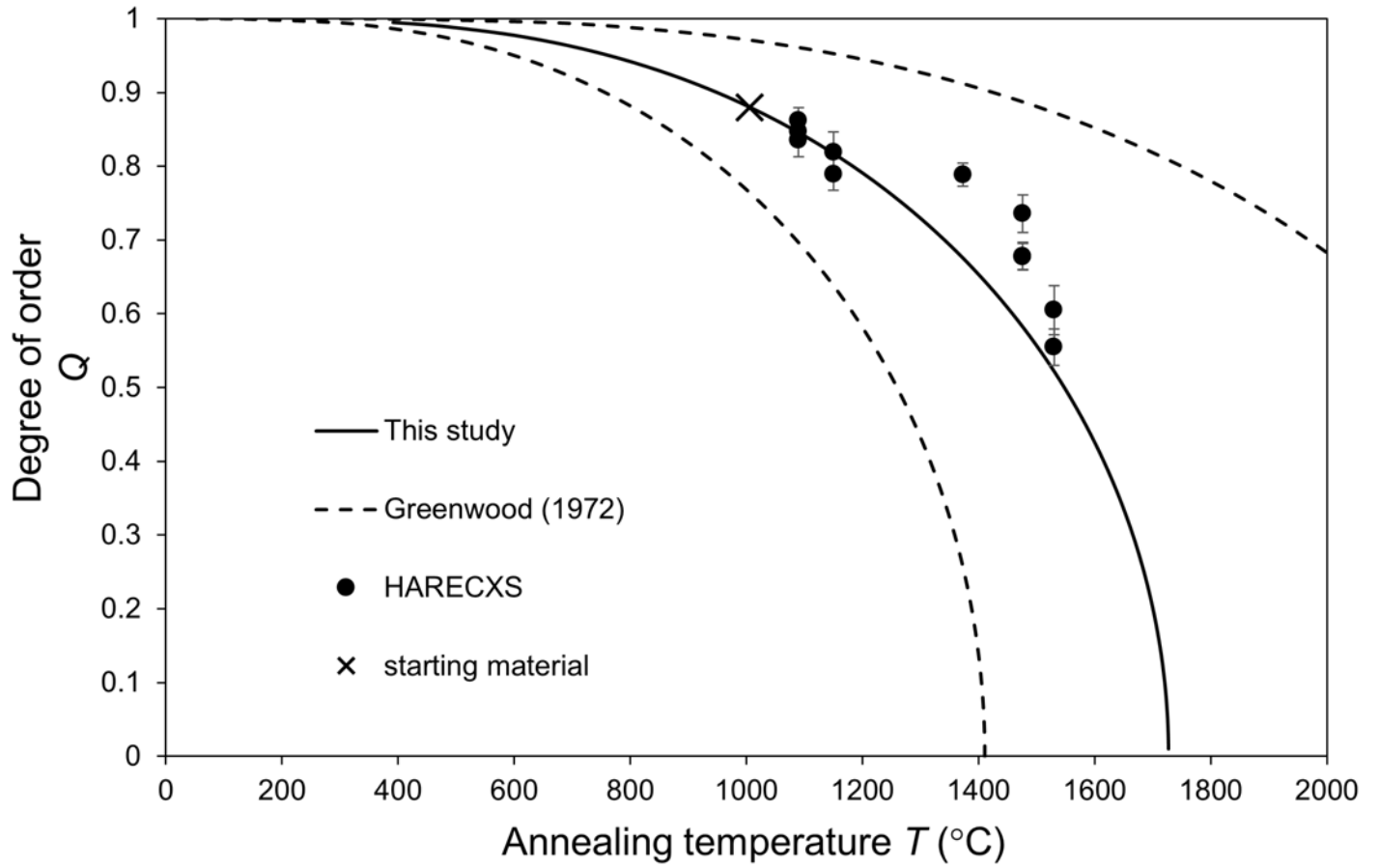


Figure 9