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1 Revision 2

2 **Determination of the full elastic tensor of single crystals using shear**
3 **wave velocities by Brillouin spectroscopy**

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13

14 **ABSTRACT**

15 Single-crystal elasticity of candidate minerals in the Earth's mantle, such as that of
16 ferropericlase and bridgmanite, etc., is very important for understanding the seismic
17 observations, geodynamic flow patterns, and testing geochemical and mineralogical
18 models of the planet's deep interior. Determination of the full elastic tensor typically
19 requires measuring both compressional and shear wave velocities (V_P and V_S) of the
20 candidate single crystal as a function of crystallographic orientations at high pressures,
21 but it has been a huge technical challenge obtaining V_P at pressures above 25 GPa
22 using Brillouin Light Scattering coupled with in a diamond anvil cell due to the
23 spectral overlap of the sample V_P with the V_S of the diamond window. In this study,
24 we present a new method to derive the full elastic tensor (C_{ij}) of single crystals using
25 only measured V_S of a given crystal platelet as a function of the azimuthal angle.
26 Experimentally determined V_P and V_S results from Brillouin measurements for cubic
27 periclase (MgO) and spinel (MgAl_2O_4), tetragonal stishovite (SiO_2), and orthorhombic
28 zoisite ($\text{Ca}_2\text{Al}_3\text{Si}_3\text{O}_{12}(\text{OH})$) at ambient conditions are used as examples to
29 demonstrate the application of our approach from theoretical analyses and

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30 experimental prospective. For high-symmetry cubic minerals, such as cubic MgO and
31 spinel, a suitable crystallographic plane with small tradeoffs between any two C_{ij} in V_S
32 is required for the method to work well such that the obtained C_{ij} using measured V_S
33 velocities alone can be within 3% of the values derived from using both V_S and V_P .
34 Our analyses show that the (-1,0.5,0.2) platelet for periclase and the (1,1,0) platelet for
35 spinel are respective optimal orientations for applying our method. For lower
36 symmetry minerals, such as tetragonal stishovite and orthorhombic zoisite, three
37 crystallographic planes, that are orthogonal to each other and are tilted at least 20
38 degrees from the principal crystallographic planes, can be used to provide reliable
39 constraints on C_{ij} using measured V_S alone. We have extended this method to derive
40 C_{ij} of the (-1,0.5,0.2) platelet for periclase at pressures of 5.8 GPa and 11.3 GPa, in a
41 high-pressure diamond anvil cell to demonstrate the usefulness of the approach in
42 studying the elasticity of Earth's mantle minerals at relevant pressure-temperature
43 conditions. Our proposed approach can be extended to all other crystal systems at
44 high pressures to overcome the constant lack of experimental V_P velocities at above
45 25 GPa, potentially providing new experimental and theoretical approaches in
46 constraining the elastic tensor of the materials in the Earth's deep interior, which will
47 be an effective strategy to solve one of the most relevant difficulties involved in the
48 experimental study of the elastic properties (especially elastic anisotropy) of minerals
49 of the lower mantle.

50 **Keywords:** Single-Crystal Elasticity, Periclase, Spinel, Zoisite, Stishovite, Brillouin
51 Light Scattering

52

53 INTRODUCTION

54 Elastic constants (C_{ij}) describe the instantaneous reversible volume and shape
55 changes of a mineral under stress (Birch 1952; Carpenter 2006; Bass et al. 2008;
56 Angel et al. 2009). Determination of the full elastic tensor of a crystal of interest
57 permits direct evaluations of a number of key elastic, thermodynamic, and mechanical
58 parameters of the crystal including adiabatic bulk modulus, shear modulus, Poisson's
59 ratio, shear stress, shear strain, compressional wave anisotropy, and shear wave

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60 splitting anisotropy, among many others (Anderson 1989; Li et al. 2004;
61 Sanchez-Valle et al. 2005; Mainprice 2007; Angel et al. 2009). Of particular
62 importance to our understanding of deep-Earth mineral physics is the elasticity of
63 minerals as a function of pressure and temperature (P-T) at conditions relevant to the
64 Earth's mantle. These results can be directly compared to seismic observations of the
65 deep Earth in order to constrain mineralogical and compositional models of the
66 Earth interior, and better understand the geodynamic behavior of the deep Earth
67 (e.g., Weidner and Carleton 1977; Weidner et al. 1978; Duffy and Anderson 1989;
68 Weidner and Zhao 1992; Zha et al. 1996; Sinogeikin et al. 2003, 2004; Li et al. 2006;
69 Li and Liebermann 2007; Bass 2008; Bass et al. 2008; Angel et al. 2009; Mao et al.
70 2012; Murakami et al. 2012; Lu et al. 2013; Yang et al. 2014). Specifically, the
71 single-crystal elasticity at deep-mantle conditions is of particular importance in
72 understanding the anisotropy and dynamic flow pattern of the deep planet (Duffy et al.
73 1995; Zha et al. 1998; Jacobsen et al. 2002; Sinogeikin et al. 2005; Chen et al. 2006;
74 Jackson et al. 2007). The ability to experimentally obtain elastic constants of minerals
75 with sufficient accuracy thus plays an indispensable role in our understanding of the
76 Earth's deep interior.

77 Brillouin Light Scattering (BLS) coupled with a diamond anvil cells (DAC) has
78 been extensively used to constrain the elasticity of earth materials at high pressures
79 and/or high temperatures (e.g., Duffy and Anderson 1989; Shimizu and Sasaki 1992;
80 Shimizu 1995; Duffy et al. 1995; Zha et al. 1996, 1998; Sinogeikin and Bass 2000;
81 Sinogeikin et al. 2006; Murakami et al. 2009a, 2009b; Mao et al. 2008, 2012; Lu et al.
82 2013; Yang et al. 2014). The frequency shifts of the BLS spectra represent a collection
83 of inelastic scattering events arising from the interactions between incident photons of
84 the laser and thermally excited acoustic phonons of the sample (Polian 2003). The
85 technique permits the measurement of one longitudinal acoustic V_P and two transverse
86 acoustic V_{S1} and V_{S2} velocities of a single crystal along a certain direction at a given
87 scattering angle (or momentum transfer) (Sinogeikin and Bass 2000). The integration
88 of the BLS technique with the DAC is well-suited for studying the single-crystal
89 elasticity of crystals at P-T conditions relevant to the deep interior of the planet. The

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90 majority of geophysical relevant materials of the deep interior of Earth are optically
91 transparent minerals (Bass et al. 2008; Speziale et al. 2014), which makes it possible
92 to use BLS to study their elastic properties to the relevant conditions of the deep Earth.
93 However, because of the overlap of the V_P of the sample with the V_S of diamond
94 anvils, it has been extremely difficult to measure V_P in order to constrain full elastic
95 moduli of a given candidate single crystal at approximately above 25 GPa (e.g., Zha et
96 al. 2000; Marquardt et al. 2009a, 2009b), especially for measuring V_P of bridgmanite,
97 ferropericlaase, post-bridgmanite by BLS at the relevant pressures of the lower mantle.
98 Such a technical problem has significantly hampered our understanding of the
99 deep-Earth physics and chemistry using experimental elasticity results. Alternative
100 approaches have been proposed to remedy this diamond window problem, but these
101 attempts have not been very successful at high P-T conditions (e.g., Jackson et al.
102 2006; Speziale et al. 2014). Examples of these studies include the use of pre-oriented
103 diamond anvils with known orientations and velocities to reduce the overlap at certain
104 orientations (Yang et al. 2014), the addition of a diamond wedge to reduce chromatic
105 aberration from the diamond anvil (Zha et al. 1998, 2000), as well as the adoption of a
106 long focusing lens to focus the laser to the sample with a small beam divergence (Zha
107 et al. 1998, 2000; Lu et al. 2013; Speziale et al. 2014; Yang et al. 2014). Nevertheless,
108 the diamond window issue remains one of the major technical hurdles that has
109 prevented mineral physicists from having a better understanding of the seismic
110 structure and the mineralogical model of the Earth's deep interior (Zha et al. 2000;
111 Bass 2007; Bass et al. 2008; Marquardt et al. 2009a, 2009b; Speziale et al. 2014).

112 Based on the expansion of the Christoffel's equation, the elastic constants of a
113 given crystal in any given crystal system are intrinsically coupled with one another,
114 although, depending on the crystal system, the coupling strength can vary in different
115 directions (Every 1980). Therefore, it is theoretically possible to derive full elastic
116 tensor of a given crystal using partial information on V_P and/or V_S datasets (Every
117 1980); though, this method has not yet been tested practically. In this study, we
118 present theoretical derivations and experimental BLS results to derive full elastic
119 tensor of a single crystal using only the V_S velocity dataset as a function of the

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120 azimuthal angle for a pre-selected crystal platelet. The validity of the results was
121 tested using derived elastic constants from both V_P and V_S results as references. Using
122 cubic periclase and spinel as well as orthorhombic zoisite and tetragonal stishovite as
123 examples, we show that the full elastic tensor of crystal in the cubic, tetragonal or
124 orthorhombic systems can be well constrained using the measured V_S velocities alone.
125 We have also derived C_{ij} of periclase with an optimal (-1,0.5,0.2) platelet at high
126 pressure using BLS measurements in a DAC to justify the use of our method at high
127 pressures. Being able to determine the full single-crystal elastic tensor of a mineral by
128 using only V_S data would allow us to largely improve our constraints on the
129 mineralogy of the deep Earth by comparing mineral physics results with seismic
130 models. In fact, despite the constant progress both in large scale geophysics models
131 and in experimental and computational mineral physics, we cannot perform precise
132 tests of proposed mineralogical models of the lowermost mantle against seismic data
133 and models based on V_P seismic velocities. Future applications of this new method at
134 high P-T conditions of the Earth's interior can help shed new light on single-crystal
135 elasticity of candidate minerals relevant to the Earth's deep interior.

136

137 THEORETICAL BACKGROUND

138 Based on the Christoffel's equation, the sound velocity (V) of a crystal can be
139 described by the following characteristic equation:

$$140 \quad |\Gamma_{ij} - \rho V^2 \delta_{ij}| = 0 \quad (1)$$

141 where ρ is the density, δ_{ij} is the Kronecker delta, and Γ_{ij} is the coefficient in the
142 Christoffel matrix. The values of the Christoffel coefficients (Γ_{ij}) depend on the
143 single-crystal constants (C_{ij}) in the reduced Voigt notation in which the propagation
144 direction of the sound velocity is described by the cosines of the velocity direction, n_i .
145 It follows that the Christoffel's equation (1) can be expanded in order to relate these
146 parameters mathematically in various directions such that one can then pre-select a
147 certain crystal plane for deriving full elastic tensor using the V_S or the V_P dataset alone.
148 Here we follow the classical paper by Every (1980) to expand equation (1) for the

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149 cubic system below. The full expansion of the Christoffel's equation for other crystal
150 systems can also be derived using similar derivation schemes given below and in
151 Every (1980).

152 For the cubic structure, Γ_{ij} is given by:

$$\begin{aligned} \Gamma_{11} &= C_{11}n_1^2 + C_{44}n_2^2 + C_{44}n_3^2 \\ \Gamma_{22} &= C_{11}n_2^2 + C_{44}n_1^2 + C_{44}n_3^2 \\ \Gamma_{33} &= C_{11}n_3^2 + C_{44}n_1^2 + C_{44}n_2^2 \\ \Gamma_{23} &= \Gamma_{32} = (C_{11} + C_{12})n_2n_3 \\ \Gamma_{13} &= \Gamma_{31} = (C_{11} + C_{12})n_1n_3 \\ \Gamma_{12} &= \Gamma_{21} = (C_{11} + C_{12})n_1n_2 \end{aligned} \quad (2)$$

158 Equation (1) can be presented in a simplified form by carrying out a linear
159 transformation that eliminates the quadratic term in the equation (3) below as follows:

$$160 \quad 3\rho V^2 = T + S \quad (3)$$

$$161 \quad T = (C_{11} + 2C_{44})(n_1^2 + n_2^2 + n_3^2) \quad (4)$$

$$162 \quad |\Lambda_{ij} - S\delta_{ij}| = 0 \quad (5)$$

$$163 \quad \Lambda_{ij} = 3\Gamma_{ij} - T\delta_{ij} \quad (6)$$

164 where T is the first invariant of Γ_{ij} , S is the real root of Γ_{ij} , Λ_{ij} is a function of the
165 elastic constants and direction cosines of the wave vector. For the cubic structure, it
166 follows that:

$$\begin{aligned} \Lambda_{11} &= 3(C_{11}n_1^2 + C_{44}n_2^2 + C_{44}n_3^2) - (C_{11} + 2C_{44})(n_1^2 + n_2^2 + n_3^2) \\ \Lambda_{22} &= 3(C_{11}n_2^2 + C_{44}n_1^2 + C_{44}n_3^2) - (C_{11} + 2C_{44})(n_1^2 + n_2^2 + n_3^2) \\ \Lambda_{33} &= 3(C_{11}n_3^2 + C_{44}n_1^2 + C_{44}n_2^2) - (C_{11} + 2C_{44})(n_1^2 + n_2^2 + n_3^2) \\ \Lambda_{23} &= \Lambda_{32} = 3n_2n_3(C_{12} + C_{44}) \\ \Lambda_{13} &= \Lambda_{31} = 3n_1n_3(C_{12} + C_{44}) \\ \Lambda_{12} &= \Lambda_{21} = 3n_1n_2(C_{12} + C_{44}) \end{aligned} \quad (7)$$

168 Expansion of the determinant in the equation (5) allows the derivation of the
169 following cubic form for S :

$$170 \quad S^3 - 3GS - 2H = 0 \quad (8)$$

171 where G and H are the second and third invariants of Λ_{ij} . Re-arranging the equation

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172 (8), the triple root of the cubic form (S_k) is represented as:

$$173 \quad S_k = 2G^{\frac{1}{2}} \cos\left(\psi + \frac{2}{3}\pi k\right) \quad (k = 0,1,2) \quad (9)$$

174 where k is the polarization index, which is only related to the absolute value of the
 175 solutions. $k=0$ always corresponds to the fastest velocity, while $k=1$ corresponds to
 176 the second intermediate, and $k=2$ to the slowest velocity, and

$$177 \quad \psi = \frac{1}{3} \arccos\left(H/G^{\frac{3}{2}}\right) \quad (10)$$

178 Based on the equation (5), variables G and H are given as:

$$179 \quad G = \frac{\Lambda_{12}^2 + \Lambda_{23}^2 + \Lambda_{31}^2 - \Lambda_{11}\Lambda_{22} - \Lambda_{22}\Lambda_{33} - \Lambda_{11}\Lambda_{33}}{3}$$

$$180 \quad = (C_{11} - C_{44})^2(n_1^4 + n_2^4 + n_3^4) - 3(C_{11} - C_{12} - 2C_{44})(C_{11} + C_{12})(n_1^2n_2^2 + n_2^2n_3^2 + n_1^2n_3^2) \quad (11)$$

$$181 \quad H = \frac{|\Lambda|}{2} = \frac{(\Lambda_{11}\Lambda_{22}\Lambda_{33} + 2\Lambda_{12}\Lambda_{23}\Lambda_{31} - \Lambda_{11}\Lambda_{23}^2 - \Lambda_{22}\Lambda_{31}^2 - \Lambda_{33}\Lambda_{12}^2)}{2}$$

$$182 \quad = (C_{11} - C_{44})^3(n_1^6 + n_2^6 + n_3^6) - \frac{9}{2}(C_{11} - C_{12} - 2C_{44})(C_{11} - C_{44})(C_{11} + C_{12}) \quad (12)$$

$$183 \quad (n_1^2n_2^2 + n_2^2n_3^2 + n_1^2n_3^2)(n_1^2 + n_2^2 + n_3^2) + \frac{27}{2}(C_{11} - C_{12} - 2C_{44})^2(C_{11} + 2C_{12} + C_{44})n_1^2n_2^2n_3^2$$

184 It follows that the velocity of a cubic crystal along a given direction (V_k) as shown in
 185 the equation (3) can be expressed in terms of the three elastic constants (C_{11} , C_{12} , C_{44}):

$$186 \quad 3\rho V_k^2 = (C_{11} + 2C_{44})(n_1^2 + n_2^2 + n_3^2) + 2G^{1/2} \cos\left(\frac{1}{3} \arccos\left(H/G^{\frac{3}{2}}\right) + \frac{2}{3}\pi k\right) \quad (13)$$

187 We note that although the off-diagonal C_{12} elastic constant is not shown explicitly
 188 in the equation (13), it has been represented in the H and G terms (see equations (11)
 189 and (12) for details). With experimentally determined C_{ij} , one can thus derive the
 190 velocities of a cubic crystal along specific directions of the crystal using equations
 191 (11), (12) and (13). Inversely, the full elastic tensor of a cubic crystal can be derived
 192 numerically from a set of measured velocities along various crystallographic
 193 orientations using the non-linear least-squares Levenberg-Marquardt method (Press
 194 1988). This method has been widely used to derive the full C_{ij} of minerals and
 195 materials from high-pressure BLS measurements in the last few decades (e.g., Duffy
 196 et al. 1995; Zha et al. 1996).

197 The aforementioned method has the advantage of resolving full elastic tensor using
 available V_S velocities from Brillouin measurements. Derivations of the longitudinal

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198 elastic constant (C_{11}) typically require having the V_P of the crystal, but the
199 longitudinal acoustic peaks in the BLS spectra normally overlap with the V_S peaks of
200 the diamond anvils at pressures above 25 GPa. Such a problem with the spectral
201 overlap is prevalent in most oxide and silicate minerals of the deep Earth, such as
202 bridgmanite and ferropericlase, when studied in a DAC (e.g., Marquardt et al. 2009a,
203 2009b; Speziale et al. 2014; Zha et al. 2000). Although the constants C_{44} and C_{12} can
204 be well constrained by measuring the V_S velocities along the principal
205 crystallographic directions ($\langle 100 \rangle$, $\langle 110 \rangle$, and/or $\langle 111 \rangle$) using BLS measurements,
206 these constants from Brillouin measurements have been combined with equation of
207 state (EoS) results from X-ray diffraction in order to evaluate the full elastic tensor of
208 the crystal, especially for the longitudinal modulus (Marquardt et al. 2009a). The use
209 of the bulk modulus (K) derived from the EoS measurements, however, greatly
210 increases the uncertainties of the derived elastic constants, therefore reducing their
211 reliability.

212 Based on our derivations of the equations above, we note that the V_S of a given
213 crystal along certain directions also carry the necessary information to retrieve the
214 longitudinal modulus. When V_S velocities of the crystal are the only available dataset
215 in high pressure BLS measurements, the equation (13) can be re-written as

$$3\rho V_{S1}^2 = (C_{11} + 2C_{44})(n_1^2 + n_2^2 + n_3^2) + 2G^{1/2} \cos\left(\frac{1}{3} \arccos(H/G^3) + \frac{2}{3}\pi\right)$$
$$3\rho V_{S2}^2 = (C_{11} + 2C_{44})(n_1^2 + n_2^2 + n_3^2) + 2G^{1/2} \cos\left(\frac{1}{3} \arccos(H/G^3) + \frac{4}{3}\pi\right) \quad (14)$$

217 where V_{S1} and V_{S2} are two shear-wave velocities with polarizations orthogonal to each
218 other, and V_{S1} is defined as having a velocity smaller than V_{S2} . In order to provide the
219 necessary constraints on the longitudinal moduli C_{11} and off-diagonal moduli C_{12} by
220 only using V_S , one must find a crystal plane with the least tradeoffs between any two
221 C_{ij} . We describe the procedure for determining a suitable crystal plane and deriving
222 reliable elastic constants in the following steps:

223 **Step 1:** With known C_{ij} , density, and the Eulerian angles (θ , ϕ , χ) which relate the
224 laboratory coordinate system to the crystal coordinate system (Shimizu, 1995), one
225 can predict a set of velocities along various crystallographic orientations of a mineral

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226 by following the equation (13).

227 **Step 2:** Considering that there are always experimental errors involved in velocity
228 measurements in BLS experiments, an error of 2% for the velocity datasets along
229 various crystal orientations can be expected for the initial evaluations. Using this
230 dataset of velocities along various crystal orientations, one can obtain C_{ij} using the V_S
231 velocities alone and then compare the derived elastic constants with literature results
232 derived from using both V_P and V_S velocities.

233 **Step 3:** If one cannot obtain C_{ij} using the V_S velocities alone, one can rotate the plane
234 by 5 degrees, and then repeat Step 1 and Step 2 until a suitable crystal plane with
235 minimal tradeoff coefficients for the derived elastic constants is found.

236 We note that the ideal planes with the least tradeoff in the C_{ij} of periclase and spinel,
237 which both are cubic in structure and belong to the same crystal class $m\bar{3}m$, are
238 different, due to their different elastic anisotropies. It is worth noting that the main
239 purpose of the proposed method is to derive full elastic tensor by using only V_S
240 velocities of a single crystal because of the constant lack of experimental V_P velocities
241 at above 25 GPa in the BLS measurements. In this case, full elastic C_{ij} , density, and
242 the Eulerian angles of the given crystal at ambient conditions need to be known in
243 advance in order to find the ideal crystal plane for further high pressure studies. Based
244 on the elasticity theory (Auld 1973; Every 1980), our study can also be expanded in
245 the future such that the use of partial sets of velocity results (V_P , V_S , or V_P and V_S)
246 from random lattice planes can also help researchers to evaluate full elastic tensor;
247 however, these require further experimental tests and theoretical elaboration in the
248 future, and are not within the scope of this article at this point in time.

249 For crystals in symmetries lower than the cubic system, such as zoisite in the
250 orthorhombic system and stishovite in the tetragonal system, we found that the
251 general method of deriving the full elastic tensor from V_S velocities is to use crystal
252 planes that are orthogonal to each other but are at least 20 degrees off the principle
253 planes. We should note that the V_S in these systems can be described by an equation
254 similar to the equation (14), but the complexity of the equation would require the
255 knowledge of the full elastic tensor of the crystal in order to derive such a plane. For

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256 example, the C_{ij} of the orthorhombic zoisite (Mao et al. 2007) exhibit the least
257 tradeoffs in $(-0.24,-0.60,0.76)$, $(0.10,0.78,0.62)$, and $(-0.95,0.10,-0.28)$ planes. In this
258 case, any three crystal planes, which are orthogonal to each other and are at an angle
259 at least 20 degrees away from the principal planes (Auld 1973; Every 1980; Mao et al.
260 2007), can be used to constrain all C_{ij} using V_S dataset alone.

261

262 SAMPLES AND EXPERIMENTAL PROCEDURES

263 Single-crystal platelets of periclase (MgO) in (100), (110), and (111)
264 crystallographic orientations and spinel (MgAl_2O_4) in (100) and (110) orientations
265 were purchased from MTI Corporation. Based on the aforementioned numerical
266 analyses of the least tradeoffs between C_{ij} , we have pre-oriented a periclase platelet in
267 $(-1,0.5,0.2)$ crystallographic orientation for testing the optimal derivations of the C_{ij} .
268 The orientations of the crystals were determined using the single-crystal X-ray
269 diffractometer at the Texas Materials Institute of The University of Texas at Austin
270 (UT-Austin). All sample platelets were double-side polished with two parallel
271 surfaces using 3M diamond lapping films with a final finishing particle size of 0.5 μm .
272 A short symmetric DAC with a large optical opening was conveniently used to hold
273 the sample platelet with dimensions typically of 200-300 μm and thickness of
274 50-70 μm . No pressure was applied to the crystals in the DAC for experiments at
275 ambient conditions.

276 The BLS measurements were conducted at the Mineral Physics Laboratory of
277 UT-Austin (Lu et al. 2013; Yang et al. 2014). The Brillouin signals were excited by a
278 Coherent Verdi V2 laser with a 532 nm wavelength and collected by a photomultiplier
279 tube (PMT) at a low dark count rate (<2 counts/s) through a JRS six-pass tandem
280 Fabry-Pérot Interferometer. The laser beam was focused down to approximately 20
281 μm in diameter at the sample position. The scattering angle of $49.6(\pm 0.02)^\circ$ for the
282 system was calibrated using previously reported elastic constants of silicate glass
283 (Polian et al. 2002), distilled water (Ostwald et al. 1977), and single-crystal MgO
284 (Sinogeikin and Bass 2000).

285 The scattered laser light in the BLS measurements consists of an

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286 elastically-scattered component with frequency, ω , as well as inelastically scattered
287 components that have undergone a frequency shift, $\Delta\omega$, due to the interaction of the
288 incident laser with thermally generated phonons in the sample crystal. In a symmetric
289 scattering geometry, the acoustic velocity, V is calculated from the relation:

$$290 \quad V = \Delta\omega\lambda/2 \sin(\theta/2) \quad (15)$$

291 where λ is the incident wavelength and θ is the scattering angle. Our BLS spectra
292 were collected from a given direction of a crystal plane and were repeated by rotating
293 the crystal platelet 5° about the axis perpendicular to the plane at every step in a total
294 of 37 directions. For each spectrum, the average collection time was approximately 30
295 minutes. The measured Brillouin spectra are of excellent quality with a high
296 signal-to-noise ratio (Fig. 1). All the measured spectra display symmetric spectral
297 pairs corresponding to the V_P and V_S modes of the sample (Fig. 1). Two V_S modes and
298 one V_P mode were observed in most of the directions, although some Brillouin spectra
299 only showed V_P and one V_S mode (Figs. 1-5). Further information about the BLS
300 technique and data analyses can be found elsewhere in previous literatures (e.g., Mao
301 et al. 2012; Lu et al. 2013; Yang et al. 2014).

302

303 **Determination of Full Elastic Tensor**

304 **1) Cubic Periclase and Spinel Crystals**

305 Brillouin measurements were obtained from six platelets [periclase in (-1,0.5,0.2),
306 (100), (110), and (111) crystallographic orientations and spinel in (100) and (110)
307 crystallographic orientations] over an azimuthal range of 180° with an angular
308 increment of 5° per step (Figs. 1-4) (see Tables 1-2 and STables 1-4). The measured
309 acoustic velocities show a systematic dependence as a function of the orientation
310 indicating that the velocities of periclase and spinel are directionally anisotropic (Figs.
311 2-4). The measured V_P and V_S velocities were modeled to derive the elastic constants
312 using the Christoffel's equations (equation 1) via the best fits to both V_P and V_S
313 velocities (Fig. 1; Fig. 4 a and b) as well as V_S velocities alone (Fig. 2; Fig. 4 c and d;
314 Fig. 5).

315 In periclase and spinel, both cubic, C_{ij} exhibit the least tradeoff coefficients in the

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316 (-1,0.5,0.2) plane for periclase and are very close to the (110) plane for spinel (Table
317 3). The tradeoff coefficients corresponding to a particular i and k represent the change
318 in an elastic constant C_i for a unit change in another elastic constant C_k . Such tradeoff
319 coefficients are formally captured by the variance-covariance matrix for the C_{ij} and
320 depend on the observations of V_S speed as functions of propagation directions. The
321 approach we identified here is to find optical crystallographic planes of a given crystal
322 with the least tradeoff in the C_{ij} at ambient conditions. In this method, the C_{ij} at
323 ambient conditions need to be known in advance in order to find the ideal crystal
324 plane, and then the full elastic moduli of the ideal crystal plane can be derived by
325 using the measured V_S velocities alone at high pressure. The derivation of the full
326 elastic tensor can be extended beyond this method by conducting iterations in the
327 derived elastic constants using V_S or V_P datasets alone via minimization of the misfits.
328 The derived values of tradeoff coefficients are very small and thus indicate that all of
329 the elastic constants are well resolved within the context of the linear relation. The
330 above inversion scheme indicates that the interdependence of the elastic constants of
331 periclase in the (-1,0.5,0.2) plane and spinel in the (110) plane are applicable and thus
332 suggest that the V_S dataset can be used to uniquely and reliably constrain all of the
333 elastic constants.

334 We have used a nonlinear inversion procedure to solve the Christoffel's equations
335 for the three independent single-crystal elastic constants (C_{ij}) of the cubic periclase
336 and spinel; this non-linear inversion procedure uses the Gauss–Newton algorithm with
337 Levenberg–Marquardt modifications for global convergence of the solutions, and has
338 been widely adopted for finding solutions to the Christoffel's equations (Weidner and
339 Carleton, 1977; Sanchez-Valle et al. 2005). The elastic constants of the periclase and
340 spinel obtained in this work are listed in Tables 4-5, together with previous results for
341 comparison. Within experimental uncertainties, our results derived from using both V_P
342 and V_S profiles for periclase in (-1,0.5,0.2), (100), (110) and (111) orientations and for
343 spinel in (110) and (100) orientations are in excellent agreement with those obtained
344 from previous measurements (Tables 4 and 5) (Jackson and Niesler 1982; Yoneda
345 1990; Askarpour et al. 1993; Sinogeikin and Bass 1999; Zha et al. 2000; Suzuki et al.

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2000). Furthermore, the derived elastic constants of periclase using the V_S dataset alone for each (100), (110) or (111) crystallographic orientation deviate from the results derived from using both V_P and V_S datasets as well as literature Brillouin results (Jackson and Niesler 1982; Yoneda 1990; Sinogeikin and Bass 2000; Zha et al. 2000) (Figs. 6-7 and Table 4), especially for C_{11} and C_{12} . For example, the difference can reach to approximately 70% for C_{12} and 25% for C_{11} between (-1,0.5,0.2) and (111) orientations (Fig. 7a). In contrast, the C_{11} , C_{12} and C_{44} constants for periclase derived from using the V_S profiles alone in the (-1,0.5,0.2) orientation are 296.7 (± 1.3), 99.6 (± 1.5), 155.0 (± 0.2) GPa, respectively, and are consistent with the literature values (Jackson and Niesler 1982; Yoneda 1990; Sinogeikin and Bass 2000; Zha et al. 2000) (Fig. 6a; Table 4). In addition, the estimated uncertainties of the derived value for the longitudinal moduli C_{11} , shear moduli C_{44} , and off-diagonal moduli C_{12} in the pre-selected (-1,0.5,0.2) crystallographic orientation is better than 1.6% (1σ level), while the estimated uncertainties are approximately 42% in the (100), (110) and (111) crystallographic orientation. Likewise, the C_{11} , C_{12} , and C_{44} of the cubic spinel derived from using the V_S profiles alone in the (100) crystallographic orientation are noticeably different from the values derived from using both V_P and V_S datasets and from previous literature values (Table 5) with the maximum difference reaching approximately 30% for C_{12} and 15% for C_{11} between (110) and (100) orientations (Fig. 7b). The derived elastic constants of the cubic spinel are: $C_{11} = 291.3$ (± 1.2), $C_{12} = 152.6$ (± 1.2), and $C_{44} = 155.7$ (± 0.4) GPa in the pre-selected (110) orientation determined from using the V_S dataset alone, which are consistent with the literature values (Yoneda 1990; Askarpour et al. 1993; Suzuki et al. 2000) (Fig. 6b; Table 5). We should note that the estimated uncertainties of the derived value for these constants is better than 0.8% (1σ level) in the (110) platelet, while the estimated uncertainties is better than 5% in the (100) platelet.

Based on the aforementioned experimental analyses, the derived elastic constants from the pre-selected (-1,0.5,0.2) orientation for periclase and (110) orientation for spinel validate the theoretical derivations for obtaining the full single-crystal elastic constants using the V_S dataset alone. Since our proposed methodology for deriving

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376 full elastic tensor is based on modeling V_S as a function of the azimuthal angle for a
377 given crystal platelet, we have also tested the sensitivity of the method on the
378 measuring intervals of the V_S values. Comparisons between the values derived from 5°
379 and 10° measuring intervals show that the elastic constants derived from the 10°
380 interval measurements slightly deviate from the 5° interval counterparts and previous
381 literature results by 8% and 7%, respectively (Fig. 8; Table 6). The deviation is most
382 significant for C_{11} at 8% and C_{12} at 7% (Fig. 8; Table 6). We thus conclude that having
383 at least a 5° measuring interval for the V_S dataset is needed to reliably derive the
384 elastic constants of the cubic crystal.

385

386 **2) Orthorhombic Zoisite and Tetragonal Stishovite**

387 The application of the method can be extended beyond simple cubic crystals as
388 shown above. Here we have used previously reported Brillouin results for
389 orthorhombic zoisite (space group: $Pnma$) and tetragonal stishovite (space group:
390 $P4_2/mnm$) as two representative examples to extend the applications of our
391 methodology. The elastic tensor of orthorhombic crystals has nine independent
392 coefficients that can be used to completely describe its elastic properties (Zha et al.
393 1996; Sanchez-Valle et al. 2005; Jackson et al. 2007; Mao et al. 2007; Mao et al.
394 2010). To determine all elastic constants of an orthorhombic crystal such as zoisite,
395 three mutually perpendicular platelets are desirable for sound velocity measurements
396 (Sanchez-Valle et al. 2005; Jackson et al. 2007; Mao et al. 2007). Using the V_S dataset
397 alone, the velocity data for all planes in BLS measurements were simultaneously
398 fitted to Christoffel's equation (Every 1980). The procedure to retrieve the elastic
399 constants has been discussed elsewhere in details (Mao et al. 2007). Analyses of the
400 measured and calculated best-fit velocities of zoisite at ambient conditions indicate
401 excellent agreements among the velocities (Fig. 5).

402 Three platelets orthogonal to each other but otherwise randomly oriented were used
403 to obtain the full elastic tensor of orthorhombic zoisite, and we also found that the C_{ij}
404 of zoisite exhibit very small tradeoffs in $(-0.24,-0.60,0.76)$, $(0.10,0.78,0.62)$ and
405 $(-0.95,0.10,-0.28)$ planes. Comparisons between the elastic constants of zoisite using

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406 both the V_P and V_S datasets and the V_S dataset alone in the pre-selected
407 crystallographic planes show that most of the elastic constants for zoisite in the
408 pre-selected crystallographic planes are reasonably consistent within experimental
409 uncertainties, except for the off-diagonal modulus C_{23} (Table 7). The estimated
410 uncertainties of the derived values is better than 2.6% (1σ level) for all other elastic
411 constants, but the estimated uncertainties for the C_{23} is approximately 11.4%. The
412 relatively large difference of 17% for C_{23} is mainly due to the much smaller value of
413 the C_{23} compared to other elastic constants; the derived value of the C_{23} is 27.5 GPa
414 which is among the smallest of all elastic constants of zoisite, and 11.4% estimated
415 uncertainties translates into a value of 22.8 GPa for its elastic constant. We should
416 note that this difference in the C_{23} constant is still acceptable when compared with
417 other elastic constants of zoisite. In fact, the value of C_{23} determined using the V_S
418 dataset alone is completely comparable to the value determined by using both V_P and
419 V_S datasets within experimental uncertainties, justifying the use of the V_S dataset
420 alone to retrieve full elastic tensor of zoisite.

421 One can also extend the application of this new approach to other crystal systems.
422 For example, stishovite crystallizes in the tetragonal system (space group: $P4_2/mnm$)
423 with six independent non-zero elastic constants: C_{11} , C_{12} , C_{13} , C_{33} , C_{44} , and C_{66}
424 (Weidner et al. 1982; Jiang et al. 2009). The elastic constants of stishovite were
425 measured previously at ambient conditions using Brillouin spectroscopy (Weidner et
426 al. 1982; Brazhkin et al. 2005). Recently, Jiang et al. (2009) also conducted acoustic
427 velocities measurements on three crystal platelets of single-crystal stishovite in a
428 forward symmetric scattering geometry using Brillouin spectroscopy; these
429 experimental results permitted them to retrieve full elastic tensor of the sample at
430 ambient conditions (Table 8). Here we have used the reported V_S dataset for the
431 crystal planes of stishovite by Jiang et al. (2009) to derive full elastic tensor using the
432 Christoffel's equation (Every 1980) (Table 8). Comparisons between the elastic
433 constants of stishovite using both V_P and V_S datasets and the V_S dataset alone show
434 that all of the elastic constants for stishovite are reasonably consistent within
435 experimental uncertainties (Table 8). The estimated uncertainties of the derived values

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436 are better than 1.9% (1σ level) for all of the elastic constants. The relatively large
437 difference of 4.0% for C_{12} and 8.3% for C_{13} mainly comes from the much smaller
438 value of each constant, respectively, as compared to other elastic constants (the value
439 of C_{13} is among the smallest constants for all elastic constants of stishovite). It should
440 be noted that the difference in the C_{12} and C_{13} constant is still acceptable when
441 compared with other elastic constants of stishovite and that the value of C_{12} and C_{13}
442 determined by V_S profile alone is also entirely comparable to the value determined by
443 both V_P and V_S profiles within experimental uncertainties. These analyses highlight
444 the application of deriving full elastic tensor from using V_S datasets alone for other
445 tetragonal crystals.

446

447 **DISCUSSION**

448 There have previously proposed methods to solve the problem in which the
449 Brillouin scattering signal corresponding to compressional velocities of samples was
450 masked at pressures above 25 GPa (Zha et al. 2000; Marquardt et al. 2009a, 2009b).
451 Marquardt et al. (2009a, 2009b) reported that in the case of cubic structure materials,
452 *in situ* X-ray diffraction data can be used to supplement the Brillouin scattering data at
453 pressures above 25 GPa where the BLS signals corresponding to V_P was masked.
454 Using the adiabatic bulk modulus, $K_S=(C_{11}+2C_{12})/3$, together with the V_S results from
455 Brillouin scattering, they had extracted the C_{11} , C_{12} , and C_{44} of a cubic ferropericlasite
456 up to 81 GPa (Marquardt et al. 2009a). However, the method used by Marquardt et al.
457 (2009a, 2009b) is more suitable for cubic structure materials and is not as useful for
458 crystals with lower symmetries. In addition, Zha et al. (1998, 2000) also introduced a
459 method that utilized a spatial filter and cylindrical lenses to suppress the diamond
460 signal (Zha et al. 1998, 2000) such that the V_P signal at relatively higher pressures of
461 approximately above 25 GPa can be detected. It still remains difficult to apply this
462 method to materials with very fast V_P as well as to other lower symmetry systems.
463 Thus far, deriving reliable C_{ij} at high pressures remains challenging. Our proposed
464 approach overcomes these difficulties and has the advantage of being suitable for
465 deriving reliable elastic constants for all crystal systems at high pressures.

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466 To test the usefulness of our method at high pressures, we have conducted high
467 pressure BLS measurements on the pre-selected (-1,0.5,0.2) platelet of cubic periclase
468 as a function of the azimuthal angles at 5.8 (4) GPa and 11.3 (5) GPa, respectively.
469 The sample platelet was loaded into a DAC containing Ne gas as pressure transmitting
470 medium and ruby spheres as the pressure calibrant (Mao et al. 1986). At each pressure,
471 Brillouin spectra were collected in 37 directions over an angular range of 180° with a
472 5° step. Since deriving full elastic tensor requires knowledge of the sample's density,
473 we have used the equation of state of the MgO and have followed a nonlinear
474 inversion procedure proposed previously in Speziale and Duffy (2002) to determine
475 the density at each given pressure.

476 Individual C_{ij} values for the cubic periclase are obtained from fitting the measured
477 velocities using Christoffel's equation (Table 9). The derived elastic constants of the
478 cubic periclase using the V_S dataset alone are: $C_{11}=352.8 (\pm 3.6)$, $C_{12}=107.7 (\pm 3.2)$,
479 $C_{44}=165.1 (\pm 2.8)$ GPa at 5.8 (4) GPa and $C_{11}=399.4 (\pm 4.4)$, $C_{12}=113.3 (\pm 3.7)$,
480 $C_{44}=171.1 (\pm 3.8)$ GPa at 11.3 (5) GPa, which are very consistent with the results from
481 using both V_P and V_S profiles ($C_{11}=348.0 (\pm 2.8)$, $C_{12}=103.1 (\pm 2.5)$, $C_{44}=162.6 (\pm 2.4)$
482 GPa at 5.8 (4) GPa and $C_{11}=394.8 (\pm 3.8)$, $C_{12}=109.1 (\pm 3.1)$, $C_{44}=168.5 (\pm 2.4)$ GPa at
483 11.3 (5) GPa) within experimental uncertainties (Fig. 9). The derived individual C_{ij}
484 values for the cubic periclase at 5.8 (4) GPa and 11.3 (5) GPa in this study are also
485 very consistent with literature values at similar pressures of 5.5 (1) GPa and 11.00 (2)
486 GPa, respectively (Sinogeikin and Bass 2000) (Fig. 9; Table 9). As shown in Figure 9
487 and Table 9, the derived elastic constants of the cubic periclase from the pre-selected
488 (-1,0.5,0.2) orientation using the V_S dataset alone are in agreement well with previous
489 Brillouin scattering measurements within their uncertainties, supporting our proposed
490 approach of using the pre-selected crystallographic orientation to derive full elastic
491 tensor at high pressures. We should also note that our proposed method can also be
492 combined with the partially available V_P dataset at high pressure to help constrain the
493 elastic tensors. The derivation of the elastic constants from the V_S dataset alone using
494 multiple platelets with different orientations for a given crystal can also help resolve
495 the elastic constants with less uncertainty.

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496

497 **IMPLICATIONS**

498 Future applications of our proposed approach can have significant implications
499 especially on understanding seismic structures, geochemistry, and mineralogy of the
500 Earth's lower mantle with pressures ranging from 23 GPa to 136 GPa and
501 temperatures from approximately 1800 K to 3500 K, at which conditions the
502 traditional BLS approach has had some limitations (e.g., Zha et al. 2000; Marquardt et
503 al. 2009a, 2009b; Speziale et al. 2014). Recently, Brillouin scattering spectroscopy
504 has been coupled with a laser-heated DAC to measure V_S of the Earth's lower mantle
505 minerals up to lowermost-mantle pressures (Murakami et al., 2012). After comparison
506 with seismic profiles of the lower mantle, these V_S results have suggested a
507 perovskitic lower-mantle composition, although V_P profiles of the candidate minerals
508 were not available for the comparison (Murakami et al., 2012). Thus far, the
509 determination of the chemical composition of the Earth's lower mantle remains
510 challenging due to the limitation of the elasticity data mentioned above (e.g.,
511 Ringwood 1975; Sun 1982; Anderson 1989; Ito and Takahashi 1989; Allègre et al.
512 1995; Fiquet et al. 2000; Murakami et al. 2004, 2012; Lin et al. 2013). Accurate
513 knowledge of sound velocities and elastic properties in the Earth's lower mantle
514 minerals, namely ferropericlase and bridgmanite, under relevant high P-T conditions
515 can thus provide essential constraints on the mineralogy and chemical compositions of
516 the region (e.g., Oganov et al. 2001; Sinogeikin et al. 2004; Jackson et al. 2005, 2006;
517 Crowhurst et al. 2008; Reichmann et al. 2008; Marquardt et al. 2009a, 2009b; Chen et
518 al. 2012; Murakami et al. 2012).

519 Particularly, the elasticity of single-crystal ferropericlase ($\text{Mg}_{0.9}\text{Fe}_{0.1}\text{O}$) has been
520 studied recently using Brillouin scattering and X-ray diffraction up to 81 GPa in the
521 DAC (Marquardt et al. 2009a). It has been shown that the spin crossover of ferrous
522 iron is accompanied by the V_P and C_{11} softening, but the BLS measurements with only
523 V_S values available do not provide direct information in deciphering this softening
524 phenomena because of the overlap of the Brillouin signal with the V_S peaks of the
525 diamond anvils in the DAC (Marquardt et al. 2009b). Our proposed approach can

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526 overcome the spectral limitation of the Brillouin scattering and can be applied to
527 investigate the effects of the spin transition on the elasticity of lower-mantle
528 ferropericlase at relevant P-T conditions (Wentzcovitch et al. 2009; Wu et al. 2013).
529 Furthermore, our approach can also be used to investigate the elasticity of bridgmanite
530 (Al-bearing silicate perovskite $\text{Al}-(\text{Mg,Fe})\text{SiO}_3$), the most abundant mineral phase in
531 Earth's lower mantle. It is somewhat surprising to note that the single-crystal elasticity
532 of bridgmanite has only been determined at ambient conditions using BLS
533 (Yeganeh-Haeri et al. 1989; Yeganeh-Haeri 1994; Jackson et al. 2004; Sinogeikin et al.
534 2004), while high pressure single-crystal elasticity of bridgmanite has not yet been
535 reported (Speziale et al. 2014). Thus far, the V_P and V_S of polycrystalline Fe-free
536 bridgmanite has been reported at pressures up to 25 GPa (Jackson et al. 2005;
537 Murakami et al. 2007; Chantel et al., 2012), and its V_S has been investigated over the
538 entire P-T range of the lower mantle (Murakami et al. 2007). The lack of the
539 single-crystal elastic constants for bridgmanite at relevant P-T conditions has limited
540 our ability to accurately model the mineralogy and seismic anisotropy of the Earth's
541 lower mantle. In addition, the potential effect of the electronic spin transition of iron
542 on the elastic properties of bridgmanite is still unknown (Lin et al. 2013). Application
543 of our new approach in studying the elasticity of single-crystal bridgmanite at high
544 P-T will quantify its elastic anisotropy. Our approach can also be used to calculate its
545 aggregate V_P and V_S , and to assess the effect of the electronic spin transition of iron on
546 the elastic properties of bridgmanite. Therefore, this approach can significantly
547 enhance our understanding of the Earth's lower mantle seismology and composition
548 (Murakami et al. 2007; Speziale et al. 2014).

549

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FAN ET AL.: DETERMINATION OF THE FULL ELASTIC TENSOR

586 **REFERENCES CITED**

- 587 Allègre, C.J., Poirier, J.P., Humler, E., and Hofmann, A.W. (1995) The chemical
588 compositions of the earth. *Earth and Planetary Science Letters*, 134, 515-526.
- 589 Anderson, D.L. (1989) *Theory of the Earth*. Blackwell Scientific Publications,
590 Boston.
- 591 Angel, R.J., Jackson, J.M., Reichmann, H.J., and Speziale, S. (2009) Elasticity
592 measurements on minerals: a review. *European Journal of Mineralogy*, 21,
593 525-550.
- 594 Askarpour, V., Manghnani, M.H., Fassbender, S., and Yoneda, A. (1993) Elasticity of
595 single-crystal $MgAl_2O_4$ spinel up to 1273 K by Brillouin spectroscopy. *Physics
596 and Chemistry of Minerals*, 19, 511-519.
- 597 Bass, J.D. (2007) *Theory and Practice – Techniques for Measuring High P/T Elasticity*.
598 In: *Treatise on Geophysics-Mineral Physics*. Vol 2. Price, G.D. (ed) Elsevier,
599 Amsterdam, p 269-291.
- 600 Bass, J.D. (2008) Recent progress in studies of the elastic properties of earth materials.
601 *Physics of the Earth and Planetary Interiors*, 170, 207-209.
- 602 Bass, J.D., Sinogeikin, S.V., and Li, B.S. (2008) Elastic properties of minerals: A key
603 for understanding the composition and temperature of Earth's Interior. *Elements*,
604 4, 165-170.
- 605 Birch, F. (1952) Elasticity and constitution of the Earth's interior. *Journal of
606 Geophysical Research*, 57, 227-286.
- 607 Brazhkin, V.V., McNeil, L.E., Grimsditch, M., Bendeliani, N.A., Dyuzheva, T.I., and
608 Lityagina, L.M. (2005) Elastic constants of stishovite up to its amorphization
609 temperature. *Journal of Physics: Condensed Matter*, 17, 1869-1875.
- 610 Carpenter, M.A. (2006) Elastic properties of minerals and the influence of phase
611 transitions. *American Mineralogist*, 91, 229-246.
- 612 Chantel, J., Frost, D.J., McCammon, C.A., Jing, Z.C., and Wang, Y.B. (2012) Acoustic
613 velocities of pure and iron-bearing magnesium silicate perovskite measured to 25
614 GPa and 1200 K. *Geophysical Research Letters*, 39, L19307.
- 615 Chen, B., Jackson, J.M., Sturhahn, W., Zhang, D.Z., Zhao, J.Y., Wicks, J.K., and

FAN ET AL.: DETERMINATION OF THE FULL ELASTIC TENSOR

- 616 Murphy, C.A. (2012) Spin crossover equation of state and sound velocities of
617 (Mg_{0.65}Fe_{0.35})O ferropericlase to 140 GPa. *Journal of Geophysical Research*, 117,
618 B08208.
- 619 Chen, P.F., Chiao, L.Y., Huang, P.H., Yang, Y.J., and Liu, L.G. (2006) Elasticity of
620 magnesite and dolomite from a genetic algorithm for inverting Brillouin
621 spectroscopy measurements. *Physics of the Earth and Planetary Interiors*, 155,
622 73-86.
- 623 Crowhurst, J.C., Brown, J.M., Goncharov, A.F., and Jacobsen, S.D. (2008) Elasticity
624 of (Mg,Fe)O through the spin transition of iron in the lower mantle. *Science*, 319,
625 451-453.
- 626 Duffy, T.S., and Anderson, D.L. (1989) Seismic velocities in mantle minerals and the
627 mineralogy of the upper mantle. *Journal of Geophysical Research*, 94,
628 1895-1912.
- 629 Duffy, T.S., Zha, C.S., Downs, R.T., Mao, H.K., and Hemley, R.J. (1995) Elasticity of
630 forsterite to 16 GPa and the composition of the upper-mantle. *Nature*, 378,
631 170-173.
- 632 Every, A.G. (1980) General closed-form expressing for acoustic waves in elastically
633 anisotropic solids. *Physical Review B*, 22, 1746-1760.
- 634 Fiquet, G., Dewaele, A., Andrault, D., Kunz, M., and Le Bihan, M. (2000)
635 Thermoelastic properties and crystal structure of MgSiO₃ perovskite at lower
636 mantle pressure and temperature conditions. *Geophysical Research Letters*, 27,
637 21-24.
- 638 Inoue, T., Weidner, D.J., Northrup, P.A., and Parise, J.B. (1998) Elastic properties of
639 hydrous Ringwoodite (gamma-phase) in Mg₂SiO₄. *Earth and Planetary Science*
640 *Letters*, 160, 107-113.
- 641 Ito, E., and Takahashi, E. (1989) Postspinel transformations in the system
642 Mg₂SiO₄-Fe₂SiO₄ and some geophysical implications. *Journal of Geophysical*
643 *Research*, 94, 10637-10646.
- 644 Jackson, I., and Niesler, H. (1982) The elasticity of periclase to 3 GPa and some
645 geophysical implications. In S. Akimoto and M.H. Manghnani, Eds., *High*

FAN ET AL.: DETERMINATION OF THE FULL ELASTIC TENSOR

- 646 Pressure Research in Geophysics, p. 93–113. Center for Academic Publishing,
647 Tokyo, Japan.
- 648 Jackson, J.M., Sinogeikin, S.V., and Bass, J.D. (2007) Sound velocities and
649 single-crystal elasticity of orthoenstatite to 1073K at ambient pressure. *Physics*
650 *of the Earth and Planetary Interiors*, 161, 1-12.
- 651 Jackson, J.M., Sinogeikin, S.V., Jacobsen, S.D., Reichmann, H.J., Mackwell, S.J., and
652 Bass, J.D. (2006) Single-crystal elasticity and sound velocities of $(\text{Mg}_{0.94}\text{Fe}_{0.06})\text{O}$
653 ferropericlasite to 20 GPa. *Journal of Geophysical Research*, 111, B09203.
- 654 Jackson, J.M., Zhang, J.Z., and Bass, J.D. (2004) Sound velocities and elasticity of
655 aluminous MgSiO_3 perovskite: implications for aluminum heterogeneity in
656 Earth's lower mantle. *Geophysical Research Letters*, 31, L10614.
- 657 Jackson, J.M., Zhang, J.Z., Shu, J.F., Sinogeikin, S.V., and Bass, J.D. (2005)
658 High-pressure sound velocities and elasticity of aluminous MgSiO_3 perovskite to
659 45 GPa: Implications for lateral heterogeneity in Earth's lower mantle.
660 *Geophysical Research Letters*, 32, L21305.
- 661 Jacobsen, S.D., Reichmann, H.J., Spetzler, H.A., Mackwell, S.J., Smyth, J.R., Angel,
662 R.J., and McCammon, C.A. (2002) Structure and elasticity of single-crystal
663 $(\text{Mg,Fe})\text{O}$ and a new method of generating shear waves for gigahertz ultrasonic
664 interferometry. *Journal of Geophysical Research*, 107, B2.
- 665 Jiang, F.M., Gwanmesia, G.D., Dyuzheva, T.I., and Duffy, T.S. (2009) Elasticity of
666 stishovite and acoustic mode softening under high pressure by Brillouin
667 scattering. *Physics of the Earth and Planetary Interiors*, 172, 235-240.
- 668 Li, B.S., Kung, J., and Liebermann, R.C. (2004) Modern techniques in measuring
669 elasticity of Earth materials at high pressure and high temperature using
670 ultrasonic interferometry in conjunction with synchrotron X-radiation in
671 multi-anvil apparatus. *Physics of the Earth and Planetary Interiors*, 143-144,
672 559-574.
- 673 Li, B.S., and Liebermann, R.C. (2007) Indoor seismology by probing the Earth's
674 interior by using sound velocity measurements at high pressures and
675 temperatures. *Proceedings of the National Academy of Sciences of the United*

FAN ET AL.: DETERMINATION OF THE FULL ELASTIC TENSOR

- 676 States of America, 104, 9145-9150.
- 677 Li, L., Weidner, D.J., Brodholt, J., Alfe, D., and Price, D.G. (2006) Elasticity of
678 Mg₂SiO₄ ringwoodite at mantle conditions. *Physics of the Earth and Planetary*
679 *Interiors*, 157, 181–187.
- 680 Lin, J.F., Speziale, S., Mao, Z., and Marquardt, H. (2013) Effects of the electronic
681 spin transitions of iron in lower-mantle minerals: Implications for deep-mantle
682 geophysics and geochemistry. *Reviews of Geophysics*, 51, 244-275.
- 683 Lu, C., Mao, Z., Lin, J.F., Zhuravlev, K.K., Tkachev, S.N., and Prakapenka, V.B.
684 (2013) Elasticity of single-crystal iron-bearing pyrope up to 20 GPa and 750 K.
685 *Earth and Planetary Science Letters*, 361, 134-142.
- 686 Mainprice, D. (2007) Seismic Anisotropy of the Deep Earth from a Mineral and Rock
687 Physics Perspective. *Treatise on Geophysics*, 2, 437-491.
- 688 Mao, H.K., Xu, J., and Bell, P.M. (1986) Calibration of the ruby pressure gauge to
689 800 kbar under quasi-hydrostatic conditions. *Journal of Geophysical Research*,
690 91, 4673-4676.
- 691 Mao, Z., Jiang, F.M., and Duffy, T.S. (2007) Single-crystal elasticity of zoisite
692 Ca₂Al₃Si₃O₁₂(OH) by Brillouin scattering. *American Mineralogist*, 92, 570-576.
- 693 Mao, Z., Jacobsen, S.D., Jiang, F., Smyth, J.R., Holl, C.M., and Duffy, T.S. (2008)
694 Elasticity of hydrous wadsleyite to 12 GPa: Implications for Earth's transition
695 zone. *Geophysical Research Letters*, 35, L21305.
- 696 Mao, Z., Jacobsen, S.D., Jiang, F., Smyth, J.R., Holl, C.M., Frost, D.J., and Duffy, T.S.
697 (2010) Velocity crossover between hydrous and anhydrous forsterite at high
698 pressures. *Earth and Planetary Science Letters*, 293, 250-258.
- 699 Mao, Z., Lin, J.F., Jacobsen, S.D., Duffy, T.S., Chang, Y.Y., Smyth, J.R., Frost, D.J.,
700 Hauri, E.H., and Prakapenka, V.B. (2012) Sound velocities of hydrous
701 ringwoodite to 16GPa and 673K. *Earth and Planetary Science Letters*, 331,
702 112-119.
- 703 Marquardt, H., Speziale, S., Reichmann, H.J., Frost, D.J., and Schilling, F.R. (2009a)
704 Single-crystal elasticity of (Mg_{0.9}Fe_{0.1})O to 81 GPa. *Earth and Planetary Science*
705 *Letters*, 287, 345-352.

FAN ET AL.: DETERMINATION OF THE FULL ELASTIC TENSOR

- 706 Marquardt, H., Speziale, S., Reichmann, H.J., Frost, D.J., Schilling, F.R., and Garnero,
707 E.J. (2009b) Elastic shear anisotropy of ferropericlase in Earth's lower mantle.
708 *Science*, 324, 224-226.
- 709 Murakami, M., Asahara, Y., Ohishi, Y., Hirao, N., and Hirose, K. (2009a)
710 Development of in situ Brillouin spectroscopy at high pressure and high
711 temperature with synchrotron radiation and infrared laser heating system:
712 Application to the Earth's deep interior. *Physics of the Earth and Planetary*
713 *Interiors*, 174, 282-291.
- 714 Murakami, M., Hirose, K., Kawamura, K., Sata, N., and Ohishi, Y. (2004)
715 Post-perovskite phase transition in MgSiO₃. *Science*, 304, 855-858.
- 716 Murakami, M., Ohishi, Y., Hirao, N., and Hirose, K. (2009b) Elasticity of MgO to 130
717 GPa: implications for lower mantle mineralogy. *Earth and Planetary Science*
718 *Letters*, 277, 123-129.
- 719 Murakami, M., Ohishi, Y., Hirao, N., and Hirose, K. (2012) A perovskite lower mantle
720 inferred from high-pressure, high-temperature sound velocity data. *Nature*, 485,
721 90-94.
- 722 Murakami, M., Sinogeikin, S.V., Hellwig, H., Bass, J.D., and Li, J. (2007) Sound
723 velocity of MgSiO₃ perovskite to Mbar pressure. *Earth and Planetary Science*
724 *Letters*, 256, 47-54.
- 725 Oganov, A.R., Brodholt, J.P., and Price, G.D. (2001) The elastic constants of MgSiO₃
726 perovskite at pressures and temperatures of the Earth's mantle. *Nature*, 411,
727 934-937.
- 728 Ostwald, J., Pazold, W., and Weis, O. (1977) High-resolution Brillouin spectroscopy
729 of water. *Applied Physics*, 13, 351-356.
- 730 Polian, A. (2003) Brillouin scattering at high pressure: an overview. *Journal of Raman*
731 *Spectroscopy*, 34, 633-637.
- 732 Polian, A., Vo-Thanh, Dung, and Richet, P. (2002) Elastic properties of α -SiO₂ up to
733 2300 K from Brillouin scattering measurements. *Europhysics Letters*, 57,
734 375-381.
- 735 Press, W.H. (1988) *Numerical recipes in C : the art of scientific computing*. xxii, 735

FAN ET AL.: DETERMINATION OF THE FULL ELASTIC TENSOR

- 736 p. p. Cambridge University Press, Cambridge Cambridgeshire, New York.
- 737 Reichmann, H.J., Sinogeikin, S.V., and Bass, J.D. (2008) Single-crystal elastic
738 properties of (Mg_{0.987},Fe_{0.013})O to 9 GPa. *American Mineralogist*, 93, 1306-1311.
- 739 Ringwood, A.E. (1975) *Composition and Petrology of the Earth's Mantle*. McGraw
740 Hill, New York.
- 741 Sanchez-Valle, C., Sinogeikin, S.V., Lethbridge, Z.A.D., Walton, R.I., Smith, C.W.,
742 Evans, K.E., and Bass, J.D. (2005) Brillouin scattering study on the
743 single-crystal elastic properties of natrolite and analcime zeolites. *Journal of*
744 *Applied Physics*, 98, 053508.
- 745 Shimizu, H. (1995) High-pressure Brillouin scattering of molecular single-crystals
746 grown in a diamond-anvil cell. In: Senoo, M., Suito, K., Kobayashi, T., Kubota,
747 H. (Eds.), *High Pressure Research on Solids*. Elsevier, Netherlands, pp. 1-17.
- 748 Shimizu, H., and Sasaki, S. (1992) High-Pressure Brillouin Studies and Elastic
749 Properties of Single-Crystal H₂S Grown in a Diamond Cell. *Science*, 257,
750 514-516.
- 751 Sinogeikin, S.V., and Bass, J.D. (2000) Single-crystal elasticity of pyrope and MgO to
752 20 GPa by Brillouin scattering in the diamond cell. *Physics of the Earth and*
753 *Planetary Interiors*, 120, 43-62.
- 754 Sinogeikin, S.V., Bass, J.D., and Katsura, T. (2003) Single-crystal elasticity of
755 ringwoodite to high pressures and high temperatures: implications for 520 km
756 seismic discontinuity. *Physics of the Earth and Planetary Interiors*, 136, 41-66.
- 757 Sinogeikin, S.V., Bass, J.D., Prakapenka, V.B., Lakshtanov, D., Shen, G.Y.,
758 Sanchez-Valle, C., and Rivers, M. (2006) Brillouin spectrometer interfaced with
759 synchrotron radiation for simultaneous x-ray density and acoustic velocity
760 measurements. *Review of scientific instruments*, 77, 103905-103911.
- 761 Sinogeikin, S.V., Katsura, T., and Bass, J.D. (1998) Sound velocities and elastic
762 properties of Fe-bearing wadsleyite and ringwoodite. *Journal of Geophysical*
763 *Research*, 103, 20819-20825.
- 764 Sinogeikin, S.V., Lakshtanov, D.L., Nicholas, J.D., Jackson, J.M., and Bass, J.D.
765 (2005) High temperature elasticity measurements on oxides by Brillouin

FAN ET AL.: DETERMINATION OF THE FULL ELASTIC TENSOR

- 766 spectroscopy with resistive and IR laser heating. *Journal of the European*
767 *Ceramic Society*, 25, 1313-1324.
- 768 Sinogeikin, S.V., Zhang, J.Z., and Bass, J.D. (2004) Elasticity of single crystal and
769 polycrystalline MgSiO_3 perovskite by Brillouin spectroscopy. *Geophysical*
770 *Research Letters*, 31, L06620.
- 771 Speziale, S., and Duffy, T.S. (2002) Single-crystal elastic constants of fluorite (CaF_2)
772 to 9.3 GPa. *Physics and Chemistry of Minerals*, 29, 465-472.
- 773 Speziale, S., Marquardt, H., and Duffy, T.S. (2014) Brillouin Scattering and its
774 Application in Geosciences. *Reviews in Mineralogy & Geochemistry*, 78,
775 543-603.
- 776 Sun, S.S. (1982) Chemical-composition and origin of the Earth's primitive mantle.
777 *Geochimica et Cosmochimica Acta*, 46, 179-192.
- 778 Suzuki, I., Ohno, I., and Anderson, O. (2000) Harmonic and anharmonic properties of
779 spinel MgAl_2O_4 . *American Mineralogist*, 85, 304-311.
- 780 Weidner, D.J. and Carleton, H.R. (1977) Elasticity of coesite. *Journal of Geophysical*
781 *Research*, 82, 1334-1346.
- 782 Weidner, D. J., and Zhao, Y. (1992) Elasticity and equation of state of perovskite:
783 implications for the Earth's lower mantle. *Geophysical Monograph*, 67, 191-196.
- 784 Weidner, D.J., Bass, J.D, Ringwood, A.E., and Sinclair, W. (1982) The single-crystal
785 elastic moduli of stishovite. *Journal of Geophysical Research*, 87, 4740-4746.
- 786 Weidner, D.J., Wang, H., and Ito, J. (1978) Elasticity of orthoenstatite. *Physics of the*
787 *Earth and Planetary Interiors*, 17, P7-P13.
- 788 Wentzcovitch, R.M., Justo, J.F., Wu, Z.Q., da Silva, C.R.S., Yuen, D.A., and Kohlstedt
789 D. (2009) Anomalous compressibility of ferropericlase throughout the iron spin
790 crossover. *Proceedings of the National Academy of Sciences of the United States*
791 *of America*, 106, 8447-8452.
- 792 Wu Z.Q., Justo J.F., and Wentzcovitch R.M. (2013) Elastic Anomalies in a
793 Spin-Crossover System: Ferropericlase at Lower Mantle Condition. *Physical*
794 *Review Letters*, 110, 228501.
- 795 Yang, J., Mao, Z., Lin, J.F., and Prakapenka, V.B. (2014) Single-crystal elasticity of

FAN ET AL.: DETERMINATION OF THE FULL ELASTIC TENSOR

- 796 the deep-mantle magnesite at high pressure and temperature. Earth and Planetary
797 Science Letters, 392, 292-299.
- 798 Yeganeh-Haeri, A. (1994) Synthesis and re-investigation of the elastic properties of
799 single-crystal magnesium silicate perovskite. Physics of the Earth and Planetary
800 Interiors, 87, 111-121.
- 801 Yeganeh-Haeri, A., Weidner, D.J., and Ito, E. (1989) Elasticity of MgSiO_3 in the
802 perovskite structure. Science, 243, 787-789.
- 803 Yoneda, A. (1990) Pressure derivatives of elastic constants of single crystal MgO and
804 MgAl_2O_4 . Journal of the Physics of the Earth, 38, 19-55.
- 805 Zha, C.S., Duffy, T.S., Downs, R.T., Mao, H.K., and Hemley, R.J. (1996) Sound
806 velocity and elasticity of single-crystal forsterite to 16 GPa. Journal of
807 Geophysical Research, 101, 17535-17545.
- 808 Zha, C.S., Duffy, T.S., Downs, R.T., Mao, H.K., and Hemley, R.J. (1998) Brillouin
809 scattering and X-ray diffraction of San Carlos olivine: direct pressure
810 determination to 32 GPa. Earth and Planetary Science Letters, 159, 25-33.
- 811 Zha, C.S., Mao, H.K., and Hemley, R.J. (2000) Elasticity of MgO and a primary
812 pressure scale to 55 GPa. Proceedings of the National Academy of Sciences of
813 the United States of America, 97, 13494-13499.
- 814
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- 816
- 817
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826 **Figure Captions**

827 **Figure 1.** Representative Brillouin spectra of the single-crystal periclase and spinel
828 at ambient conditions. The orientation of each crystal platelet is labeled on the
829 corresponding spectrum. Open circles: experimental data; solid lines: fitted V_P and V_S
830 peaks.

831

832 **Figure 2.** Acoustic V_P and V_S velocities of the single-crystal periclase as a function
833 of the azimuthal angle in four representative crystallographic planes. a: (-1,0.5,0.2); b:
834 (100); c: (110); d: (111). Open circles: experimental data; solid lines: modeled fits for
835 deriving C_{ij} from using both V_P and V_S .

836

837 **Figure 3.** Acoustic V_S velocities of the single-crystal periclase as a function of the
838 azimuthal angle in four representative crystallographic planes. The velocities are the
839 same as shown in Figure 2, but the full elastic tensor were derived from the modeled
840 fits (solid lines) using experimentally measured V_S velocities alone (open circles).

841

842 **Figure 4.** Acoustic V_P and V_S velocities of the single-crystal spinel as a function of
843 the azimuthal angle in two representative crystallographic planes. a, b: measured and
844 modeled velocities for deriving C_{ij} using both V_P and V_S ; c, d: measured and modeled
845 velocities for deriving C_{ij} using V_S velocities alone. Open circles: experimental data;
846 solid lines: modeled fits.

847

848 **Figure 5.** Acoustic V_S velocities of the single-crystal zoisite as a function of the
849 azimuthal angle in three representative crystallographic planes. Open circles:
850 experimental data (Mao et al. 2007); solid lines: modeled fits for deriving C_{ij} using V_S
851 velocities alone.

852

853 **Figure 6.** Derived elastic constants of the single-crystal periclase and spinel in the
854 representative crystallographic planes at ambient conditions. These constants and their
855 uncertainties were derived using the experimentally measured V_S velocities alone. The

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856 (-1,0.5,0.2) plane for periclase and (110) plane for spinel show the smallest
857 uncertainties ($\pm 1\sigma$) and mismatches from the reference elastic constants taken from
858 the literatures (dash line) for periclase (Sinogeikin and Bass 2000) and for spinel
859 (Askarpour et al. 1993). a: (-1,0.5,0.2) plane for periclase; b: (110) plane for spinel.

860

861 **Figure 7.** Uncertainties in the derived C_{ij} for single-crystal periclase and spinel
862 using V_S profiles as well as both V_P and V_S profiles in the representative
863 crystallographic planes at ambient conditions. The (-1,0.5,0.2) plane for periclase and
864 (110) plane for spinel show the smallest uncertainties ($\pm 1\sigma$). a: periclase; b: spinel.

865

866 **Figure 8.** Derived elastic constants of the single-crystal periclase and spinel using 5°
867 and 10° measuring interval at ambient conditions. The measuring interval was
868 determined from the rotation angle of the crystal platelet on the rotatory stage. The
869 (-1,0.5,0.2) plane for periclase and (110) plane for spinel were used for the study, and
870 the elastic constants and their uncertainties were derived using the V_S velocities alone.
871 The dataset with a 5° measuring interval shows very small uncertainties ($\pm 1\sigma$) and
872 mismatches to the reference literature values (dash line) (Sinogeikin and Bass 2000);
873 Askarpour et al. (1993)). a: (-1,0.5,0.2) plane for periclase; b: (110) plane for spinel.

874

875 **Figure 9.** Derived elastic constants of the single-crystal periclase at high pressure.
876 These constants and their uncertainties were derived from the representative
877 crystallographic (-1,0.5,0.2) plane using the experimentally measured V_S velocities
878 alone. The black symbols with error bars are the present data points, the blue and red
879 symbols with error bars are the reference elastic constants taken from Sinogeikin and
880 Bass (2000) and Zha et al. (2000), respectively. Open squares: C_{11} ; Open triangles:
881 C_{44} ; Open circles: C_{12} .

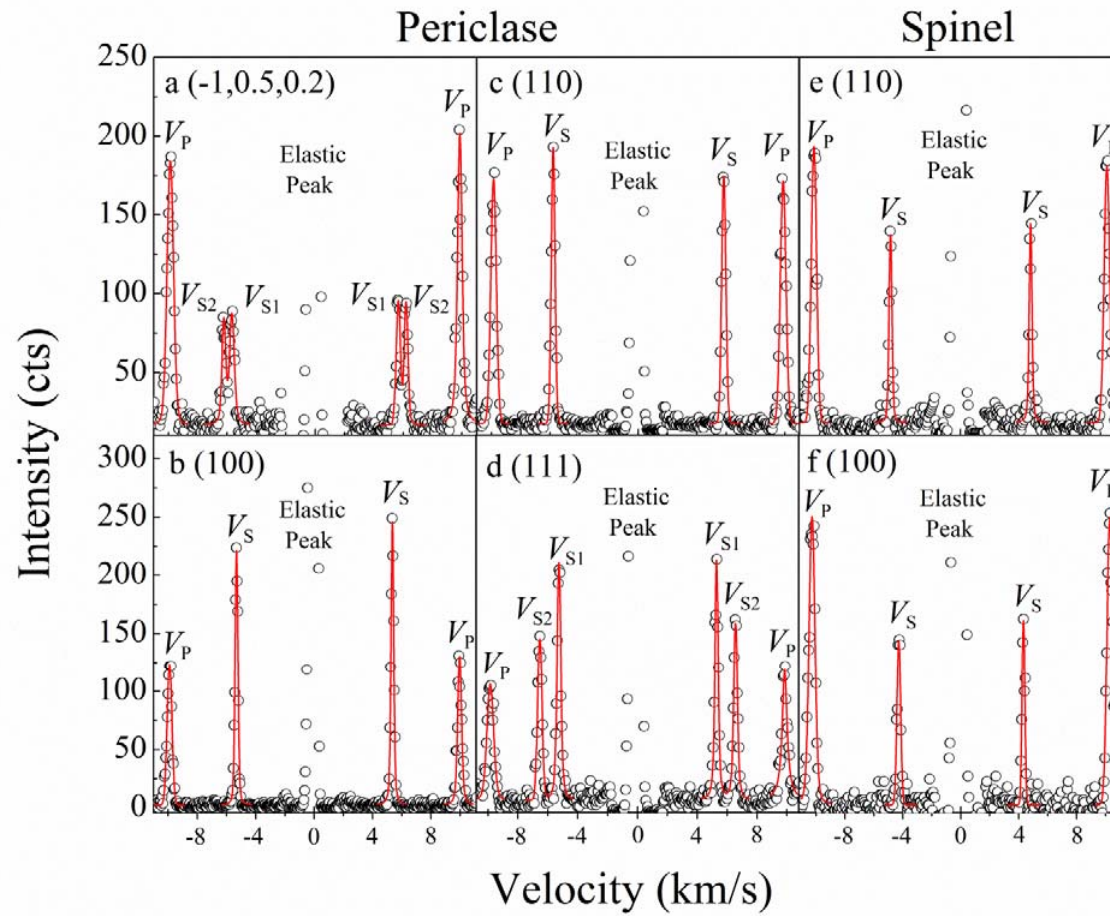


Fig. 1

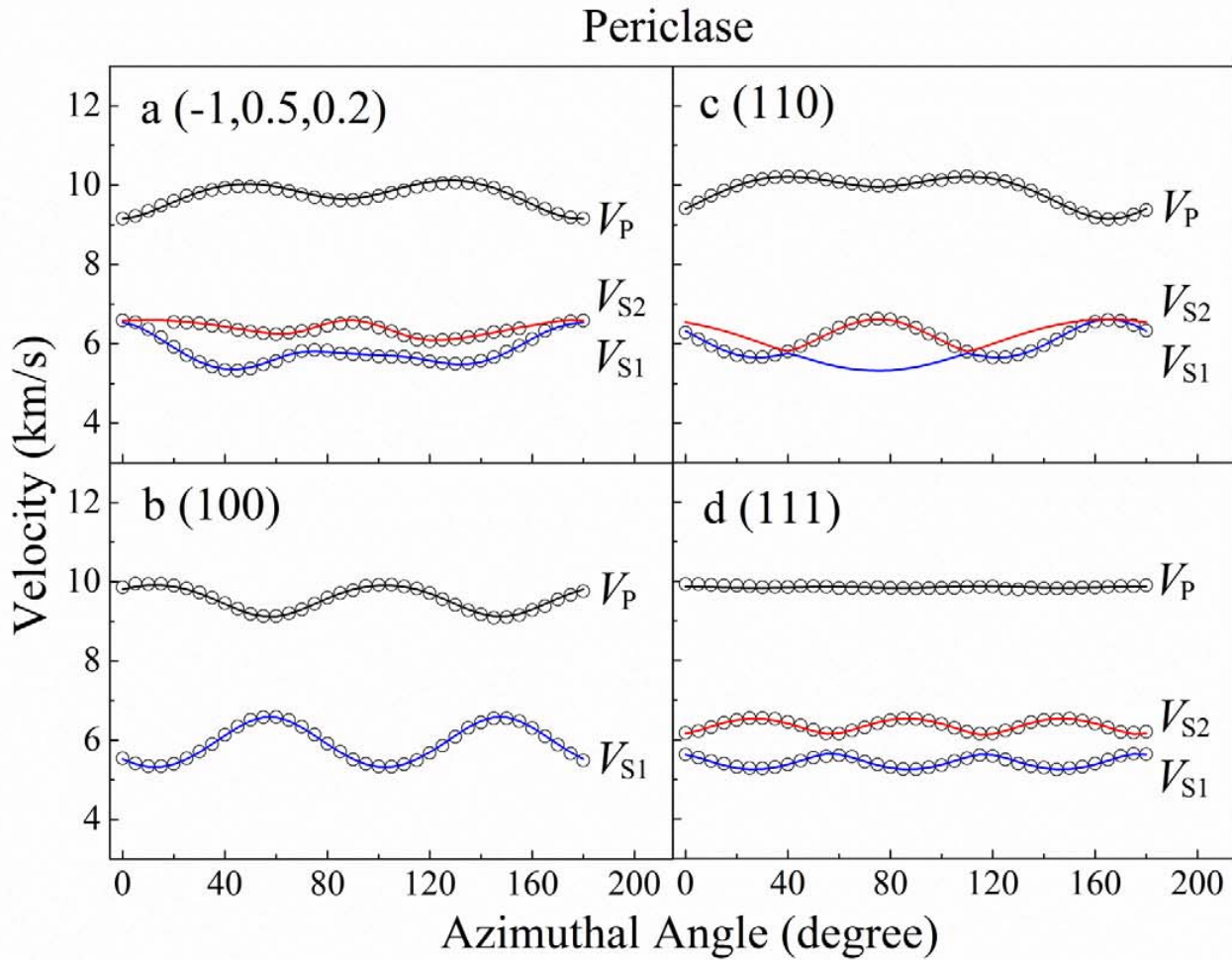


Fig. 2

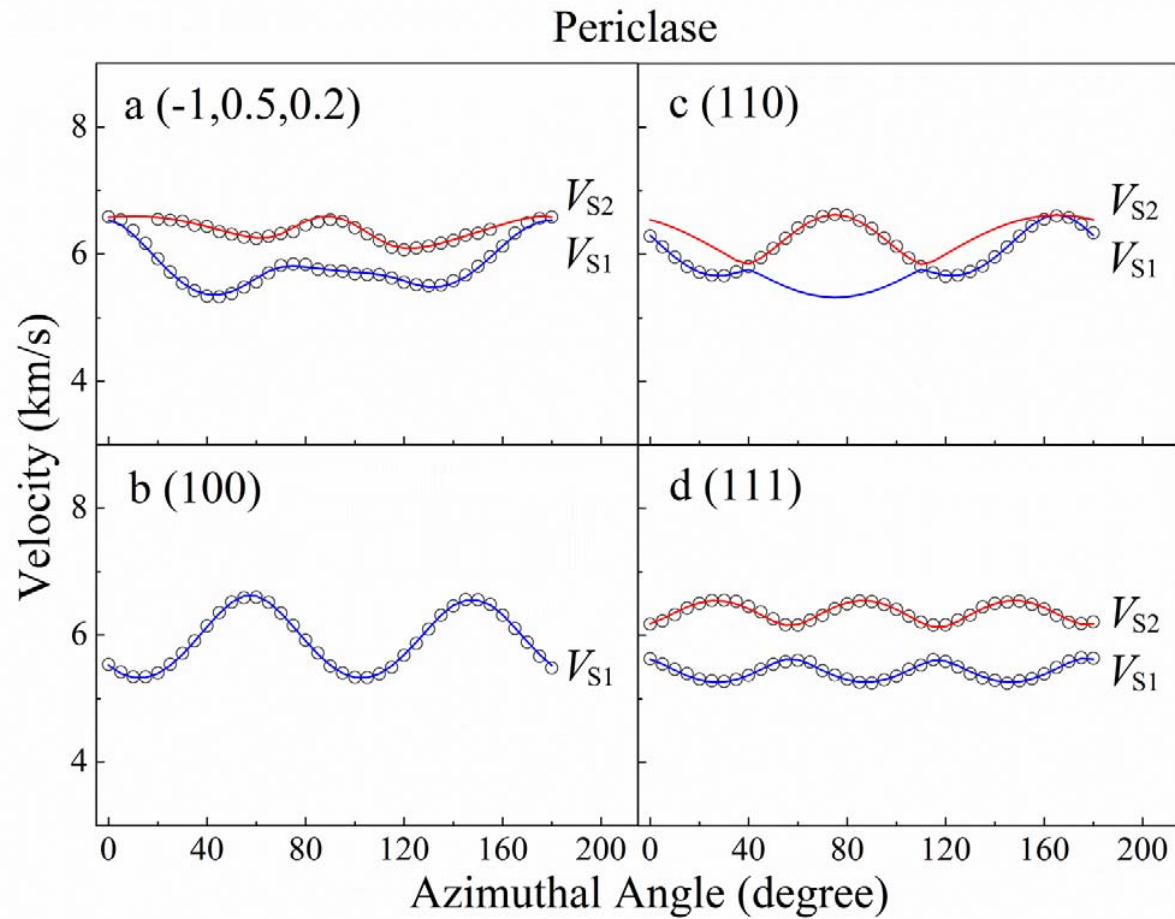


Fig. 3

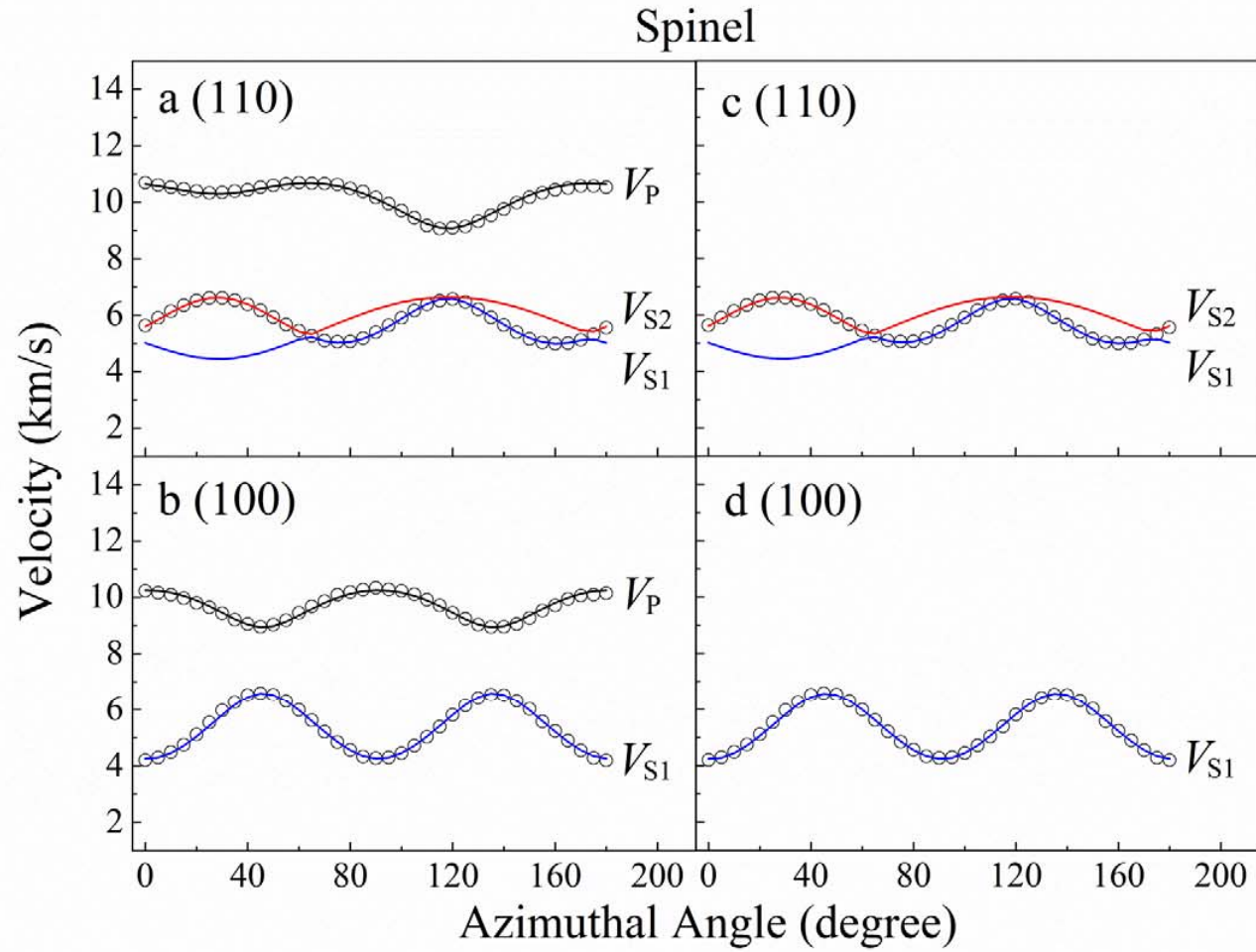


Fig. 4

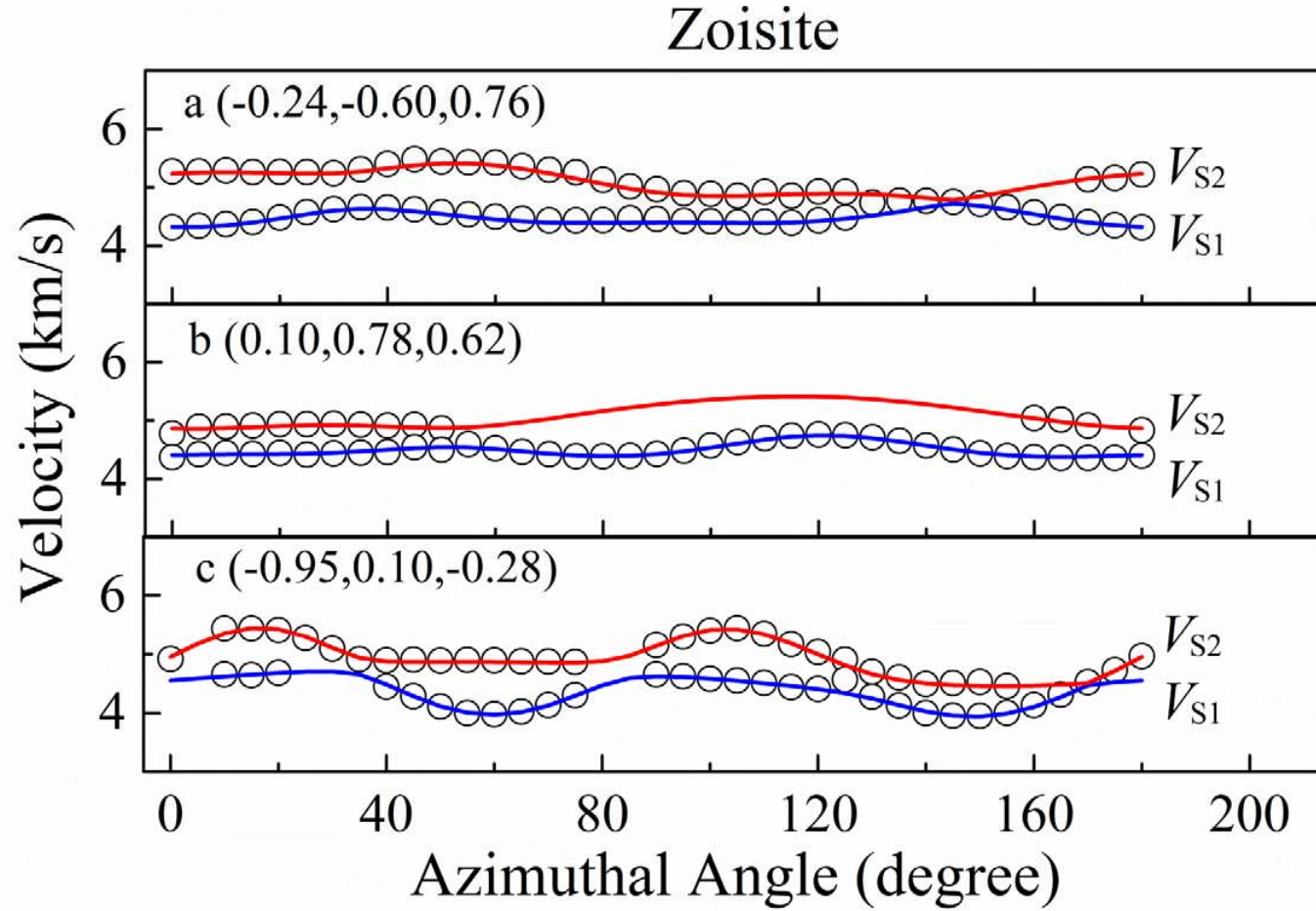


Fig. 5

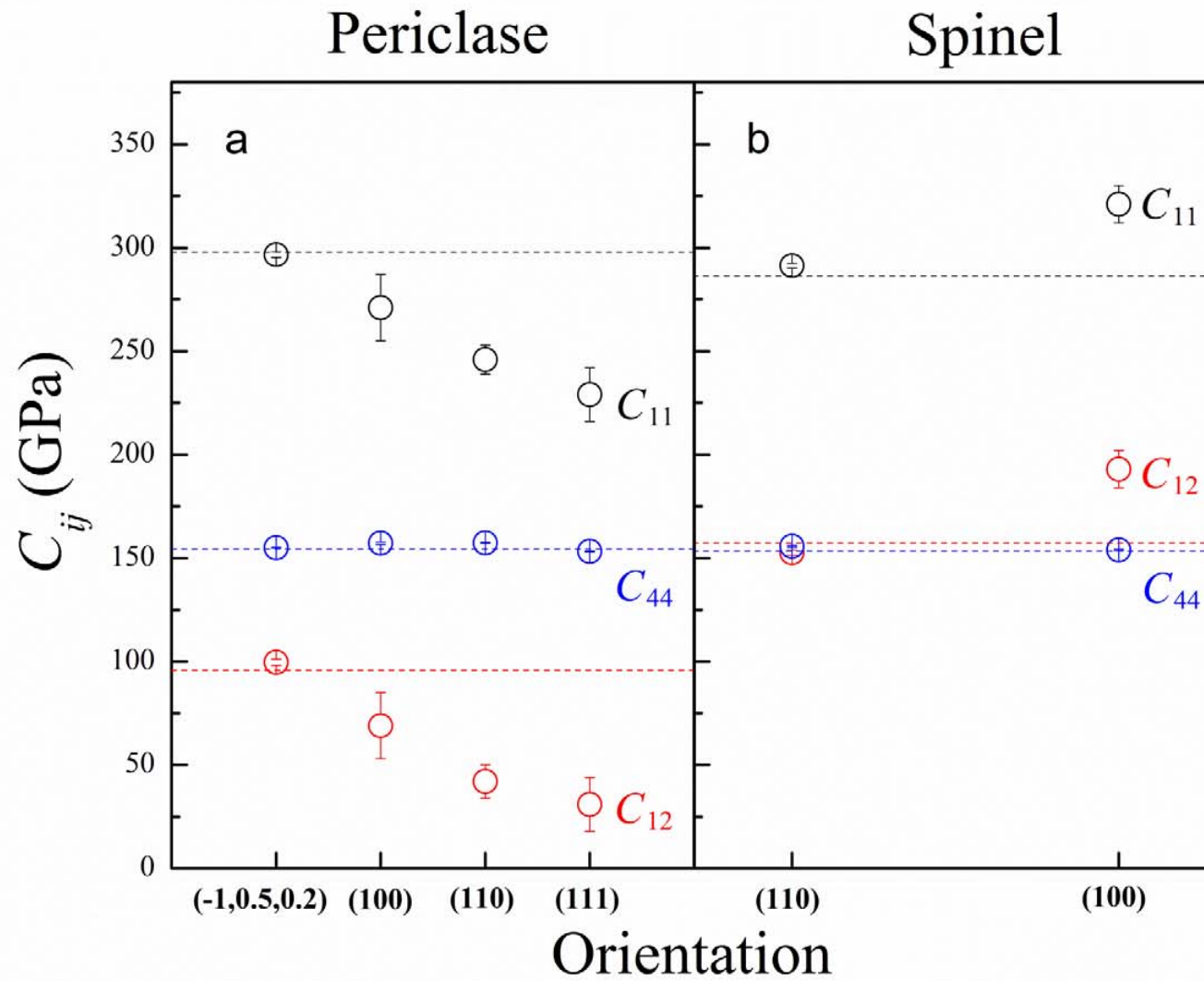


Fig. 6

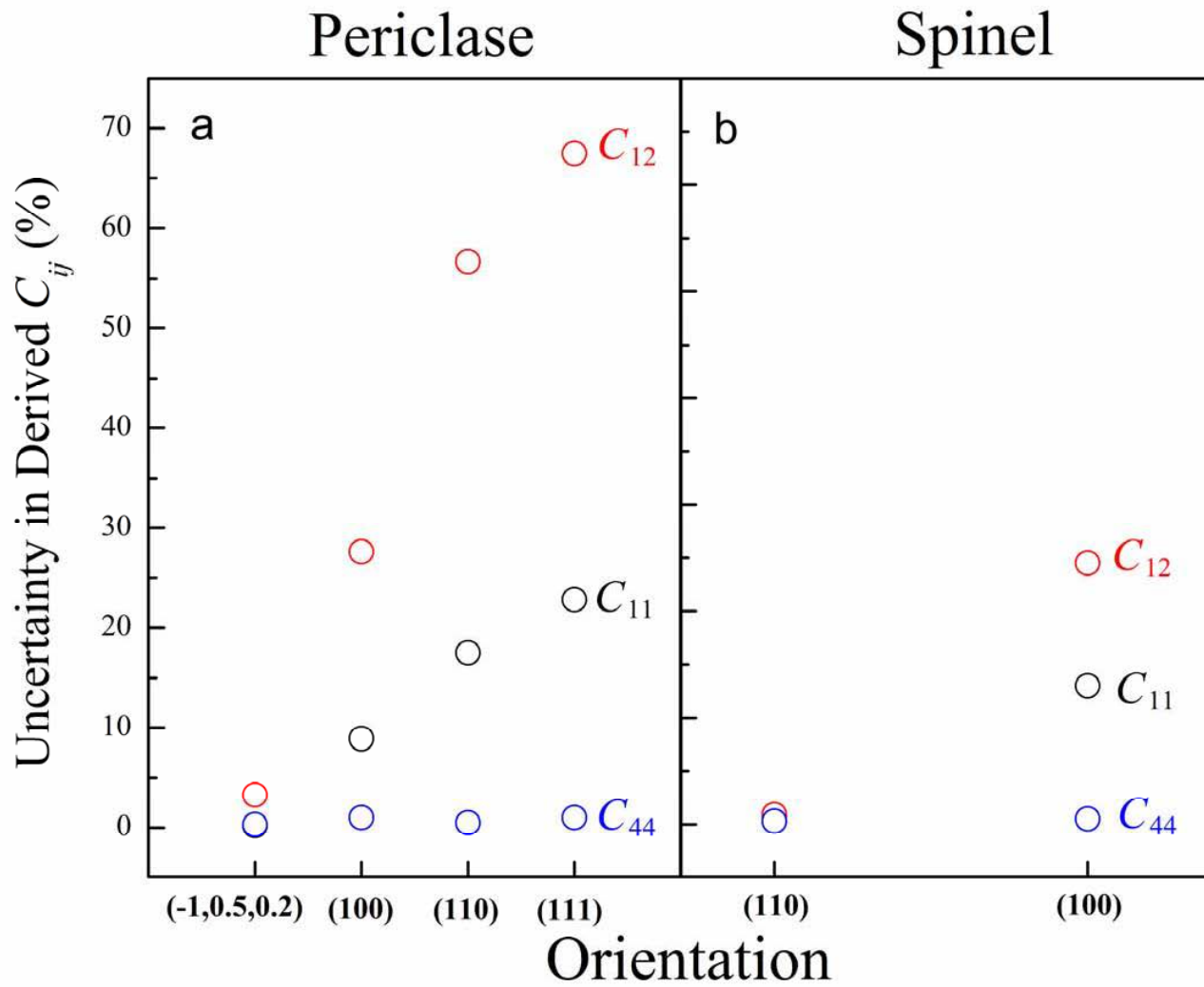


Fig. 7

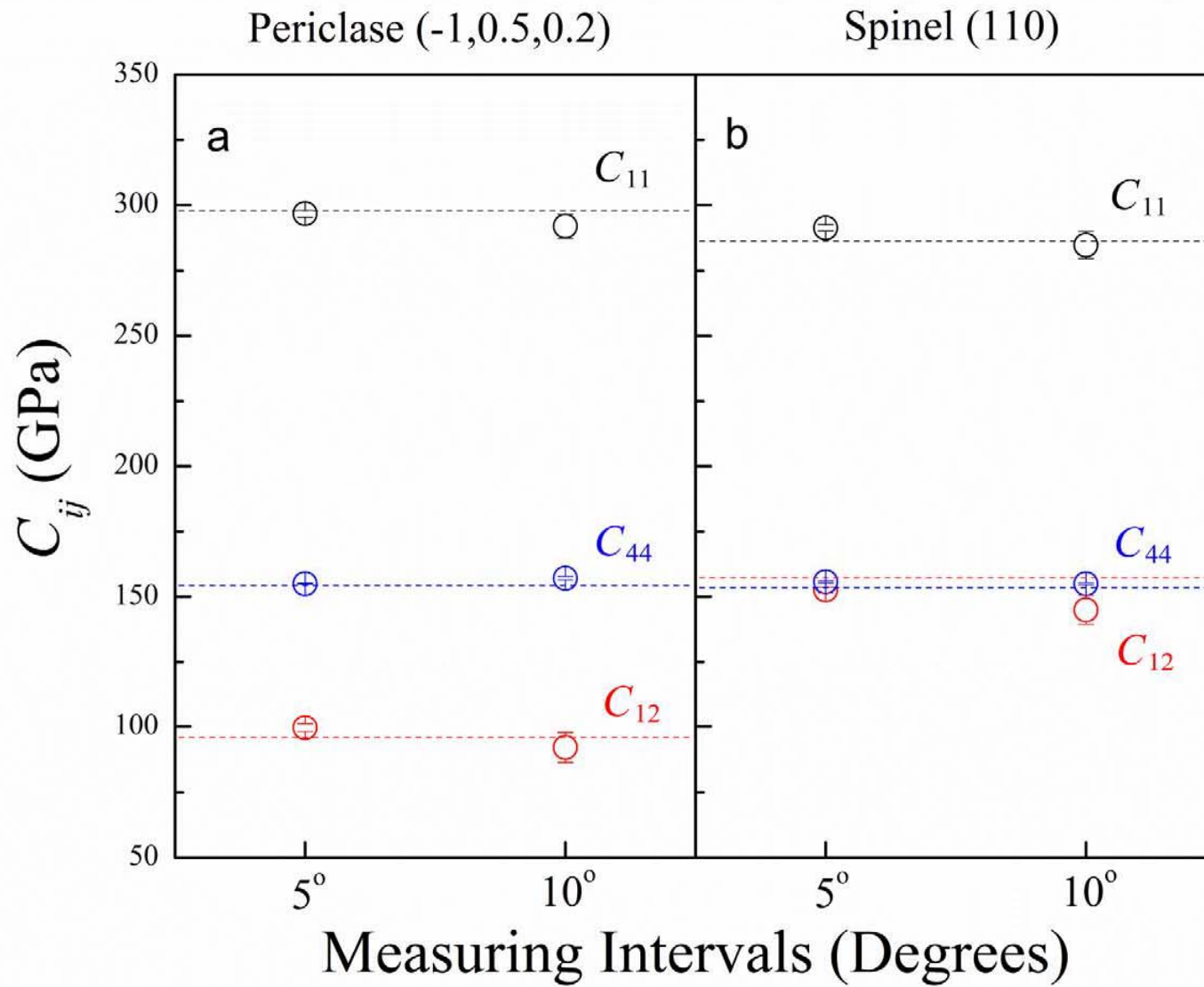


Fig. 8

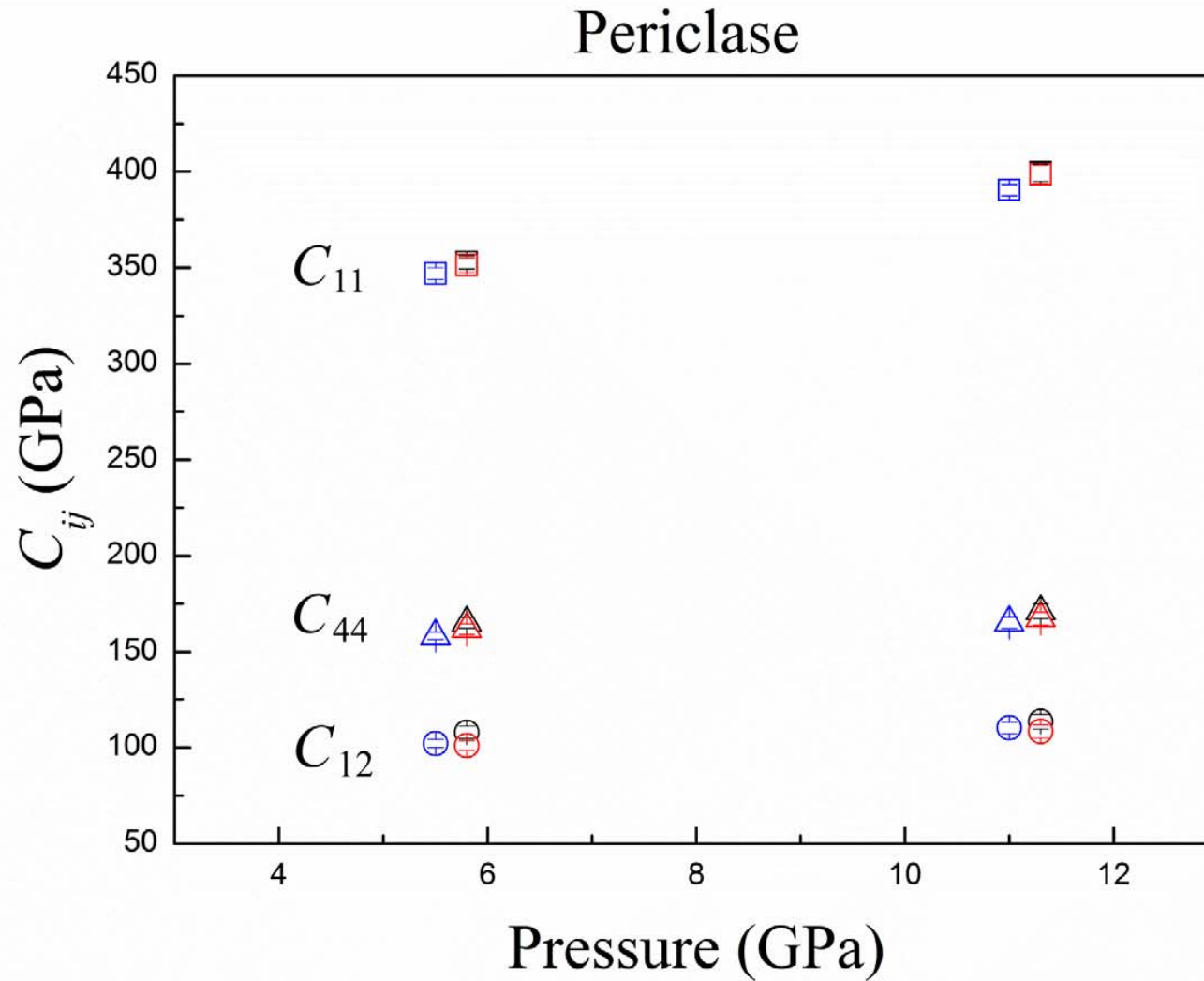


Fig. 9

Table 1 Measured acoustic V_P and V_S velocities of the single-crystal periclase (MgO) as a function of the crystallographic orientation within the (-1,0.5,0.2) plane at ambient conditions. The azimuthal angle is expressed as the rotated angle with respect to an arbitrary reference direction within the plane.

Azimuthal angle (degree)	V_P (km/s)	V_{S1} (km/s)	V_{S2} (km/s)
0	9.276 (± 0.004)	6.538 (± 0.010)	
5	9.302 (± 0.005)	6.472 (± 0.007)	
10	9.393 (± 0.006)	6.327 (± 0.008)	
15	9.489 (± 0.008)	6.165 (± 0.005)	
20	9.600 (± 0.009)	5.926 (± 0.005)	6.542 (± 0.008)
25	9.735 (± 0.007)	5.714 (± 0.004)	6.532 (± 0.006)
30	9.806 (± 0.007)	5.541 (± 0.003)	6.471 (± 0.008)
35	9.880 (± 0.009)	5.422 (± 0.004)	6.426 (± 0.011)
40	9.931 (± 0.008)	5.338 (± 0.006)	6.402 (± 0.009)
45	9.966 (± 0.014)	5.326 (± 0.006)	6.343 (± 0.007)
50	9.956 (± 0.012)	5.378 (± 0.011)	6.301 (± 0.005)
55	9.929 (± 0.011)	5.476 (± 0.006)	6.267 (± 0.005)
60	9.855 (± 0.008)	5.562 (± 0.007)	6.241 (± 0.006)
65	9.759 (± 0.005)	5.681 (± 0.007)	6.277 (± 0.004)
70	9.694 (± 0.005)	5.749 (± 0.009)	6.320 (± 0.007)
75	9.614 (± 0.005)	5.763 (± 0.012)	6.371 (± 0.009)
80	9.549 (± 0.005)	5.753 (± 0.013)	6.453 (± 0.011)
85	9.537 (± 0.008)	5.727 (± 0.006)	6.535 (± 0.011)
90	9.524 (± 0.006)	5.706 (± 0.012)	6.540 (± 0.008)
95	9.588 (± 0.005)	5.696 (± 0.006)	6.506 (± 0.008)
100	9.676 (± 0.006)	5.686 (± 0.005)	6.412 (± 0.006)
105	9.760 (± 0.007)	5.674 (± 0.010)	6.314 (± 0.007)
110	9.869 (± 0.005)	5.636 (± 0.007)	6.212 (± 0.007)
115	9.952 (± 0.006)	5.603 (± 0.006)	6.104 (± 0.006)
120	10.007 (± 0.013)	5.555 (± 0.005)	6.064 (± 0.006)
125	10.044 (± 0.010)	5.517 (± 0.004)	6.095 (± 0.007)
130	10.065 (± 0.012)	5.481 (± 0.003)	6.124 (± 0.006)
135	10.055 (± 0.008)	5.484 (± 0.003)	6.169 (± 0.010)
140	10.004 (± 0.008)	5.558 (± 0.005)	6.214 (± 0.006)
145	9.930 (± 0.006)	5.668 (± 0.004)	6.284 (± 0.007)
150	9.838 (± 0.006)	5.804 (± 0.005)	6.315 (± 0.008)
155	9.713 (± 0.005)	5.955 (± 0.005)	6.376 (± 0.006)
160	9.576 (± 0.006)	6.121 (± 0.009)	
165	9.462 (± 0.006)	6.326 (± 0.009)	
170	9.368 (± 0.006)	6.455 (± 0.010)	
175	9.299 (± 0.006)	6.523 (± 0.010)	
180	9.291 (± 0.007)	6.554 (± 0.010)	

Numbers in parentheses are standard deviations ($\pm 1\sigma$).

Table 2 Measured acoustic V_P and V_S velocities of the single-crystal spinel ($MgAl_2O_4$) as a function of the crystallographic orientation within the (110) plane at ambient conditions. The azimuthal angle is expressed as the rotated angle with respect to an arbitrary reference direction within the plane.

Azimuthal angle (degree)	V_P (km/s)	V_S (km/s)
0	10.681 (± 0.004)	5.637 (± 0.003)
5	10.605 (± 0.006)	5.896 (± 0.003)
10	10.522 (± 0.004)	6.137 (± 0.002)
15	10.464 (± 0.002)	6.353 (± 0.003)
20	10.393 (± 0.003)	6.513 (± 0.002)
25	10.329 (± 0.003)	6.606 (± 0.002)
30	10.348 (± 0.004)	6.609 (± 0.002)
35	10.390 (± 0.002)	6.524 (± 0.002)
40	10.437 (± 0.006)	6.375 (± 0.002)
45	10.523 (± 0.003)	6.169 (± 0.003)
50	10.599 (± 0.002)	5.918 (± 0.002)
55	10.659 (± 0.002)	5.666 (± 0.003)
60	10.688 (± 0.004)	5.440 (± 0.004)
65	10.686 (± 0.005)	5.274 (± 0.005)
70	10.656 (± 0.002)	5.096 (± 0.003)
75	10.628 (± 0.008)	5.067 (± 0.005)
80	10.481 (± 0.008)	5.077 (± 0.004)
85	10.373 (± 0.009)	5.182 (± 0.004)
90	10.165 (± 0.005)	5.390 (± 0.008)
95	9.933 (± 0.008)	5.643 (± 0.006)
100	9.703 (± 0.008)	5.902 (± 0.007)
105	9.455 (± 0.011)	6.168 (± 0.007)
110	9.178 (± 0.009)	6.388 (± 0.006)
115	9.062 (± 0.007)	6.512 (± 0.005)
120	9.093 (± 0.008)	6.557 (± 0.005)
125	9.149 (± 0.006)	6.459 (± 0.007)
130	9.325 (± 0.004)	6.208 (± 0.004)
135	9.533 (± 0.006)	5.921 (± 0.006)
140	9.739 (± 0.005)	5.650 (± 0.006)
145	9.987 (± 0.007)	5.399 (± 0.005)
150	10.175 (± 0.005)	5.168 (± 0.006)
155	10.328 (± 0.006)	5.035 (± 0.005)
160	10.440 (± 0.006)	4.991 (± 0.005)
165	10.519 (± 0.009)	5.011 (± 0.006)
170	10.575 (± 0.008)	5.138 (± 0.006)
175	10.586 (± 0.014)	5.323 (± 0.005)
180	10.561 (± 0.009)	5.549 (± 0.005)

Numbers in parentheses are standard deviations ($\pm 1\sigma$).

Table 3 Trade-off coefficients for elastic constants of single-crystal periclase (MgO) within (-1,0.5,0.2) plane and spinel (MgAl₂O₄) within (110) plane.

<i>i</i> ↓	<i>k</i> →		
	11	12	44
periclase			
11	0.044147	0.001501	-0.000149
12	0.001501	0.013785	-0.002397
44	-0.000149	-0.002397	0.018163
spinel			
11	0.014740	0.004580	-0.002577
12	0.004580	0.019508	-0.005696
44	-0.002577	-0.005696	0.088773

Table 4 Single-crystal elastic constants of periclase (MgO) at ambient conditions. Literature results are also listed for comparison.

Orientation and References	Data Used	C_{11} (GPa)	C_{12} (GPa)	C_{44} (GPa)
(-1,0.5,0.2) (This study)	V_P and V_S	296.0 (± 0.4)	96.8 (± 0.4)	154.6 (± 0.2)
	V_S	296.7 (± 1.3)	99.6 (± 1.5)	155.0 (± 0.2)
(100) (This study)	V_P and V_S	297.5 (± 0.3)	95.2 (± 0.3)	155.6 (± 0.2)
	V_S	271 (± 16)	69 (± 16)	157.2 (± 0.6)
(110) (This study)	V_P and V_S	298.6 (± 0.5)	96.8 (± 0.4)	156.5 (± 0.2)
	V_S	246 (± 7)	42 (± 8)	157.3 (± 0.2)
(111) (This study)	V_P and V_S	296.6 (± 0.4)	94.7 (± 0.4)	154.8 (± 0.2)
	V_S	229 (± 13)	31 (± 13)	153.3 (± 0.2)
(100) Jackson and Niesler (1982)	V_P and V_S	296.8 (± 1.5)	95.3 (± 0.2)	155.8 (± 0.2)
(100) Yoneda (1990)	V_P and V_S	297.8 (± 1.5)	95.1 (± 1.0)	155.8 (± 1.5)
(100) Sinogeikin and Bass (2000)	V_P and V_S	297.9 (± 1.5)	95.8 (± 1.0)	154.4 (± 2.0)
(01-1) Zha et al. (2000)	V_P and V_S	297.0 (± 0.1)	95.2 (± 0.7)	155.7 (± 0.5)

Numbers in parentheses are standard deviations ($\pm 1\sigma$).

Table 5 Single-crystal elastic constants of spinel (MgAl_2O_4) at ambient conditions. Literature results are also listed for comparison.

Orientation and References	Data used	C_{11} (GPa)	C_{12} (GPa)	C_{44} (GPa)
(110) (This study)	V_p and V_s	288.0 (± 0.5)	154.2 (± 0.5)	156.2 (± 0.3)
	V_s	291.3 (± 1.2)	152.6 (± 1.2)	155.7 (± 0.4)
(100) (This study)	V_p and V_s	284.4 (± 0.3)	155.0 (± 0.4)	154.7 (± 0.2)
	V_s	321 (± 9)	193 (± 9)	153.9 (± 0.3)
(110) Yoneda (1990)	V_p and V_s	282.9*	155.4*	154.8*
(100) Askarpour et al. (1993)	V_p and V_s	286.3 (± 5.3)	157.2 (± 3.4)	153.5 (± 2.7)
Suzuki et al. (2000) [§]	V_p and V_s	281.3 (± 0.1)	155.4 (± 0.1)	154.6 (± 0.1)

Numbers in parentheses are standard deviations ($\pm 1\sigma$).

*: uncertainties of the elastic constants were not given in the literature.

[§]: The orientation of the sample in Suzuki et al. (2000) is not specified because the elastic tensor was determined by resonant ultrasound spectroscopy performed on a spherical sample.

Table 6 Single-crystal elastic constants of periclase (MgO) and spinel (MgAl₂O₄) derived using the V_S velocities alone data set having 5° and 10° measuring intervals at ambient conditions.

Sample and Orientation	Interval	Data used	C_{11} (GPa)	C_{12} (GPa)	C_{44} (GPa)
Periclase (-1,0.5,0.2)	5°	V_S	296.7 (± 1.3)	99.6 (± 1.5)	155.0 (± 0.2)
	10°	V_S	291.9 (± 4.6)	92.0 (± 5.7)	157.1 (± 0.7)
Spinel (110)	5°	V_S	291.3 (± 1.2)	152.6 (± 1.2)	155.7 (± 0.4)
	10°	V_S	284.7 (± 5.2)	145.0 (± 5.6)	155.0 (± 0.4)

Numbers in parentheses are standard deviations ($\pm 1\sigma$).

Table 7 Single-crystal elastic constants of zoisite ($\text{Ca}_2\text{Al}_3\text{Si}_3\text{O}_{12}(\text{OH})$) at ambient conditions. The difference in percentage is calculated as the difference between the elastic constants of zoisite using V_P and V_S velocities and V_S velocities alone using Mao et al. (2007) as the reference.

Modulus	Data used*	Data used*	Difference
	V_P and V_S	V_S	%
C_{11} (GPa)	279.8 (± 0.6)	276.8 (± 1.7)	1.1
C_{12} (GPa)	94.7 (± 1.1)	90.1 (± 2.3)	4.9
C_{13} (GPa)	88.7 (± 1.0)	85.2 (± 1.4)	3.9
C_{22} (GPa)	249.2 (± 0.6)	243.4 (± 0.8)	2.3
C_{23} (GPa)	27.5 (± 0.7)	22.8 (± 2.6)	17.2
C_{33} (GPa)	209.4 (± 0.9)	205.5 (± 3.2)	1.8
C_{44} (GPa)	51.8 (± 0.3)	51.8 (± 0.2)	0.1
C_{55} (GPa)	81.4 (± 0.3)	81.6 (± 0.4)	0.2
C_{66} (GPa)	66.3 (± 0.3)	66.2 (± 0.5)	0.1

Numbers in parentheses are standard deviations ($\pm 1\sigma$).

* Data were taken from Mao et al. (2007).

Table 8 Single-crystal elastic constants of stishovite (SiO₂) at ambient conditions. The difference in percentage is calculated as the difference between the elastic constants of stishovite using V_P and V_S velocities and V_S velocities alone using Jiang et al. (2009) as the reference.

Modulus	Data used*	Data used*	Difference
	V_P and V_S	V_S	%
C_{11} (GPa)	455 (± 1)	447 (± 4)	1.8
C_{12} (GPa)	199 (± 2)	207 (± 4)	4.0
C_{13} (GPa)	192 (± 1)	208 (± 3)	8.3
C_{33} (GPa)	762 (± 1)	755 (± 3)	0.9
C_{44} (GPa)	258 (± 1)	260 (± 4)	0.8
C_{66} (GPa)	321 (± 1)	319 (± 5)	0.6

Numbers in parentheses are standard deviations ($\pm 1\sigma$).

* Data were taken from Jiang et al. (2009).

Table 9 Single-crystal elastic constants of periclase (MgO) at high pressures. Literature results from Sinogeikin and Bass (2000) are also listed for comparison.

Orientation and References	Pressure (GPa)	Data Used	C_{11} (GPa)	C_{12} (GPa)	C_{44} (GPa)
(-1,0.5,0.2) (This study)	5.8 (± 0.4)	V_P and V_S	348.0 (± 2.8)	103.1 (± 2.5)	162.6 (± 2.4)
		V_S	352.8 (± 3.6)	107.7 (± 3.2)	165.1 (± 2.8)
	11.3 (± 0.5)	V_P and V_S	394.8 (± 3.8)	109.1 (± 3.1)	168.5 (± 2.4)
		V_S	399.4 (± 4.4)	113.3 (± 3.7)	171.1 (± 3.8)
(100) Sinogeikin and Bass (2000)	5.5 (± 0.1)	V_P and V_S	346.9 (± 3.0)	102.0 (± 2.0)	158.4 (± 2.0)
	11.00 (± 0.02)	V_P and V_S	390.4 (± 3.0)	110.1 (± 3.0)	165.2 (± 3.0)

Numbers in parentheses are standard deviations ($\pm 1\sigma$).