

2

3 **Current limitations of molecular dynamic simulations as probes of thermo-physical**  
4 **behavior of silicate melts**

5 **Revision 1**

6 Jean-Philippe Harvey<sup>1,\*</sup> and Paul D. Asimow<sup>1</sup>

7 <sup>1</sup>Division of Geological and Planetary Sciences, California Institute of Technology, Pasadena  
8 CA 91125 USA

9 \*jpharvey@caltech.edu

10 **Abstract**

11 Molecular dynamic simulations offer promise as an essential tool, complementary to  
12 experiments, for expanding the reach of computational thermodynamics in igneous petrology by  
13 evaluating excess thermodynamic properties of multicomponent silicate melts. However, we  
14 present evidence suggesting that current practices in simulation may not achieve the precision  
15 needed to predict complex phase equilibria relevant to modeling the Earth's interior evolution.  
16 We highlight the importance of quantification of the chemical short range order in terms of  
17 cation-cation pairs in the melt and its impact on different kinetic aspects of molecular dynamic  
18 simulations. We analyze published molecular dynamic simulation studies of silicate melts in  
19 order to identify specific criteria and best practices for achieving and demonstrating equilibrium  
20 and producing accurate results. Finally, we propose a list of experimental and numerical  
21 investigations that need to be performed in the future to ensure full consistency between these  
22 two approaches in order to reduce the gap in our fundamental understanding of silicate melts  
23 between the atomic level and the macroscopic scale.

24 **Keywords:** molecular dynamics, ab initio calculations, classical thermodynamics, silicate melts

## 25 **1. Introduction**

26 Our ability to describe the range of evolutionary pathways for terrestrial planets and to identify,  
27 through interpretation of available data, the particular pathway that the Earth followed depends  
28 on our understanding of the thermo-physical properties of Earth materials under a wide range of  
29 conditions. Multi-component liquid solutions of oxides, metals and silicates, in particular, are the  
30 defining materials of magmatic processes that have driven chemical differentiation throughout  
31 planetary history and that continue to modulate ongoing dynamic processes and natural volcanic  
32 hazards. Questions whose answers depend on such liquid properties include (but are not limited  
33 to): temperature and heat flow profiles across the earth's mantle over time (Davies, 2006, 2008);  
34 bulk chemical composition and both radial and lateral heterogeneity of mantle and crust  
35 (Solomatov and Stevenson, 1993); and evolution of oxidation state and the speciation and  
36 outgassing (and ingassing) of volatile components, an essential part of the story of Earth's  
37 habitability over time (Dasgupta and Hirschmann, 2006; Hirschmann, 2006).

38 In turn, the properties of melts and partial melts that must be predicted accurately as functions of  
39 temperature ( $T$ ), pressure ( $P$ ) or volume ( $V$ ), and composition include elastic and thermodynamic  
40 quantities as well as dynamic transport properties such as viscosity, thermal conductivity and  
41 chemical diffusivities (Stixrude et al. 2009). The densities of silicate melts, solids and glasses  
42 define the driving forces by which gravity leads to physical flow and differential transport of  
43 phases (Agee, 1998; de Koker et al. 2013; Lange and Carmichael, 1987; McKenzie, 1984, 1985;  
44 Sparks and Parmentier, 1991; Stolper et al. 1981). Standard-state thermodynamic quantities and  
45 activity-composition relationships govern phase equilibria and provide the driving forces for  
46 chemical transport and differential transport of species within phases (Fei et al. 1990; Ghiorso,

47 1985). Viscosity of melts controls many aspects of magma transport and chemical evolution of  
48 magmas (Clemens and Petford, 1999; Dingwell, 1996, 1998; Vetere et al. 2008; Whittington et  
49 al. 2009), although texture of multiphase aggregates is important as well (Faul et al. 1994; von  
50 Bargen and Waff, 1986). Chemical and self-diffusivity of various species lead to stable isotope  
51 fractionation (Lacks et al. 2012; Lundstrom et al. 2005; Richter et al. 2003) and are often rate-  
52 limiting in crystal growth in igneous rocks (Faul and Scott, 2006) and more generally in the  
53 achievement of thermodynamic equilibrium (Leshner, 2010; Liang, 2003).

54 The behavior and distribution of silicate liquids can be addressed near the surface and in both the  
55 present and the past through collection and analysis of igneous rocks (Klein and Langmuir, 1987;  
56 Lee et al. 2009; Lehnert et al. 2000; Niu et al. 2011; Plank and Langmuir, 1988), whereas the  
57 current distribution of melts at depth can be inferred from seismological (Forsyth, 1996; Karato  
58 and Jung, 1998; Lay et al. 2004; Williams and Garnero, 1996) or magnetotelluric (Evans et al.  
59 2005; Matsuno et al. 2012) measurements. However, many melting-related phenomena occur at  
60 extreme  $P$  and  $T$ , where observational and even experimental data are available only at sparsely  
61 sampled compositions and ( $P$ ,  $T$ ) or are lacking altogether. This creates an obvious need for  
62 theoretical or computational approaches in order to construct and calibrate a successful, general  
63 model of liquids, applicable to all relevant compositions and conditions and capable of predicting  
64 all thermodynamic and kinetic properties. The question is how to construct such a model. We  
65 propose that major progress can be achieved through a combination of approaches from Earth  
66 science and from engineering thermodynamics that have yet to be brought together.

### 67 *1.1 The goal*

68 An ultimate strategy to self-consistently represent and predict the thermo-physical behavior of  
69 silicate melts should link together the internal energy ( $U$ ), the structure and the equilibrium

70 molar volume in order to provide a predictive and accurate equation of state (EOS). A rigorous  
71 classical treatment would combine a robust interatomic potential (including electronic and  
72 magnetic effects) for  $U$  of the solution as a function of  $V$ , an accurate configurational entropy of  
73 mixing ( $S_{mix}$ ) expression accounting for structure, and the introduction of  $T$  via vibrational  
74 phonon frequency theory (Fultz, 2010). However, classical interatomic potentials need to be  
75 parameterized and cannot capture all subtle energetic effects; exact three-dimensional  $S_{mix}$   
76 expressions are not available and must be approximated (e.g., the 1D chain of Ising (1925) or the  
77 coordination-number-dependent approximation of Pelton et al. (2000)); there is no fully  
78 predictive treatment of anharmonic effects for solids (Fultz, 2010) and our knowledge of atomic  
79 vibrations in liquids is limited (Bolmatov et al. 2012). In geochemistry, robust analytical  
80 treatments of condensed phases accounting for each contribution to the Helmholtz energy are  
81 generally not available. Instead one common empirical approach to  $P$ - $V$  relations begins with  
82 analytical derivation of a compact interatomic potential (Vinet et al. 1987). However, most self-  
83 consistent approaches to date avoid an explicit interatomic potential via a quasi-harmonic finite  
84 strain free energy formalism (Davies, 1973). Such approaches depend on the quantity and quality  
85 of thermo-physical data for parameterization. In de Koker and Stixrude (2009), parameterization  
86 of such a model for single-component liquid phases using numerical experiments requires at  
87 least 16 adjustable parameters (for MgO) and at most 19 adjustable parameters (for SiO<sub>2</sub>).

## 88 *1.2 The challenge*

89 In section 2 we will discuss the importance and limitations of experimental observations for  
90 parameterizing EOS models and will introduce the need for complementary constraints from  
91 numerical simulations. We will then discuss the level of accuracy that simulations must achieve  
92 in order to enable pure and multi-component phase equilibrium calculations. Chemical phase

93 equilibria are sensitive to remarkably small energy differences, such that effects that may appear  
94 negligible in physical treatments of a single phase EOS become important. We will highlight the  
95 special challenges associated with simulating liquids and point out where the techniques  
96 commonly used in performing and interpreting ab initio molecular dynamics (AIMD)  
97 simulations, well-established in the study of solids, may not describe liquids well. In particular,  
98 the chemical complexity of silicate melts requires large numbers of atoms in the simulation and  
99 long run times. Ergodicity, a prerequisite for obtaining accurate transport properties, may take a  
100 long time to establish and, worse, apparently ergodic states with poorly equilibrated short range  
101 order may persist over even longer run times. It is difficult to establish criteria for complete  
102 sampling of the partition function except by (computationally expensive) approaches from  
103 multiple initial conditions with different degrees of internal order. Even though a simple test to  
104 judge the ergodicity of MD simulation data exists — plotting the logarithm of the mean square  
105 displacement as a function of the logarithm of time (Morgan and Spera 2001a, 2001b; Tikunoff  
106 and Spera 2014) — it is unclear how numerically sensitive internal energy and volume are to  
107 slightly non-ergodic conditions. As a result, the distinction between a deeply supercooled  
108 metastable liquid and a glass needs further exploration which goes beyond the scope of the  
109 present paper. We will close with a summary of recommendations for experimental,  
110 computational, and thermodynamic workers in this field.

## 111 **2. Context and Motivation**

### 112 *2.1 The experimental approach*

113 Ideally, all available experimental techniques would be used to provide sets of self-consistent  
114 thermo-physical data for the silicate melts and solids relevant to the Earth's interior. Experiments  
115 spanning adequate ranges of  $T$ ,  $P$  and composition would optimally constrain the parameters of

116 an EOS and mixing relationships for each potential stable phase. Then parameterized Helmholtz  
117 or Gibbs energy models for those phases would be considered in global constrained  
118 minimization algorithms such as MELTS (Ghiorso et al. 2002) or FACTSAGE (Bale et al. 2009)  
119 to predict complex phase equilibria in the Earth. Transport properties may also be fitted to  
120 experimental data using relatively simple forms such as Arrhenius or Adam-Gibbs laws.

121 Indeed, a great variety of experimental data on properties of natural and synthetic silicate melts,  
122 solids and glasses are available for this purpose. Chemical order and local chemical environment  
123 (a few nm) around cations or anions can be studied using nuclear magnetic resonance  
124 spectroscopy (Putnis, 1996), extended X-ray absorption fine structure (Simon et al. 2013) and X-  
125 ray diffraction (Sugiyama et al. 1996). Chemical diffusivity of many species has been measured,  
126 generally at intermediate scales ( $\mu\text{m}$  to  $\text{mm}$ ); see compilations of Brady (2013), Zhang and Ni  
127 (2010); Zhang et al. (2010) or Lesher (2010). Density of silicate melts, solids and glasses is  
128 studied at macroscopic scale ( $\text{mm}$  to  $\text{cm}$ ) by Archimedean methods (Lange and Carmichael,  
129 1987), shock compression (Asimow and Ahrens, 2010), and X-ray absorption (Sakamaki et al.  
130 2009, 2010a, 2010b, 2011). Viscosity is studied at macroscopic scale using devices such as  
131 rotational or parallel-plate viscometers (Del Gaudio and Behrens, 2009; Urbain et al. 1982) and  
132 creep apparatus (Neuville and Richet, 1991). Relaxation effects, i.e. the time-dependence of all  
133 of these properties, can be studied with techniques such as dilatometry, differential thermal  
134 analysis, calorimetry and viscometry (see Sipp and Richet (2002)). Chemical potentials or  
135 thermodynamic activities of species in silicate melts can be determined by electromotive force  
136 measurements (Sterten and Maeland, 1985), Knudsen effusion cell and mass spectrometry  
137 (Zaitsev et al. 2000), or via heterogeneous equilibrium between a melt and a mineral or a probe  
138 phase such as liquid (Rein and Chipman, 1965) or solid (Chamberlin et al. 1994) metallic alloy.

139 Considering specifically the thermo-physical behavior of silicate melts at  $P \geq 10$  GPa, however,  
140 fewer experiments are available. Shock compression of silicate melts, following Rigden et al.  
141 (1988), defines the  $P$ - $V$ -energy EOS via the Rankine-Hugoniot equations. This technique has  
142 been extended to  $\sim 150$  GPa and  $\sim 10000$  K (Asimow and Ahrens, 2010) using gas guns and in a  
143 few cases to TPa pressures using laser compression (Hicks et al. 2006; McWilliams et al. 2012;  
144 Spaulding et al. 2012). The  $P$ - $T$  paths of these experiments are governed by the physics of shock  
145 compression and manipulation of the initial condition or application of novel drive paths are  
146 needed for states hotter or colder than the principal Hugoniot. The extent to which this technique  
147 explores fully equilibrated states requires careful evaluation, although derived equations of state  
148 generally extrapolate consistently from known low- $P$  values and the very high  $T$  of shock states  
149 suggests rapid relaxation (Rigden et al. 1988). Nevertheless, shock rise times are short ( $\sim 10^{-10}$  s  
150 for liquids relevant to the earth's mantle) and this may be insufficient for complete relaxation of  
151 the chemical arrangement at the atomic level (called hereafter chemical short range order, SRO),  
152 particularly in highly multicomponent or topologically complex liquids. The extent to which all  
153 possible cation-cation second nearest neighbor (2NN) distributions are explored, and the  
154 equilibrium distribution located, can be estimated from the Einstein relation for diffusion and the  
155 self-diffusivity of Si. Experiments (Poe et al. 1997) and AIMD simulations (de Koker et al. 2008;  
156 Karki et al. 2007) show that this self-diffusivity strongly decreases as the degree of  
157 polymerization of the silicate melt increases. AIMD simulations also predict decreasing self-  
158 diffusivity with increased  $P$ . If the duration of shock compression and the resulting migration  
159 distances of Si cations do not allow enough permutations of the cation distribution, then  
160 chemical SRO cannot evolve; the chemical structure becomes frozen and does not reach the  
161 equilibrium state. As internal structure, quantified by the amplitude of the SRO, may evolve in

162 non-ideal solutions as a function of composition,  $T$  and  $P$ , this suggests that shock compression  
163 measurements may not be able to capture the configurational entropy correctly.

164 Considering the ensemble of available data, then, empirical calibration of self-consistent  
165 thermodynamic properties of silicate phases is not yet possible at the level of accuracy to which  
166 one might aspire. Experimental limitations and cost lead to scarcity and inconsistency in thermo-  
167 physical data for silicate melts, solids and glasses, specifically at high  $T$  and  $P$ . Missing,  
168 imprecise or inaccurate experimental data in turn limit our ability to constrain the parameters of  
169 complex functional forms for the EOS over the entire range of relevant compositions. We must  
170 turn to simplified mixing rules to interpolate among studied compositions of liquid, solid and  
171 glass solutions. Various mixing rules for different thermo-physical properties of solutions at  
172 constant  $P$  and  $T$  have been proposed in the literature. It is common practice in geochemistry (de  
173 Koker et al. 2013; de Koker and Stixrude, 2009; Ghiorso et al. 2002) to model the energetic  
174 behavior of silicate solutions assuming that the  $S_{mix}$  in these multicomponent systems is ideal  
175 according to some selected definition. Excess heat capacity and excess molar volume of liquid  
176 solutions upon mixing are commonly taken to be zero and, in general, linear interpolation of the  
177 molar heat capacity and volume between available chemical compositions appears to describe  
178 the behavior of silicate liquids (Ghiorso et al. 2002; Lange and Carmichael, 1987). This leaves  
179 the enthalpy of mixing ( $H_{mix}$ ) as the sole excess property to capture the non-ideal solution  
180 properties of such liquids. Laboratory data itself can have systematic and composition-dependent  
181 errors, so this general thermodynamic approach may become rather inexact unless coupled to  
182 constraints from numerical experiments. When many excess parameters (both enthalpy and  
183 entropy parameters) are required to accurately fit the set of available thermodynamic data (see  
184 for example the work of de Koker et al. (2013) on the MgO-SiO<sub>2</sub> system), it is unclear whether



185 this reflects internal inconsistency in the calibration data, poor choice of functional forms for  
186 interpolation, or true complexity of the studied solutions.

## 187 *2.2 The computational approach*

188 Given gaps and shortcomings in experimental data, a promising way to refine and parameterize  
189 theories of phase properties is through numerical experiments, i.e. simulations of the individual  
190 and collective behavior of atoms via molecular dynamics or Monte Carlo methods. In these  
191 simulations,  $U$  of a given structure can be evaluated using classical interatomic potentials or  
192 quantum physics via density functional theory (DFT). Classical potentials contain empirical  
193 parameters and therefore hardly represent a replacement for experiments; on the other hand, DFT  
194 calculations are often referred to as ab initio techniques since in principle they do not depend on  
195 experimental input. In practice, the adjustment of some parameters is needed to represent the  
196 physical behavior of condensed structures. The cutoff energy, valence state, and cutoff radii for  
197 the partial waves are needed when building pseudo-potentials as well as in the projector  
198 augmented-wave (PAW) method to mimic all-electron wave functions (Kresse and Joubert,  
199 1999). The convergence test or parameterization methodology used to adjust these parameters is  
200 typically not fully objective (see the methodology of Garrity et al. (2014) and of Jollet et al.  
201 (2014)).

202 Technically, such convergence tests should be conducted for each studied structure, as the  
203 convergence behavior appears to be structurally/chemically dependent according to benchmark  
204 testing of pseudo-potentials applied to several unary and binary compounds of different  
205 crystallographic structures by Garrity et al. (2014). The evident popularity of DFT in the  
206 scientific literature as a numerical tool to understand and explore the thermo-physical behavior  
207 of condensed phases relies on an optimum tradeoff between accuracy, transferability and

208 computational efficiency, which has motivated compilation of optimal sets of these parameters.  
209 Examples of such libraries for several elements of the periodic table can be found, for example,  
210 in the recent work of Jollet et al. (2014) for the PAW method and in the work of Garrity et al.  
211 (2014) for the generation of pseudo-potentials. It is to be noted that there could exist cases where  
212 pseudo-potentials are used under conditions of strong compression (perhaps at terrestrial mantle  
213 pressures) that induce very short bond lengths. In this scenario, the transferability of the pseudo-  
214 potentials and the final precision of the resulting calculations could be severely impaired, which  
215 is not a topic frequently covered in the geochemical literature. As an example, the frozen core  
216 approximation typically used when building pseudo-potentials failed in the work of Pickard and  
217 Needs (2011) where they studied the behavior of solid lithium at pressures up to 2000 GPa. In  
218 this case, the 1S core electrons overlap on neighboring atoms under strong compression. For  
219 these reasons, very high quality pseudo-potentials in which all three electrons are treated  
220 explicitly are required in this case.

221 Moreover, the parameterization of a correlation energy function (Perdew and Wang, 1992) for  
222 charged bosons and fermions is required to account for the non-interacting electron-gas  
223 hypothesis of DFT; this is done, for example, by the quantum Monte Carlo method of Ceperley  
224 and Alder (1980).

225 Static calculations allowing the evaluation of the equilibrium  $V$ , the bulk modulus and its  
226 pressure derivative for various structures in their ground state are often used to determine the  
227 validity of DFT calculations through comparison with available experimental data. Wentzcovitch  
228 et al. (2010) presented an extensive literature review of DFT calculations applied to mantle  
229 minerals and highlighted the importance of correcting ground state calculations to account for  
230 thermal effects even with 300 K experimental data. Semi-classical lattice dynamic methods such

231 as the quasi-harmonic approximation (Born and Huang, 1969) introduce the effect of atomic  
232 vibrations in the description of the Helmholtz energy, providing a useful and relatively robust  
233 method of estimating vibrational entropy and enthalpy for  $P$ - $T$  ranges relevant to Earth's mantle  
234 (Wentzcovitch et al. 2010). For solids, the phonon dispersion needed to evaluate the vibrational  
235 excess energy is obtained from density-functional perturbation theory (Baroni et al. 2001).

236 The effect of  $T$  on thermo-physical properties can be accounted for more explicitly, up to and  
237 above the melting point, using Molecular Dynamics (MD). In this simulation scheme, Newton's  
238 equations of motion are solved iteratively to describe the paths of individual atoms in a structure  
239 (solid, liquid or glass) subject to periodic boundary conditions by determining the forces acting  
240 on them. The exploration of thermodynamic ensembles such as the isobaric-isothermal (NPT)  
241 ensemble and the canonical (NVT) ensemble requires the implementation of a thermostat,  
242 typically the Nosé-Hoover thermostat (Evans and Holian, 1985). Forces acting on atoms required  
243 in MD simulations can be described by either classical interatomic potentials or DFT.

244 The need for and promise of such numerical experiments is clear, and reviewing the entire  
245 literature of both classical and ab initio MD simulations relevant to Earth's mantle is beyond the  
246 scope of this work. However, conclusions of recent AIMD studies that "*they are now capable of*  
247 *treating chemically rich systems with multiple stable phases and extensive solution*" (de Koker et  
248 al. 2013) motivate the present discussion. As explained by Schubert et al. (2001), the earth's bulk  
249 composition is constrained, in part, by the cosmic material believed to be the primordial building  
250 blocks of the earth, e.g. the CI chondrites. It is generally accepted that the mantle is mostly  
251 peridotite and that the core is mostly iron as discussed in detail by McDonough and Sun (1995).  
252 The original pyrolite model of Ringwood (1962), refined by McDonough and Sun (1995),  
253 informs us that the earth's mantle is dominated by the  $\text{SiO}_2$ - $\text{MgO}$ -" $\text{FeO}$ "- $\text{Al}_2\text{O}_3$ - $\text{CaO}$  5-major

254 component system. However, the  $\text{Cr}_2\text{O}_3$ ,  $\text{Na}_2\text{O}$ ,  $\text{TiO}_2$ ,  $\text{MnO}$  and other minor as well as volatile  
255 components may also drastically alter the thermo-chemical properties of the different mantle  
256 phases, the stability relations among those phases, and hence the geochemical evolution of the  
257 silicate Earth. Moreover, classical thermodynamics shows that phase equilibria are sensitive to  
258 small shifts in any of the free energy functions of the system. A difference of  $\sim 10^2$  J/mol (1  
259 meV/atom) in the thermodynamic description of a phase can shift a eutectic reaction by  $\sim 10^2$  K  
260 or change the computed phase assemblage for imposed equilibrium conditions.

261 It is therefore important to explore whether such precision has been (or can be) achieved. We  
262 argue here that the chemically-rich terrestrial mantle silicate system and all its associated  
263 potentially stable phases cannot be studied accurately with the presently available numerical  
264 tools. The basis for this claim lies both in the theoretical and practical limits of standard DFT  
265 itself — including difficulty handling van der Waals type forces (Vuilleumier et al., 2009),  
266 hydrogen bonds (Dyer and Cummings, 2006), and compositions rich in transition metals such as  
267 Fe (requiring hybrid functionals to handle strongly correlated d-orbitals, e.g. Alfredsson et al.,  
268 2005) — as well as the theoretical and practical limits of molecular dynamic simulations based  
269 on DFT-derived forces. MD simulations endeavor to define the thermo-physical behavior of  
270 macroscopic, real solutions through the behavior of a limited number of atoms sampled for a  
271 limited period of time. It is of fundamental importance to analyze how picosecond ( $10^{-12}$  s)  
272 numerical experiments on hundreds of atoms ( $\sim 10^{-22}$  mol) with periodic boundary conditions can  
273 be scaled up to represent the macroscopic behavior of condensed phases (moles) which can be  
274 observed evolving towards equilibrium on timescales of hours ( $\sim 10^3$  s) to years ( $\sim 10^7$  s).

275 Based on ideas from classical thermodynamics and results presented in the literature, this work  
276 examines the likely limitations of current state-of-the-art atomistic simulations of silicate melts.

277 We believe that a critical review is needed at this time, with several goals in mind. We hope to  
278 motivate the reassessment of important systems using both classical MD and AIMD simulations,  
279 to identify key experimental investigations that can better constrain theories, to encourage  
280 progress in the definition and parameterization of interatomic potentials, to encourage authors to  
281 report more of their statistical analysis and resulting errors when performing MD simulations,  
282 and to suggest the need for hybrid strategies that build on the power of MD while overcoming  
283 computational limitations. We focus on three aspects of MD simulations that have received  
284 relatively limited scrutiny: 1) general kinetic limitations inherent to the classical treatment of  
285 atomic motions under periodic conditions; 2) the consequence of the extreme cooling/heating  
286 rates of MD simulations for potential phase transitions; and 3) the homogenization times  
287 sufficient for liquid structures, especially at low  $T$  or high  $P$ , to equilibrate the local chemical  
288 environments of the atoms, quantified by the amount of short range order.

### 289 **3. Configurational entropy of mixing in the chemical sense**

290 It is important at the outset to define some important concepts underlying the idea of chemical  
291 SRO. First, let us attempt to define the configurational entropy of mixing, i.e. the entropy  
292 (distinct from and in addition to vibrational entropy) arising from the availability of numerous  
293 distinct arrangements of atoms in a mixed solution phase. Figures 1a and 1b show the theoretical  
294 range of possible  $S_{mix}$  as a function of composition in the binary atomic systems Mg-O and Si-O.  
295 The reference basis is set by the total number of atoms needed to define the MgO (2 atom basis)  
296 and the SiO<sub>2</sub> (3 atom basis) pure oxides. At the stoichiometric compositions MgO and SiO<sub>2</sub>,  $S_{mix}$   
297 can be no smaller than  $0 \text{ J mol}^{-1} \text{ K}^{-1}$  (inverted triangles) — which describes the limit of only one  
298 possible configuration of the atoms — and no larger than the ideal maximum (stars)  $S_{mix}$   
299 corresponding to random permutation of all the atoms. The minimum is achieved for solid MgO-

300 periclase and SiO<sub>2</sub>-low quartz at 0 K, where the equilibrium concentration of all defects with  
301 positive substitution energy goes to zero. The maximum  $S_{mix}$  for each oxide implies that the  
302 internal structure of the solution is equivalent to that of an ideal gas; the atoms are not interacting  
303 with each other and there is no correlation between atomic positions in the solution. Chemical  
304 interactions between cations and anions in liquid MgO and SiO<sub>2</sub> must necessarily yield  $S_{mix}$  for  
305 each system between these two limits.

306 In Figure 2,  $S_{mix}$  of the MgO-SiO<sub>2</sub> system is presented for two different reference states. In  
307 Figure 2a, Mg, Si and O atoms define the reference state. The maximum configurational entropy  
308 in this case is 20 J mol<sup>-1</sup> K<sup>-1</sup>, but this can only be equated to  $S_{mix}$  along the binary if the  
309 configurational entropies of pure MgO and SiO<sub>2</sub> are both equal to 0. On the other hand, if fully  
310 disordered MgO and SiO<sub>2</sub> are used as reference state (figure 2b), then a maximum of 6.05 J mol<sup>-1</sup>  
311 K<sup>-1</sup> can be reached. Finally, if MgO and SiO<sub>2</sub> are treated as associates a configurational entropy  
312 of mixing of 5.76 J mol<sup>-1</sup> K<sup>-1</sup> is obtained at the equimolar composition. This is the so-called  
313 ideal  $S_{mix}$  relative to the oxide end members. In this case, MgO and SiO<sub>2</sub> entities are assumed to  
314 be indistinguishable in the chemical sense. For silicate melts, first nearest neighbors are  
315 exclusively strong cation-anion energetic interactions. Therefore, it is more reasonable to assume  
316 that the amplitude of  $S_{mix}$  will be in the range of values obtained from the associate model.

317 These important notions regarding the limiting behavior of  $S_{mix}$  appear to have been neglected in  
318 recent AIMD studies. A maximum  $S_{mix}$  of about 18 J mol<sup>-1</sup> K<sup>-1</sup>, proposed by de Koker et al.  
319 (2013) using their coordination state model (de Koker and Stixrude, 2009) requires both that the  
320 configurational entropies of pure MgO and SiO<sub>2</sub> liquid relative to the atoms are close to zero and  
321 that the mixing of these two liquid oxides results in a completely randomized solution ( $S_{mix}$  of an  
322 ideal gas), which seems unreasonable. While cations in different coordination states yield more

323 possible configurations than fixed coordination numbers, the resulting number of configurations  
324 surely should not approach that of totally random and uncorrelated atomic positions. It should be  
325 emphasized here that AIMD simulations do not provide direct access to the entropy of the  
326 studied structure. As explained by de Koker et al. (2013),  $S_{mix}$  of the liquid phase in the MgO-  
327 SiO<sub>2</sub> system must be parameterized by fitting experimental observations such as the 1 atm  
328 liquidus temperatures for periclase and forsterite together with the immiscible liquid field.

329 Another clear indication that the general concept of  $S_{mix}$  and its limiting behavior are not  
330 consistently applied in the MD simulation literature is provided by Karki et al. (2013), where  
331 configurational entropy is evaluated using the Adam-Gibbs theory (Adam and Gibbs, 1965) to  
332 represent the  $T$ -dependence of the melt viscosity obtained from the same AIMD simulations. The  
333 Adam-Gibbs theory gives a maximum  $S_{mix}$  of 2.5 J mol<sup>-1</sup> K<sup>-1</sup> close to  $X_{SiO_2} = 0.4$  (Karki et al.  
334 2013) (Figure 3). The amplitude of the maximum  $S_{mix}$  inferred from viscosity is more than 7  
335 times less than that inferred from the coordination state model applied to the same simulations.  
336 This discrepancy could be an issue with the simulations themselves (the results for viscosity at  
337 different  $T$  might not be comparable and might lead to anomalous fits to Adam-Gibbs theory) or  
338 it might be an issue with the thermodynamic model and its definition of configurational entropy.  
339 Obviously, the underlying assumptions leading to the Adam-Gibbs theory, i.e. that the relaxation  
340 time is a function of the macroscopic configurational entropy and that the viscosity follows an  
341 Arrhenius-like behavior, must be valid in order to estimate this thermodynamic property from  
342 viscosity data alone, as discussed by Dyre et al. (2009).

343 These observations suggest a need to test the precision of the AIMD simulation results and the  
344 resulting thermodynamic model of de Koker et al. (2013) to see whether it accurately predicts  
345 experimental measurements for this system not considered in the parameterization process,

346 especially in the SiO<sub>2</sub>-rich region. Available data include the SiO<sub>2</sub> activity in silicate melts at 1  
347 atm (Kambayashi and Kato, 1983, 1984); the evolution of the consolute point of the miscibility  
348 gap in the silica-rich region as a function of  $P$ , indirectly measured by Hudon et al. (2004, 2005);  
349 and the liquidus surface for crystallization of coesite in this system at 5 GPa, measured by Dalton  
350 and Presnall (1997).

#### 351 **4. General considerations about kinetic limitations of MD simulations**

##### 352 *4.1 Immiscibility*

353 The most recent AIMD study of the MgO-SiO<sub>2</sub> system by de Koker et al. (2013) serves to  
354 introduce some technical issues in exploring the partition function of a multicomponent solution  
355 by MD simulation. First, note that equilibrium miscibility gaps are determined solely by the  
356 excess properties of the considered solution. Figure 4 shows raw enthalpy of mixing ( $H_{mix}$ ) data  
357 from AIMD simulations of de Koker et al. (2013) at 2000 K and 0 GPa for different silicate  
358 melts. The “s” shape of the enthalpy of mixing (which is also obtained at 3000 K) is typical for a  
359 solution that tends to form immiscible liquids. Combined with the total entropy of mixing  
360 expression in the de Koker et al. (2013) thermodynamic model, this enthalpy of mixing induces a  
361 small miscibility gap in the range  $0.8 \leq X_{SiO_2} \leq 1.0$  (Figure S15 of their work). Comparison of the  
362 predicted miscibility gap and experimental results of Hageman and Oonk (1986) and Ol’shanskii  
363 (1951) shows that AIMD simulations and thermodynamic model of de Koker et al. (2013)  
364 greatly underestimate the width of the solvus, although the consolute  $T$  at 1 atm is close to that of  
365 Hageman and Oonk (1986). Nevertheless, both the experimental and predicted miscibility gaps  
366 show that MgSi<sub>5</sub>O<sub>11</sub>, one of the simulated compositions, should be inside the two-liquid region.  
367 Simulation of this composition, however, yields a single liquid phase with a positive enthalpy of



368 mixing even though the equilibrium state should be a two-liquid assemblage. This is of concern,  
369 whether or not this composition lies inside the spinodal region: if inside the spinodal region, the  
370  $\text{MgSi}_5\text{O}_{11}$  melt should spontaneously decompose into silica-rich and MgO-rich liquids, which  
371 clearly did not occur. If outside the spinodal region, the  $\text{MgSi}_5\text{O}_{11}$  melt is metastable and should  
372 have been studied using an initial configuration that accounts for the immiscible behavior. In  
373 either case, the AIMD simulations do not reflect the true equilibrium state of this melt.

374 This is a clear example showing that AIMD simulations, applied to silicate liquids using short  
375 simulation times (3-6 ps) and  $\sim 100$  atoms per studied supercell, may not explore the entire  
376 partition function. There are several examples in the literature that show that MD simulations are  
377 neither designed nor capable of ensuring complete sampling of partition functions, even after  
378 long simulation runs originating from a given initial configuration. The work of Schön et al.  
379 (2006) on ab initio computation of low- $T$  phase diagrams exhibiting miscibility gaps show that,  
380 at elevated  $T$ , the locally ergodic regions of phase space are not necessarily localized around a  
381 single local minimum (Helmholtz energy surface), but may contain many such local minima.  
382 Likewise, Zhang (2011), performing AIMD simulations of MgO-SiO<sub>2</sub> liquids, found a standard  
383 deviation in enthalpy among runs using different initial configurations fully as large as the  
384 amplitude of the excess enthalpy of solution reported by de Koker et al. (2013). This does not  
385 imply that the simulations are non-ergodic; on the contrary it emphasizes the highly probable  
386 existence of several locally-ergodic minima that have to be explored in order to define the  
387 equilibrium state of the system correctly.

#### 388 *4.2 Evolution of Short Range Order (SRO)*

389 Obtaining, in a simulation, a reasonable sampling of equilibrium configurations with the correct  
390 structural and physical properties requires the achievement of the correct degree of short-range

391 order. We suggest that important insights concerning the procedures generally used in MD  
392 simulations of liquids and the extent to which they explore the SRO energy landscape can be  
393 found by examining well-documented studies of analogous phenomena surrounding a well-  
394 known solid-solid order-disorder phase transition in a simple binary system. From an internal  
395 chemical structure perspective, an order-disorder phase transition is highly relevant to our  
396 discussion of liquids, since it is driven by the vanishing of short-range chemical order; that a  
397 solid has long-range chemical order and a liquid does not is unimportant for this discussion. In  
398 particular, let us consider the example of the Au-Cu system, where the equimolar composition  
399 undergoes a first-order phase transition from a face-centered cubic (FCC)-L1<sub>0</sub> ordered structure  
400 to a FCC-disordered solution at a critical temperature  $T_{O \rightarrow D}$  of  $\sim 685\text{K}$  (Cao et al. 2007), which is  
401 about  $0.58T_{liquidus}$  at this composition. Similar order/disorder phase transitions also occur for the  
402 Cu<sub>3</sub>Au and the CuAu<sub>3</sub> FCC-L1<sub>2</sub> ordered structures in this system.

403 All thermodynamic models that allow the representation of such a phase transition — e.g., the  
404 cluster variation method (Kikuchi, 1951) or the cluster site approximation (Oates and Wenzl,  
405 1996; Oates et al. 1999) — predict the presence of some residual SRO above the phase  
406 transition. This SRO decreases to some extent with further increase of  $T$  beyond the transition  
407 point, whereas SRO in the disordered phase is expected to increase with increasing  $P$  for this  
408 system, at least in the low compression regime. As pointed out by Franzblau and Gordon (1967),  
409 the initial slope of the evolution of  $T_{O \rightarrow D}$  as a function of  $P$  is defined by the Clapeyron

410 equation:  $\frac{\partial T_{O \rightarrow D}}{\partial P} = \frac{T_{O \rightarrow D} \Delta V_{O \rightarrow D}}{\Delta H_{O \rightarrow D}}$ . A thermodynamic literature review of the Au-Cu system

411 (Okamoto *et al*, 1987) then predicts that  $\frac{\partial T_{O \rightarrow D}}{\partial P} > 0$  at 1 atm; indeed  $T_{O \rightarrow D}$  increases by  $\sim 10\%$   
412 from 1 atm to 2 GPa for Cu<sub>3</sub>Au (Franzblau and Gordon, 1967). Under strong compression, the

413 Clapeyron relation cannot be applied easily because  $\Delta V_{O \rightarrow D}$  and  $\Delta H_{O \rightarrow D}$  at elevated  $P$  are not  
414 available in the literature. Franzblau and Gordon (1967) also argue that the energy barrier  
415 (kinetic limitations) to disorder the  $\text{Cu}_3\text{Au}$  structure in this case should increase with  $P$ .

416 When starting from a perfectly ordered structure, MD simulations of FCC AuCu on a periodic  
417 lattice do not allow identification of the transition temperature even for the slowest heating rates.  
418 Experimentally, the order/disorder transitions in the Au-Cu system probably nucleate by  
419 macroscopic heterogeneous mechanisms such as surface segregations or antiphase boundaries, as  
420 discussed by Hayoun et al. (1998) in  $\text{Cu}_3\text{Au}$ . Numerically, however, the atomic permutations  
421 needed to modify the chemical order and induce this order/disorder transition face major kinetic  
422 limitations: to permute two nearest neighbor atoms in a periodic defect-free lattice requires an  
423 energetically unfavorable Frenkel defect (substitutional vacancy + interstitial atom). When  $T$   
424 increases, the structure generally expands, reducing the energy cost of a Frenkel defect. As the  
425 energetic cost of a Frenkel defect approaches  $k_B T$ , atoms start to permute, but this is typically far  
426 above the observed critical  $T$  of solid-solid order-disorder transitions. As defects accumulate in  
427 this  $T$  range, the solid structure loses mechanical stability and undergoes mechanical fusion. For  
428 FCC gold, e.g., mechanical fusion occurs in MD simulations using the interatomic potential  
429 parameterized by Lee et al. (2003) at around  $1.3T_{\text{liquidus}}$  (compare to  $T_{O \rightarrow D} \approx 0.58T_{\text{liquidus}}$  in AuCu).  
430 This implies that MD simulations can explore a superheated FCC structure up to this  $T$ .

431 Above the mechanical fusion temperature, liquid state MD simulations of metallic systems  
432 should not be too sensitive to the initial configuration because permutations of first nearest  
433 neighbor pair fractions that define the chemical order are energetically inexpensive, even when  
434 performed in a random motion fashion. For the same reason, it is a common practice to use high  
435  $T$  and large volume to melt initial solid structures in MD simulations. However, applying high  $P$

436 to mechanically melted structures should be done with care: imposing high  $P$  increases the  
437 energetic cost of modifying short-range chemical order, as discussed above. In this case, MD  
438 simulations might not be able to explore enough chemical configurations to populate the  
439 partition function adequately. Monte Carlo simulations (Metropolis et al. 1953) are therefore  
440 preferred when thermodynamic properties based on statistical analysis are to be obtained for  
441 solid and liquid structures, as the technique allows one to span the partition function much more  
442 efficiently.

443 The analogy of the energetic barrier kinetic limitation to disordering of solid structures extends  
444 to silicate melts as follows. It is evident that silicate melts are defect-rich, low-symmetry  
445 structures less prone to this problem. However, when  $P$  increases or  $T$  decreases, there is no  
446 general rule to predict how the kinetic energy barrier between two states of internal energy  
447 compares to  $k_B T$  (even if the equilibrium  $U$  difference between the states is small), especially  
448 considering the fact that the atoms follow a random walk inside the simulation box (Brownian  
449 motion). In other words, atoms are not specifically seeking minimum energy transition states.  
450 This is an important issue that was raised in early MD simulations works (see for example Alder,  
451 1964). For this reason, it is of fundamental importance to start several MD simulations with  
452 initial configurations differing in their SRO to test how the initial structure affects the derived  
453 thermo-physical properties. To our knowledge, such a systematic study for silicate melts under  
454 pressure has not been published in the literature yet. However, the provocative MD study of  
455 MgO-SiO<sub>2</sub> melts by Zhang (2011) justifies further work; they examine three distinct initial  
456 configurations for each composition and obtain average properties with standard deviations from  
457 10 to 50 kJ/mol. Even though the total simulation time might have been too small for silica-rich  
458 melts to adequately relax their structures (only 6 ps), it appears evident from this study that the

459 uncertainty due to initial configuration in the evaluation of a mixing property could be of the  
460 same order of magnitude or even larger than the property itself.

### 461 4.3 *Self-diffusivity*

462 Another kinetic limitation inherent to the chemical nature of MD simulated silicate melts is the  
463 limited self-diffusivity of the various species. In the NVT or NPT ensembles that are typically  
464 explored in earth science studies, the homogenization or evolution toward the equilibrium state  
465 of simulated melts is ensured solely by self-diffusion of the atoms (the system is not sensitive to  
466 chemical potentials in these ensembles). If the total MD simulation time at a given applied  $P$  and  
467  $T$  is too short compared to self-diffusion time-scales, then the system will not have enough time  
468 to reach the equilibrium state. The severity of this kinetic limitation in the approach to  
469 equilibrium for high  $P$  (or low  $T$ ) silicate melts can be characterized using self-diffusivities from  
470 the MD simulations themselves. Ghosh and Karki (2011) presented AIMD simulations of  
471  $\text{Mg}_2\text{SiO}_4$  melt at high  $P$  in order to evaluate self-diffusivity and viscosity. These authors noted  
472 two simulations, at compressed volumes of  $0.6V_X$  (3000 K and 67 GPa;  $V_X$  is the volume at 3000  
473 K and 0 GPa) and  $0.5V_X$  (4000 K and 159 GPa) that displayed glass-like rather than liquid-like  
474 dynamics, in which mean square displacements remained below  $10 \text{ \AA}^2$  for all species even for  
475 runs of 300 ps. This observation makes perfect sense from a self-diffusivity perspective. The  
476 self-diffusivity of Si is about  $5 \times 10^{-11} \text{ m}^2/\text{s}$  under these conditions (Ghosh and Karki, 2011). To  
477 permute two Si-Si second nearest neighbor cations (distance of  $3 \text{ \AA}$  according to Karki et al.,  
478 2007), a time of about 300 ps is required:

$$479 \quad (6 \cdot D_{\text{Si-Si}} \cdot t)^{1/2} = \left( 6 \cdot 5 \times 10^{-11} \frac{\text{m}^2}{\text{s}} \cdot 300 \times 10^{-12} \text{s} \right)^{1/2} = 3 \text{ \AA} \quad (1)$$

480 Indeed, 300 ps is the maximum simulation time the authors used. Under these simulation  
481 conditions, the SRO defined by 2NN pairs remains almost constant. Similar kinetic limitation  
482 examples can be found in the work of Karki et al. (2007) and Lacks et al. (2007) for liquid SiO<sub>2</sub>.  
483 In these studies, equilibrium conditions inducing kinetic limitations can be identified by looking  
484 at the slope of log-log plots of the mean square displacement as a function of time, which should  
485 be equal to one in the diffusive regime. The systematic application of such a “kinetic” test to  
486 identify the state (i.e. liquid or glassy) of the supercell in MD simulations can be found, for  
487 example, in the work of Morgan and Spera (2001a, 2001b). In general, however, MD simulations  
488 do not present such evaluations of the kinetic relaxation, even when computing transport  
489 properties.

490 Another clear example showing the importance of self-diffusivity on homogenization and  
491 relaxation of simulated structures in MD can be observed for liquid MgO. In MgO melts, the  
492 self-diffusivity of Mg is  $\sim 8 \times 10^{-9} \text{ m}^2/\text{s}$  at 3000 K and 0 GPa according to Karki et al. (2006) (a  
493 factor of 160 higher than Si in liquid SiO<sub>2</sub> at 3000 K). There is a clear agreement (Karki et al.  
494 2006, 2013; Lacks et al. 2007) that MD simulations of 2-5 ps are sufficient to generate accurate  
495 results ( $d_{\text{Mg-Mg}} \approx 3 \text{ \AA}$ ), which is consistent with the application of Equation 1. Similarly, Martin et  
496 al. (2012) set a self-diffusion cut-off of  $10^{-9} \text{ m}^2/\text{s}$  in their study of the 1-bar eutectic melt in the  
497 system CaAl<sub>2</sub>Si<sub>2</sub>O<sub>8</sub>-CaMgSi<sub>2</sub>O<sub>6</sub> in order to ensure ergodicity, which induces a mean travel  
498 distance for oxygen of 3.2 Å (twice the anion-cation separation) in 50 ps, which appears to be  
499 equivalent to the present criterion.

500 The kinetic limitations to achievement of equilibrium SRO presented in this section could be the  
501 main cause of the apparently non-equilibrium results presented in Figures 3 and 4.

502 *4.4 Effect of the cooling rate in MD simulations on the resulting studied structure*

503 Silica glasses were studied by Vollmayr et al. (1996) using MD simulations. The authors  
504 mentioned that the studied cooling rates were between  $1.14 \times 10^{15}$  and  $4.44 \times 10^{12}$  K/s, which are  
505 many orders of magnitude larger than laboratory rates. In the work of de Koker et al. (2008) on  
506  $\text{Mg}_2\text{SiO}_4$  liquid, the “slow cooling rate” of their AIMD simulations is  $1.0 \times 10^{15}$  K/s (3000 K in  
507 3000 fs), which is again about 10 orders of magnitude higher than typical cooling rates used to  
508 produce silicate glasses. As an example, Revcolevschi (1976) estimated the cooling rate of an  
509 eutectic NiO-CaO liquid in a splat quenching apparatus to be  $2.6 \times 10^5$  K/s.

510 According to the equation parameterized by Vollmayr *et al.* (1996) for the glass transition  
511 temperature of silica, the cooling rate of de Koker et al. (2008) would induce a glass transition  
512 for pure silica above 4000 K at atmospheric pressure. Furthermore, increasing pressure for a  
513 given cooling rate will increase the glass transition temperature (see Martin et al. 2012). This  
514 glass transition temperature for silica is higher than any observable glass transition temperature  
515 (higher even than the liquidus temperature) but the important point is that structural relaxation  
516 may not continue below 4000 K or higher in these simulations, and the state of order recorded at  
517 lower temperature seems unlikely to represent equilibrium.

## 518 **5. Large-scale classical MD simulations vs. small-scale AIMD simulations**

519 Classical MD simulations exploit the fast evaluation of forces to achieve much longer simulation  
520 times and numbers of simulated atoms. Each of these advantages contributes to the higher  
521 precision of classical MD in distinct ways. Concerning the number of atoms, Ghiorso and Spera  
522 (2010) presented all the advantages of using large scale simulations to access the most accurate  
523 thermo-physical properties of silicate melts. It is clear that large MD simulations allow better  
524 statistics: sampling over more atoms yields more countable first nearest neighbors, second  
525 nearest neighbors, etc. For a closed packed FCC-like structure, with 70 atoms (typical of AIMD

526 simulations), there would be 420 distinct 1NN pairs to analyze; for a system of  $10^4$  atoms  
527 (typical of classical MD), there would be  $6 \times 10^4$  1NN pairs. Furthermore, number of atoms  
528 contributes to uncertainty in evaluating both  $T$  (induced by the thermostat) and  $P$ : McQuarrie  
529 (2000) reported that this uncertainty scales as  $N^{1/2}$ . Finite-size effects caused by a small number  
530 of atoms are often argued to be insignificant in AIMD studies (de Koker and Stixrude, 2009;  
531 Karki et al. 2006, 2007), although the evidence supporting this claim is generally difficult to  
532 evaluate quantitatively. On the contrary, several classical MD simulations quantified this effect  
533 (Horbach et al. 1996; Saksengwijit and Heuer, 2007; Yeh and Hummer, 2004; Zhang et al.  
534 2004). The general consensus of these studies is that, while thermodynamic properties appear to  
535 be insensitive to system size, dynamic properties such as self-diffusivity and viscosity can be  
536 quite sensitive. However, studies that suggest limited system size is adequate for assessing  
537 thermodynamic properties are all applied to pure chemical systems. In multicomponent systems,  
538 the number of different types of 2NN pairs to explore and the swap distances needed to exchange  
539 different cations are much larger. The system size effects in multicomponent systems have not  
540 been comprehensively evaluated, either with regard to major components or dilute components.

541 Reaching the equilibrium state in a pseudo-unary system is fast (no kinetic limitations related to  
542 self-diffusivity), even when the high symmetry of a solid structure has to be broken in order to  
543 reach equilibrium. The classical solid-liquid interface MD simulation study of Soules et al.  
544 (2011) demonstrates this principle, showing that the phase boundary interface propagates much  
545 faster than atomic displacements caused by self-diffusivity. For pseudo-unary systems such as  
546 liquid  $\text{SiO}_2$ , dominated by indistinguishable tetrahedral entities, distinct chemical species make  
547 little contribution to configurational entropy. The situation might be slightly different when  
548 pressure is increased, as different entities become significant in the melt. As reported by Karki et



549 al. (2007), the Si-O coordination number shifts from 4 at ambient pressure to 6.5 at 150 GPa.  
550 Therefore, there might be equilibrium conditions for which distinctions between different stable  
551 entities have to be accounted for in the description of the configurational entropy of the solution.  
552 However, conversion among differently coordinated cations requires only small displacements of  
553 the oxygen anions and so coexistence of different coordination states probably does not imply a  
554 substantial increase in necessary simulation time. Hence, relatively short simulation times should  
555 allow exploration of the partition function for such unary/pseudo-unary systems. To evaluate  
556 dynamic properties, the shortest simulation time should allow at least one “equivalent” cation-  
557 cation permutation. For liquid silica, all authors in the literature (for both classical MD and  
558 AIMD studies) (Karki et al. 2007, 2013; Lacks et al. 2007) appear to agree that a simulation of  
559 0.3-1 ns is required to evaluate the viscosity and self-diffusivity ( $5 \times 10^{-11} \text{ m}^2/\text{s}$  at 3000 K and 0  
560 GPa) in liquid silica, which is about the minimum time to allow one Si-Si cation-cation  
561 permutation ( $\sim 3 \text{ \AA}$ ) (see equation 1).

562 For MD simulations of multicomponent systems, the situation is quite different. A sampling of  
563 2NN pair fractions sufficient to obtain the average equilibrium proportions, given the long  
564 distances between cations that arise (ex.:  $d_{\text{Si-Si}}$ ,  $d_{\text{Mg-Mg}}$ ,  $d_{\text{Mg-Si}} > 3 \text{ \AA}$  according to Zhang (2011) for  
565  $\text{MgSiO}_3$  melt at 0 GPa and 4000 K), should require a simulation of at least several ns, especially  
566 for silica-rich melts (based on self-diffusivity of Si in liquid  $\text{SiO}_2$  at 3000 K). In this case, the  
567 thermo-physical data required to parameterize a silicate melt EOS would be accurately obtained,  
568 as several cation-cation permutations would occur in the course of the simulation.

569 For shorter simulations, the accuracy depends on the energetic differences among configurations.  
570 If chemical ordering is a minor contribution to  $U$ , the average internal energy from MD

571 simulations may accurately describe the true equilibrium  $U$  of the system. Also in this case, the  
572 assumption of ideal  $S_{mix}$  (suitably defined) would be fully consistent with the MD simulations  
573 and the resulting Gibbs (or Helmholtz) energy would also be accurate.

574 In contrast, if chemical ordering significantly affects  $U$ , then it is unclear if MD simulations of  
575 several ns are sufficient to accurately parameterize an EOS of silicate melts. Imagine a system  
576 where short range order is important enough to result in  $S_{mix}$  appreciably lower than the ideal  
577 value. Energy differences between distinct configurations could be large. Energy fluctuations  
578 during a single MD simulation run with thousands of atoms traveling for several ns might not  
579 capture this particularity of the system. The average  $U$  obtained from this MD simulation might  
580 still approximate thermodynamic  $U$  (if the initial configuration reflects on average the  
581 equilibrium chemical ordering of the system) but properties related to fluctuations (instantaneous  
582 heat capacity, instantaneous thermal expansion, Grüneisen parameter, etc) would not be captured  
583 well. This is one reason why 2<sup>nd</sup>-order derivative parameters are generally better estimated from  
584 thermodynamic models fitted to many simulation points, rather than directly extracted from  
585 time-dependent simulations. In this context, starting with different initial solid structures (all  
586 ordered in some way, by definition) and subsequently melting them at high  $T$  to see if the liquid  
587 retains memory of the initial state (see Stixrude and Karki (2005) and de Koker et al. (2008)) will  
588 not diagnose these issues. This is a different test from starting with different liquid  
589 configurations with distinct chemical ordering to see if MD simulations present similar statistics,  
590 as proposed by Zhang (2011).

591 Finally, in intervals of phase space where small  $T$  or  $P$  changes induce large increases in  
592 equilibrium chemical ordering, significant extra time may be needed to reach equilibrium after

593 perturbation of  $T$ ,  $V$  or  $P$ . In this case, running MD simulations of silicate melts even of several  
594 ns (typical for classical MD) might not be sufficient. Consider an MD simulation performed  
595 close to a consolute point (whose critical  $T$  is not known *a priori*). Cooling (or quenching, in MD  
596 simulations) a disordered liquid structure across an equilibrium exsolution boundary would  
597 reveal the miscibility gap only for simulation times long enough to allow species to segregate. To  
598 homogenize a simulation box for these conditions (relevant phase transitions generally occur at  
599 low  $T$  / high  $P$ ) only by self-diffusivity and a random walk of atoms, long simulation times will  
600 be needed. With kinetic limitations (e.g., small free volumes available at elevated  $P$ ), it may not  
601 ever be reasonable to use MD simulations alone to explore the partition function in this case.  
602 Monte Carlo simulations should be better able to study phase transitions driven by evaluation of  
603 chemical order (e.g. miscibility gaps, fragile-to-ductile liquid transition, order-disorder transition  
604 in solids, etc.).

## 605 **6. Cautions concerning LDA pressure corrections in multicomponent liquid systems**

606 It is well known that neither of the common exchange-correlation functionals used in DFT, the  
607 local density approximation (LDA) and the generalized gradient approximation (GGA), yields  
608 precise matches to the volume and compressibility of all phases (Karki et al. 2001). There are,  
609 however, well-established corrections, for example an empirical pressure correction to account  
610 for the under-estimation of molar volume due to overbinding in LDA (van de Walle and Ceder,  
611 1999). Here we consider the magnitude of these corrections applied to multicomponent liquid  
612 systems and question whether uncertainty in such corrections is large enough to prevent useful  
613 definition of energetic mixing relations in materials of the Earth's mantle.

614 There is actually no physical theory specifying that LDA systematically induces overbinding or  
615 consequently underestimates equilibrium molar volume. Rather such a trend often arises in

616 comparisons between ab initio calculations and experimental data. Thus there is no fundamental  
617 understanding of how the error varies with composition or structure. Furthermore, in  
618 consideration of oxide solids, Wentzcovitch et al. (2010) argued that the error is simply a matter  
619 of applying static (0 K) LDA calculations to 300 K conditions. Combining quasi-harmonic  
620 theory for vibrational effects with static DFT calculations (0 K) yields an almost perfect match to  
621 experimental 300 K volumes of the solids examined in that study. In this scenario, the correction  
622 to the static energetic stability of a structure,  $\Delta U_{LDA \rightarrow exp.}$ ,

$$623 \quad \Delta U_{LDA \rightarrow exp.} = \int_0^{S^{vib.}} T dS - \int_{V_0}^{V_{exp.}} P dV \quad (2)$$

624 arises from vibrational entropy effects alone; no mechanical work contribution (second term in  
625 Equation 2) is needed. Likewise, AIMD simulations performed by Karki et al. (2006) for MgO  
626 and by Karki et al. (2007) for SiO<sub>2</sub> using LDA, incorporating entropy effects via molecular  
627 dynamics rather than quasi-harmonic theory, appear not to suffer from overbinding.

628 On the other hand, the same authors in more recent studies (de Koker et al. 2008, 2013; de Koker  
629 and Stixrude, 2009) present a different and incompatible observation regarding the precision of  
630 LDA-MD calculation of silicate melts, whose differences from experiment are once again  
631 attributed to LDA overbinding. In this treatment, the mechanical work term in Equation (2)  
632 rather than the vibrational entropy contribution to static LDA simulations is taken as the source  
633 of LDA errors. Thus, in MD simulations where vibrational entropy effects are folded into the  
634 internal energy contribution ( $T$  is mimicked by a thermostat), a systematic  $P$ -shift is applied to  
635 the  $P$ - $V$  EOS to account for overbinding effects (de Koker et al. 2008, 2013; de Koker and  
636 Stixrude, 2009). This  $P$ -shift is calibrated as a function of composition only for those

637 compositions where experimental solid volumes at 1 atm can be compared to NVT-MD  
638 simulations at the same volume. This  $P$ -shift function is taken to be independent of structure and  
639 applied to both solids and liquids of equal chemical composition. Moreover, the  $P$ -shift is  
640 assumed to be adiabatic (no entropy contribution) and independent of the elastic moduli of the  
641 material. The energy correction  $\Delta U_{emp}$  simply becomes (de Koker and Stixrude, 2009):

$$642 \quad \Delta U_{emp} = -P_{emp} (V - V_0) \quad (3)$$

643 Applying such a systematic correction to MD simulations is of uncertain validity and can have a  
644 large effect on derived properties such as enthalpy of mixing. First, it is unclear that LDA does  
645 systematically underestimate volume for geologically relevant systems when the zero-point  
646 anharmonic expansion is considered, especially for ionic solids or condensed phases with partly  
647 ionic chemical interactions, according to the study of Wentzcovitch et al. (2010). On the other  
648 hand, it appears evident from the calculations performed by Haas et al. (2009) on 60 cubic  
649 structures that LDA considerably underestimates molar volumes of solids at 0 K when compared  
650 to available experimental data. Second, if the degree of overbinding depends on the nature of  
651 chemical bonding in a material, then the changes in bonding character in silicate melts with  $P$  (de  
652 Koker et al. 2008, 2013) renders it highly improbable that a constant  $P$  correction term based on  
653 a 1 atm solid structure would be sufficient to account for the gradual shift toward differently  
654 coordinated and more metallic-like chemical interactions in increasingly compact liquids.

655 Finally, the application of the  $P$ -shift scheme to multicomponent systems may introduce serious  
656 errors. In their work, de Koker et al. (2013) estimated the  $P$ -shift as follows. Four anchor points  
657 — solids of known volume at 1 atm — are available (MgO, Mg<sub>2</sub>SiO<sub>4</sub>, MgSiO<sub>3</sub>, and SiO<sub>2</sub>). Of  
658 these, only SiO<sub>2</sub> is on the silica-rich side of the join. A quadratic polynomial fit to the four points

659 was assumed by de Koker et al. (2013) (Figure 5). This is reasonable, but a  $P$ -shift that increases  
660 linearly from MgO up to MgSi<sub>5</sub>O<sub>11</sub> (open circle in Figure 5) and then sharply decreases towards  
661 SiO<sub>2</sub> is equally plausible. This would lead to a “V-shape”  $P$ -shift, which resembles the V-shaped  
662 curves for several important mixing properties in de Koker et al. (2013).

663 Table 1 presents an estimate of volumes at 0 GPa,  $V_0^{FST}$ , using the finite-strain EOS of de Koker  
664 et al. (2013) for each MgO-SiO<sub>2</sub> melt composition, with the pressure correction term  $P_{emp}$  applied  
665 by these authors removed. That is, the  $V_0$  values in de Koker et al. (2013) are volumes at  $P$   
666 empirically corrected to 0 GPa, whereas here we fit instead the actual 0 GPa MD simulation  
667 results. A short FORTRAN algorithm and parameter files are provided in the supplement to  
668 implement the EOS of de Koker et al. (2013).

669 This exercise allows us to estimate the corrections to both internal energy,  $\Delta U_{emp}$ , and enthalpy,  
670  $\Delta H_{emp}$ , that should be applied for any functional form of the  $P$ -shift. Table 1 shows these results  
671 for all compositions studied by de Koker et al. (2013) using the quadratic composition  
672 dependence of  $P_{emp}$  and (in italics) results for MgSi<sub>5</sub>O<sub>11</sub> liquid using the V-shaped  $P$ -shift from  
673 Figure 5. This particular composition demonstrates the potential effect of the  $P$ -shift uncertainty  
674 on the resulting thermo-physical properties because 1) it lies inside the miscibility gap at low  $T$   
675 and 2) its  $P$ -shift must be predicted from a certain arbitrary functional form.

676 Figure 6 shows how such a systematic correction induces a non-negligible excess enthalpy  
677 correction  $h_{correction}^{xs}$  defined as follows:

$$678 \quad h_{correction}^{xs} = \left( \Delta H_{emp} \right)_{X_{SiO_2}} - \left[ X_{SiO_2} \Delta H_{emp}^{SiO_2} + (1 - X_{SiO_2}) \Delta H_{emp}^{MgO} \right] \quad (4)$$

679 with a shape mirroring the shape of the  $P$ -shift function. A series of MD simulations whose raw  
680 enthalpy values suggest a substantial negative enthalpy of mixing (-8 kJ/mol) could be  
681 transformed into a positive enthalpy of mixing entirely through the application of the  $P$ -shift. In  
682 multicomponent melts, this could potentially invert the tendency to favor heterogeneous 2NN  
683 pairs, favoring Mg-Mg and Si-Si pairs. The internal structure and  $S_{mix}$  should be modified in turn,  
684 which is not accounted for in this simple correction scheme. Thus, while a small and constant  
685 energetic LDA overbinding correction might be helpful in unary systems, extension of this  
686 principle to multicomponent systems and to phases with different chemical bonding character is  
687 unsupported. Moreover, differences among functions that might define the composition  
688 dependence introduce differences in enthalpy corrections as large as the enthalpy of mixing  
689 itself. Such uncertainties might lead to excess properties that are not accurate enough to predict  
690 phase equilibria.

691 Besides the uncertain composition dependence of the LDA correction, there is direct evidence  
692 that it also depends on the structure of silicate phases and therefore on the volume applied to  
693 liquid silicates. Hamann (1996) showed that LDA-based DFT predicts that stishovite is slightly  
694 more stable than quartz ( $\Delta U_{Quartz \rightarrow Stishovite} = -0.05 eV$ ) in the ground state at 0 GPa. Experimental  
695 calorimetry by Holm et al. (1967) and Akaogi and Navrotsky (1984), however, show that quartz  
696 is the more stable phase ( $\Delta U_{Quartz \rightarrow Stishovite} \approx +0.52 eV$ ). The most likely explanation is that the  
697 error in LDA is not invariant to coordination number change and the energetics of the 4-fold  
698 (quartz) to 6-fold (stishovite) transition do not benefit from the same error cancellation as EOS  
699 curves applied to a fixed crystal structure.

700 There are other concerns with the use of LDA for multicomponent mantle materials. An  
701 electronic entropy contribution to the Helmholtz or Gibbs energy may need to be considered at

702 high  $T$  (de Koker et al. 2013). Yet it is known that the band gap in LDA calculations is typically  
703 too small, up to 40% in several cases (Perdew, 1985). For MgO-periclase, Tran and Blaha (2009)  
704 predicted, using LDA, a band gap energy of 4.70 eV compared to the experimental value of 7.83  
705 eV. Anisimov and Kozhevnikov (2005) explained how the one-electron eigenvalues in LDA  
706 calculations are Kohn–Sham energies that do not directly correspond to excitation energies. In  
707 theory, such a difference in the energy band gap could significantly alter the energetic properties  
708 of MgO under extreme conditions of temperature only (between about 54500 K and 91000 K if  
709 the band gap scales to  $k_B T$ ). In this context, the presence of an electronic contribution in the  
710 definition of the Gibbs energy of MgO-SiO<sub>2</sub> melts in the 3000 – 10000 K range as reported by de  
711 Koker et al. (2013) is surprising, even if the band gap was reduced upon melting and mixing with  
712 SiO<sub>2</sub> in various amounts. This electronic contribution should be taken with care as it could easily  
713 be an artifact from the LDA when predicting the band gap of these melts. De La Pierre et al.  
714 (2011) studied in detail the performance of 6 functionals in the simulation of the vibrational and  
715 dielectric properties of solid forsterite. They concluded that the LDA is the worst performing  
716 functional. A direct observation of the imprecision of electronic band gaps from LDA can be  
717 found in the work of Cococcioni et al. (2003), which predicts a metallic ground state for solid  
718 fayalite, contrary to experimental evidence that indicates an insulating, possibly Mott-Hubbard,  
719 behavior.

720 For these reasons, exploration of the accuracy of GGA-based MD simulations of silicate melts at  
721 high  $P$  and  $T$  seems warranted. Moreover, it is well documented (Hautier et al. 2012) that  
722 transition metals such as Fe, Mn and Ni in oxide materials require the use of GGA and other  
723 corrections such as the Hubbard parameter (Anisimov et al. 1991) to account for strong  
724 correlation of electrons in the d-states (Cococcioni and de Gironcoli, 2005). This will be an issue



725 when trying to extend binary MgO-SiO<sub>2</sub> calculations and compatibly merge them with ternary  
726 and higher systems, especially when incorporating iron, the third-most abundant cation in Earth's  
727 mantle. Spera et al. (2011) pointed out important differences in thermo-physical properties for  
728 liquid MgSiO<sub>3</sub> at high *T* and *P* between corrected LDA MD simulations (Stixrude and Karki  
729 2005) and GGA MD simulations (Wan et al. 2007). There are other differences between these  
730 two studies (e.g. the Car and Parrinello (1985) scheme vs. the Born and Oppenheimer (1927)  
731 approximation), so the difference may not be entirely LDA vs. GGA. Spera et al. (2011)  
732 concluded that key experimental measurements are needed to identify the most accurate  
733 approach in this specific case.

## 734 **7. Cautions concerning other aspects of DFT implementation**

735 In this section we consider several additional issues related to the choices and approximations  
736 that are adopted by DFT practitioners, specifically as applied to calculating forces for ab initio  
737 MD simulation of multicomponent silicate liquids: the choice of exchange-correlation functional,  
738 the approximation adopted to parameterize core electron behavior, and the demonstration of  
739 convergence with respect to numerical details such as plane-wave basis cut-off and k-point  
740 sampling.

### 741 *7.1 Functionals*

742 The selection of an adequate functional to define the exchange-correlation electronic  
743 contribution is of course a critical step when using DFT. Computational resource limitations  
744 often govern this choice when simulating complex geological systems using AIMD. This  
745 explains the popularity of LDA, the cheapest option, allowing longer MD simulations and bigger  
746 supercells for given computational resources. However, LDA is often not the most accurate

747 functional for oxide systems. De La Pierre et al. (2011) reported key studies on this topic and  
748 highlighted the importance of introducing a fraction of Hartree-Fock (exact) exchange in hybrid  
749 functionals to accurately model calcite, quartz, katoite, and garnets. They also performed DFT  
750 calculations for forsterite and confirmed that a hybrid functional such as B3LYP (Becke 1993;  
751 Lee et al. 1988; Vosko et al. 1980; Stephens et al. 1994) provides the most accurate results.  
752 Kotomin et al. (2008) concurred that B3LYP is needed to predict correct band gaps of  
753 perovskites, while Strassner and Taige (2005) concluded that a hybrid functional is required for  
754 accurate representation of compounds containing transition metals.

755 An efficient and popular choice of hybrid functional is the Hubbard U correction. LDA+U and  
756 GGA+U methods, following the work of Anisimov et al. (1991, 1997), can improve the ability of  
757 DFT to quantitatively (and even, sometimes, qualitatively) reproduce experimental observations.  
758 This approach was applied, for example, by Stackhouse et al. (2010) to study the high-pressure  
759 properties of fayalite. They showed that iteration between predicted thermo-physical properties  
760 and available experimental data is required to set the value of the U parameter. As for the LDA  
761 pressure correction, there is no physical justification to assume that the U parameter is  
762 independent of the chemical environment and structure of the considered phase. Therefore, it is  
763 not clear how the U parameter should be treated across variable compositions when studying  
764 silicate melts under pressure. Also, without any prior knowledge about the physico-chemical  
765 behavior of the considered phase obtained from laboratory data, it is not possible to parameterize  
766 this empirical correction. Stackhouse et al. (2010) also concluded that the mixed success of the  
767 GGA+U approach when studying fayalite justifies the continued development of more exact  
768 methods such as Quantum Monte Carlo (Foulkes et al. 2001).

769 7.2 Core electrons

770 Practical DFT generally does not compute the density of all electrons in a system self-  
771 consistently. The rapidly oscillating wavefunctions close to atomic nuclei make all-electron  
772 calculations very expensive, and at the same time efficient shielding of closed-shell core  
773 electrons typically reduces their influence over chemical bonding behavior. So explicit  
774 calculation of their electron density is replaced by simplified effective contributions. The most  
775 popular methods are pseudo-potential and projector augmented wave (PAW) approaches. Note  
776 that PAW methods are built from ultra-soft pseudo-potentials. This in each case, a  
777 parameterization is required, as detailed by Jollet et al. (2014). This should be aimed not at  
778 fitting empirical data but at converging to the all-electron result. All-electron calculations,  
779 however are rarely available for system chemical environments and pressures relevant to  
780 multicomponent silicate liquids in planetary mantles. The objective procedure proposed by  
781 Lejaeghere et al. (2013) is used in this case to quantify the error of a given pseudo-potential  
782 compared to all-electron calculations. In this method, the integrated Birch-Murnaghan EOS (i.e.  
783  $E(V)$  curve) is fit both to pseudo-potential and all-electron DFT results spanning a volume range  
784  $\Delta V$ ; the difference between these fits is  $\Delta E(V)$ , and the error is expressed by  
785  $\Delta = \sqrt{\int \Delta E^2(V) dV} / \Delta V$ . This transferability test is typically performed for elements and binary  
786 cubic compounds and will provide sensible results for materials that follow the Birch-Murnaghan  
787 EOS only. In some plane wave codes such as VASP (Kresse and Furthmüller, 1996a, b; Kresse  
788 and Hafner, 1993, 1994), the user cannot evaluate the impact of these parameters on the  
789 precision of results since they cannot be freely modified as in more flexible codes like ABINIT  
790 (Gonze et al. 2009), SIESTA (Soler et al. 2002), or CASTEP (Clark et al. 2005). Of course, the  
791 parameterization of pseudo-potentials is a difficult task which requires expertise in electronic  
792 structure theory. Even though geoscientists should not be required to play with these parameters

793 when modeling systems using DFT, there should be efforts to generate benchmark testing for  
794 evaluating the transferability and accuracy of these pseudo-potentials using structures and  
795 conditions more suited for geological and planetary applications, i.e. more chemically-rich  
796 systems subjected to high pressure.

797 At high  $P$ , the pseudo-potential cutoff radius can affect the accuracy of calculations. Correa et al.  
798 (2006) show the impact of cutoff radius on total energy of liquid, diamond and BC8 carbon  
799 phases in AIMD simulations using GGA and norm-conserving pseudo-potentials to describe core  
800 electrons. A change of pseudo-potential cutoff radius from  $0.74\text{\AA}$  to  $0.60\text{\AA}$  at very high  $P$   
801 significantly alters the total energy of carbon structures, decreasing the inferred melting point of  
802 carbon by 200 K at 1000 GPa. At standard  $P$  and  $T$  the single bond length for carbon in diamond  
803 is  $\sim 1.54\text{\AA}$  (Ocellì et al. 2003). This is close to the onset of core overlap at  $1.48\text{\AA}$  with the  
804 pseudo-potential cutoff radius of Correa et al. (2006). The single bond length of carbon in  
805 diamond decreases to about  $1.19\text{\AA}$  at 1000 GPa according to the Vinet EOS of Ocellì et al.  
806 (2003), again close to the  $1.2\text{\AA}$  overlap threshold for the high- $P$  cutoff radius of Correa et al.  
807 (2006). The sensitivity of very high- $P$  DFT calculations to pseudo-potential cutoff radius reflects  
808 the inability of the pseudo-potential functional form to extrapolate the “exact” orbital wave  
809 functions below the radius cutoff. In the case of norm-conserving pseudo-potentials, the core  
810 overlap violates the norm-conserving condition, so that integrated charge per orbital deviates  
811 from one electron. This is a serious error since these pseudo-potentials rely on norm conservation  
812 to guarantee the correct logarithmic eigenvalue derivative that ensures good transferability. For  
813 ultra-soft pseudo-potentials, the consequence of overlap is much less severe as they rely on  
814 augmentation charge and multiple projectors. However to our knowledge there is no study that  
815 attempts to quantify this transferability error as a function of pressure. This emphasizes the

816 importance of customizing the pseudo-potential benchmark tests for geological applications in  
817 order to ensure the highest achievable precision of the DFT in these equilibrium conditions.

818 As another example, Karki et al. (2007) applied the  $3s^23p^2$  valence state reference and a cutoff  
819 radius of 0.82 Å for silicon atoms in silica liquid. With the ultra-soft pseudo-potential of oxygen  
820 (0.82 Å and  $2s^22p^4$ ), a minimal Si-O distance of 1.64 Å can be studied without electronic core  
821 overlap. This is virtually equal to the AIMD simulation value obtained for pure liquid silica by  
822 these authors. In another AIMD study on  $Mg_2SiO_4$  liquid, de Koker et al. (2008) applied an ultra-  
823 soft pseudo-potential for Si having 0.95 Å cutoff radius and  $3s^23p^2$  reference valence state, with  
824 the same oxygen pseudo-potential. The predicted Si-O distance ( $\approx 1.63$  Å) in their simulations is  
825 smaller than the distance (1.77 Å) for onset of electronic core overlapping based on the reported  
826 cutoff radii. The Si-O bond-length alone, although virtually equal in both simulations, cannot  
827 warrant the exact same transferability precision of these two distinct pseudo-potentials when  
828 evaluating any thermo-physical property. In this context, it should be explained and quantified  
829 whether ultra-soft pseudo-potential calculations can be applied to interactions requiring core  
830 overlap, and importantly whether errors will become worse as  $P$  increases and bonds get shorter.

### 831 *7.3 Numerical precision*

832 Finally, a convergence test with respect to the selected basis set (i.e plane-wave cut-off) and the  
833 density of k-point sampling over the Brillouin-zone must be performed for at conditions specific  
834 enough to each calculation in a study to provide a quantified estimate of errors due to these  
835 simplifications for that calculation in particular. An example of such a careful convergence test is  
836 presented by de Koker et al. (2008) for  $Mg_2SiO_4$  liquid. They found in this case an uncertainty of  
837 8.8 kJ/mol on the internal energy and 1.5 GPa on the total pressure of their AIMD simulations. In  
838 other words, they provide not only the selected cut-off radius and k-point density but also the

839 residual uncertainty they tolerate at the chosen level of convergence. It should be pointed out  
840 here that this convergence test cannot be replaced by selection of a different exchange-  
841 correlation function. A badly converged simulation with respect to these parameters cannot  
842 provide accurate results even with an accurate functional such as B3LYP.

## 843 **8. Cautions concerning van der Waals interactions**

844 Another important consideration regarding conventional DFT based simulations is that they  
845 cannot account for van der Waals interactions arising from mutual dynamical polarization of the  
846 interacting atoms, as highlighted by Vuilleumier et al. (2009). Langreth et al. (2005) introduce  
847 van der Waals DFT to correct this problem. More recent and widespread adoptions of the so-  
848 called DFT-D method to overcome these limitations are presented by Grimme (2006) and  
849 Tkatchenko et al. (2009) respectively.

850 However, classical interatomic force-field potentials are another possible solution to this  
851 problem. In this approach, an analytical function describing energetic interaction between atoms  
852 as a function of their inter-distance  $r_{ij}$  is proposed based on fundamental physics theories. In the  
853 specific case of condensed silicate phases, a standard approach to the interionic potential  $\phi_{ij}$   
854 includes Coulomb, repulsive and attractive terms, e.g., (Bukowinski and Akber-Knutson, 2005):

$$855 \quad \phi_{ij} = \frac{I_i I_j}{r_{ij}} + b_{ij} \exp\left(\frac{-r_{ij}}{\rho_{ij}}\right) - \frac{c_{ij}}{r_{ij}^6} \quad (5)$$

856 Where  $I_i$  is the charge on the  $i^{\text{th}}$  ion, and  $b_{ij}$ ,  $\rho_{ij}$ , and  $c_{ij}$  are parameters that describe a given pair  
857 of atoms. In silicate melts, Van der Waals forces are generally introduced in the description of  $U$   
858 ( $c_{ij}$  parameters) for some (Guillot and Sator, 2007; Lacks et al. 2007) or all interionic interactions

859 (Matsui, 1998). Empirical parameters of such a classical interionic potential for silicate systems  
860 are typically obtained by reproducing experimental data for both solid and liquid phases. Hence,  
861 classical interatomic potentials can only be used if enough thermo-physical data are available  
862 and that their predictive abilities depend on the physical complexity of the functional form used  
863 to describe the energetics of the considered structure.

864 Moreover, a pairwise-type potential is only an effective description of the energetic interactions  
865 in condensed phases, since many-body interactions may be present. These higher-order  
866 interactions should logically be accounted for in a rigorous analytical treatment of condensed  
867 matter energetics. For example, there should be an energetic perturbation induced by a third ion  
868 on a given ionic pair (influence of the chemical environment). Salanne et al. (2012) proposed a  
869 general formalism for improving interionic potentials like Equation 5, to account for the most  
870 important many-body effects in the overlap repulsion component and the polarization part of  
871 their potential, based on the aspherical ion model (Aguado et al. 2003; Aguado and Madden,  
872 2004; Jahn and Madden, 2007). This complex functional form is parameterized by the force-  
873 fitting or force-matching method (Laio et al. 2000), and only DFT stress and force calculations  
874 are used as input data. These authors also highlighted the conundrum induced by this procedure:  
875 as there is no dynamical polarization in conventional DFT calculations, results of these  
876 simulations do not constrain the dispersion interaction parameters of their functional, which are  
877 therefore not included in the fitting procedure. However, dispersive terms are introduced in their  
878 functional a posteriori based on other considerations (Salanne et al. 2012).

879 The approach of Salanne et al. (2012) is a clear improvement over the conventional classical  
880 fitting of empirical pairwise interionic potentials to a limited set of experimental data. Use of  
881 DFT results spanning a range of  $P$ ,  $T$ , and atomic configurations leads to a more transferable

882 potential and encompasses energetic contributions not explicitly defined in their functional via  
883 correlated terms. There are still important limitations to such an approach. First, the original  
884 force-fitting approach proposed by Laio et al. (2000) was intended for the parameterization of  
885 interionic potentials for a specific set of ( $P$ ,  $T$ ) equilibrium conditions only and may not be fully  
886 transferable to other equilibrium conditions. Even the more complex many-body interionic  
887 functional form does not explicitly represent important phenomena such as metallization of ionic  
888 interactions at high  $T$  and  $P$  (de Koker and Stixrude, 2009). Without an explicit treatment of the  
889 effect of  $P$  on the electronic density of states in these interionic potentials, one can only hope to  
890 fold these effects into correlated terms for a given set of ( $P$ ,  $T$ ) conditions. Determining a single  
891 set of interionic parameters for a range of  $T$ ,  $P$  and composition will inevitably result in a loss of  
892 precision, as observed by Salanne et al. (2012) for silica-rich melts.

## 893 **9. Implications**

894 We have presented several limitations of current MD simulations, inherent to the technique  
895 (kinetic limitations, limited computing resources, etc.) or to our limited knowledge of atomistic  
896 phenomena underlying the evolution of many-body interactions in silicate solutions as a function  
897 of  $T$ ,  $P$  and composition. These constructive criticisms are not intended to discourage the reader  
898 from applying classical or ab initio MD simulation methods to Earth science problems. Instead,  
899 the intent is to stimulate interdisciplinary discussions among physicists and geochemists, to  
900 refine our fundamental understanding of silicate melts, solids and glasses. Another goal of the  
901 present communication is to provide scientists unfamiliar with the detail of quantum simulation  
902 methods with a better appreciation of the potential errors and uncertainties.



903 For the MD simulation expert, this work highlights that many suspicions about the robustness  
904 and statistical significance of their work could be allayed simply by more systematic publication  
905 of raw simulation data. We would encourage the scientific community to:

- 906 1. Present all raw simulation data in appendices or online supplements in order to allow data  
907 post-treatment as well as a precise evaluation of the error/uncertainty.
- 908 2. Provide the finite-size and time-equilibration data of each simulation.
- 909 3. In the case of AIMD simulations, provide the results of the convergence test with respect  
910 to the basis set and the k-points for Brillouin zone sampling in terms of stress and force  
911 errors.
- 912 4. Evaluate instantaneous heat capacity and thermal expansion, when possible, from the  
913 variance of the statistical ensemble for comparison to derivatives of fitted thermodynamic  
914 models, to quantify the ability of the simulation to span the entire partition function.
- 915 5. Study the impact of the initial configuration on the resulting equilibrium state, especially  
916 for systems containing miscibility gaps, and including both ordered and disordered initial  
917 states. This would ensure that MD simulation parameters used for imposed equilibrium  
918 conditions allow the exploration of the true equilibrium state.
- 919 6. Design a series of MD simulations to quantifiably observe the evolution of the SRO.

920 For the experimentalist, we highlight information that ought to be developed in collaboration  
921 with MD simulation experts:

- 922 1. Provide experiments, such as diffuse X-ray scattering and nuclear magnetic resonance  
923 spectroscopy, to quantify the 2NN pair fraction environment in silicate melts, solids and  
924 glasses, for validation of current MD simulations and development of future methods.

925 2. Target equilibrium conditions of temperature, pressure and composition of fundamental  
926 importance for the improvement of physical theories of silicate solutions.

927 For the theoretician, the goals highlighted by this work are:

928 1. Incorporate self-consistent thermo-physical data obtained from MD simulations in  
929 classical thermodynamic models to see if they allow accurate prediction of complex  
930 phase equilibria for which experimental data are available. This will serve to validate or  
931 falsify some of the suspicions raised here.

932 2. Use both simulations and experiments to test new approaches to linking molar volume,  
933 internal energy, configurational entropy, and the microscopic internal configuration of  
934 melts, to improve predictive abilities of classical thermodynamic models.

935 3. Quantify the importance of Van der Waals forces in silicate melts, perhaps using classical  
936 interatomic potentials, to judge if they limit accuracy of AIMD simulations.

937 4. Parameterize more robust interionic potentials that account for many-body interactions.

938

### 939 **Reference list**

940 Adam, G., and Gibbs, J.H. (1965) On the temperature dependence of cooperative relaxation  
941 properties in glass-forming liquids. *The Journal of Chemical Physics*, 43, 139-146.

942 Agee, C.B. (1998) Crystal-liquid density inversions in terrestrial and lunar magmas. *Physics of*  
943 *the Earth and Planetary Interiors*, 107, 63-74.

944 Aguado, A., and Madden, P.A. (2004) Fully transferable interatomic potentials for large-scale  
945 computer simulations of simple metal oxides: Application to MgO. *Physical Review B*, 70,  
946 245103.

- 947 Aguado, A., Bernasconi, L., Jahn, S., and Madden, P.A. (2003) Multipoles and interaction  
948 potentials in ionic materials from planewave-DFT calculations. *Faraday Discussions*, 124,  
949 171-184.
- 950 Akaogi, M., and Navrotsky, A. (1984) The quartz-coesite-stishovite transformations: new  
951 calorimetric measurements and calculation of phase diagrams. *Physics of the Earth and*  
952 *Planetary Interiors*, 36, 124-134.
- 953 Alder, B.J. (1964) Studies in molecular dynamics. III. A mixture of hard spheres. *The Journal of*  
954 *Chemical Physics*, 40, 2724-2730.
- 955 Alfredsson, M., Brodholt, J.P., Wilson, P.B., Price, G.D., Cora, F., Calleja, M., Bruin, R.,  
956 Blanshard, L.J., and Tyer, R.P. (2005) Structural and magnetic phase transitions in simple  
957 oxides using hybrid functionals. *Molecular Simulation*, 31, 367-377.
- 958 Anisimov, V.I., and Kozhevnikov, A.V. (2005) Transition state method and Wannier functions.  
959 *Physical Review B*, 72, 075125.
- 960 Anisimov, V.I., Zaanen, J., and Andersen, O.K. (1991) Band theory and Mott insulators:  
961 Hubbard  $U$  instead of Stoner  $I$ . *Physical Review B*, 44, 943-954.
- 962 Anisimov, V. I., Aryasetiawan, F., and Lichtenstein, A. I. (1997) First-principles calculations of  
963 the electronic structure and spectra of strongly correlated systems: the LDA +  $U$  method.  
964 *Journal of Physics: Condensed Matter*, 9, 767-808.
- 965 Asimow, P.D., and Ahrens, T.J. (2010) Shock compression of liquid silicates to 125 GPa: The  
966 anorthite-diopside join. *Journal of Geophysical Research: Solid Earth*, 115, B10209.
- 967 Bale, C.W., Belisle, E., Chartrand, P., Deckerov, S.A., Eriksson, G., Hack, K., Jung, I.H., Kang,  
968 Y.B., Melancon, J., Pelton, A.D., Robelin, C., and Petersen, S. (2009) FactSage  
969 thermochemical software and databases - recent developments. *CALPHAD*, 33, 295-311.

- 970 Baroni, S., de Gironcoli, S., Dal Corso, A., and Giannozzi, P. (2001) Phonons and related crystal  
971 properties from density-functional perturbation theory. *Review of Modern Physics*, 73, 515-  
972 562.
- 973 Becke, A.D. (1993) Density-functional thermochemistry. III. The role of exact exchange. *The*  
974 *Journal of Chemical Physics*, 98, 5648-5652.
- 975 Bolmatov, D., Brazhkin, V.V., and Trachenko, K. (2012) The phonon theory of liquid  
976 thermodynamics. *Scientific Reports*, 2, srep00421-1 srep00421-6.
- 977 Born, M., and Huang, K. (1969) *Dynamical Theory of Crystal Lattices*, Oxford University Press.
- 978 Born, M., and Oppenheimer, R. (1927) Zur Quantentheorie der Molekeln (in German). *Annalen*  
979 *der Physik*, 84, p. 457-484.
- 980 Brady, J.B. (2013) Diffusion Data for Silicate Minerals, Glasses, and Liquids, in *Mineral Physics*  
981 *& Crystallography: A Handbook of Physical Constants*. American Geophysical Union, 269-  
982 290.
- 983 Bukowinski, M.S.T., and Akber-Knutson, S. (2005) The Role of Theoretical Mineral Physics in  
984 *Modeling the Earth's Interior, Earth's Deep Mantle: Structure, Composition, and Evolution*.  
985 American Geophysical Union, 137-163.
- 986 Cao, W., Chang, Y.A., Zhu, J., Chen, S., and Oates, W.A. (2007) Thermodynamic modeling of  
987 the Cu-Ag-Au system using the cluster/site approximation. *Intermetallics*, 15, 1438-1446.
- 988 Car, R., and Parrinello, M. (1985) Unified approach for molecular dynamics and density-  
989 functional theory. *Physical Review Letters*, 55, 2471-2474.
- 990 Ceperley, D.M., and Alder, B.J. (1980) Ground state of the electron gas by a stochastic method.  
991 *Physical Review Letters*, 45, 566-569.

- 992 Chamberlin, L., Beckett, J.R., and Stolper, E. (1994) Pd-oxide equilibration: a new experimental  
993 method for the direct determination of oxide activities in melts and minerals. *Contributions to*  
994 *Mineralogy and Petrology*, 116, 169-181.
- 995 Clark, S.J., Segall, M.D., Pickard, C.J., Hasnip, P.J., Probert, M.I.J., Refson, K., and Payne, M.C.  
996 (2005) First principles methods using CASTEP. *Zeitschrift für Kristallographie*, 220, 567-  
997 570.
- 998 Clemens, J.D., and Petford, N. (1999) Granitic melt viscosity and silicic magma dynamics in  
999 contrasting tectonic settings. *Journal of the Geological Society* 156, 1057-1060.
- 1000 Cococcioni, M., Dal Corso, A., and de Gironcoli, S. (2003) Structural, electronic, and magnetic  
1001 properties of Fe<sub>2</sub>SiO<sub>4</sub> fayalite: Comparison of LDA and GGA results. *Physical Review B*,  
1002 67, 094106.
- 1003 Cococcioni, M., and de Gironcoli, S. (2005) Linear response approach to the calculation of the  
1004 effective interaction parameters in the LDA+U method. *Physical Review B* 71, 035105.
- 1005 Correa, A.A., Bonev, S.A., and Galli, G. (2006) Carbon under extreme conditions: Phase  
1006 boundaries and electronic properties from first-principles theory. *Proceedings of the National*  
1007 *Academy of Sciences of the United States of America*, 103, 1204-1208.
- 1008 Dalton, J.A., and Presnall, D.C. (1997) No liquid immiscibility in the system MgSiO<sub>3</sub>-SiO<sub>2</sub> at 5.0  
1009 GPa. *Geochimica et Cosmochimica Acta*, 61, 2367-2373.
- 1010 Dasgupta, R., and Hirschmann, M.M. (2006) Melting induced extraction of C–O–H volatiles at  
1011 mid-ocean ridges. *Geochimica et Cosmochimica Acta*, 70, A127.
- 1012 Davies, G.F. (1973) Quasi-harmonic finite strain equations of state of solids. *Journal of Physics*  
1013 *and Chemistry of Solids*, 34, 1417-1429.

- 1014 Davies, G.F. (2006) Early mantle dynamics: Depletion, plates and a revised cooling history.  
1015 *Geochimica et Cosmochimica Acta*, 70, A132.
- 1016 Davies, G.F. (2008) Episodic layering of the early mantle by the ‘basalt barrier’ mechanism.  
1017 *Earth and Planetary Science Letters*, 275, 382-392.
- 1018 de Koker, N., and Stixrude, L. (2009) Self-consistent thermodynamic description of silicate  
1019 liquids, with application to shock melting of MgO periclase and MgSiO<sub>3</sub> perovskite.  
1020 *Geophysical Journal International*, 178, 162-179.
- 1021 de Koker, N.P., Stixrude, L., and Karki, B.B. (2008) Thermodynamics, structure, dynamics, and  
1022 freezing of Mg<sub>2</sub>SiO<sub>4</sub> liquid at high pressure. *Geochimica et Cosmochimica Acta*, 72, 1427-  
1023 1441.
- 1024 de Koker, N., Karki, B.B., and Stixrude, L. (2013) Thermodynamics of the MgO–SiO<sub>2</sub> liquid  
1025 system in Earth's lowermost mantle from first principles. *Earth and Planetary Science*  
1026 *Letters*, 361, 58-63.
- 1027 de La Pierre, M., Orlando, R., Maschio, L., Doll, K., Ugliengo, P., and Dovesi, R. (2011)  
1028 Performance of six functionals (LDA, PBE, PBESOL, B3LYP, PBE0, and WC1LYP) in the  
1029 simulation of vibrational and dielectric properties of crystalline compounds. The case of  
1030 forsterite Mg<sub>2</sub>SiO<sub>4</sub>. *Journal of Computational Chemistry*, 32, 1775-1784.
- 1031 Del Gaudio, P., and Behrens, H. (2009) An experimental study on the pressure dependence of  
1032 viscosity in silicate melts. *The Journal of Chemical Physics*, 131, 044504.
- 1033 Dingwell, D.B. (1996) Volcanic Dilemma--Flow or Blow? *Science*, 273, 1054-1055.
- 1034 Dingwell, D.B. (1998) Melt viscosity and diffusion under elevated pressures. *Reviews in*  
1035 *Mineralogy and Geochemistry*, 37, 397-424.

- 1036 Dyer, P.J., and Cummings, P.T. (2006) Hydrogen bonding and induced dipole moments in water:  
1037 predictions from the Gaussian charge polarizable model and Car-Parrinello molecular  
1038 dynamics. *The Journal of Chemical Physics*, 125, 144519.
- 1039 Dyre, J.C., Hechsher, T., and Niss, K. (2009) A brief critique of the Adam–Gibbs entropy model.  
1040 *Journal of Non-Crystalline Solids*, 355, 624-627.
- 1041 Evans, D.J., and Holian, B.L. (1985) The Nose–Hoover thermostat. *The Journal of Chemical*  
1042 *Physics*, 83, 4069-4074.
- 1043 Evans, R.L., Hirth, G., Baba, K., Forsyth, D., Chave, A., and Mackie, R. (2005) Geophysical  
1044 evidence from the MELT area for compositional controls on oceanic plates. *Nature*, 437,  
1045 249-252.
- 1046 Faul, U.H., and Scott, D. (2006) Grain growth in partially molten olivine aggregates.  
1047 *Contributions to Mineralogy and Petrology*, 151, 101-111.
- 1048 Faul, U.H., Toomey, D.R., and Waff, H.S. (1994) Intergranular basaltic melt is distributed in  
1049 thin, elongated inclusions. *Geophysical Research Letters*, 21, 29-32.
- 1050 Fei, Y., Saxena, S.K., and Navrotsky, A. (1990) Internally consistent thermodynamic data and  
1051 equilibrium phase relations for compounds in the system MgO-SiO<sub>2</sub> at high pressure and  
1052 high temperature. *Journal of Geophysical Research: Solid Earth*, 95, 6915-6928.
- 1053 Forsyth, D.W. (1996) Partial melting beneath a Mid-Atlantic Ridge Segment detected by  
1054 teleseismic PKP delays. *Geophysical Research Letters*, 23, 463-466.
- 1055 Foulkes, W.M.C., Mitas, L., Needs, R.J., and Rajagopal, G. (2001) Quantum Monte Carlo  
1056 simulations of solids. *Reviews of Modern Physics*, 73, 33-83.
- 1057 Franzblau, M.C., and Gordon, R.B. (1967) The order-disorder transformation in Cu<sub>3</sub>Au at high  
1058 pressure. *Journal of Applied Physics* 38, 103-110.

- 1059 Fultz, B. (2010) Vibrational thermodynamics of materials. *Progress in Materials Science*, 55,  
1060 247-352.
- 1061 Garrity, K.F., Bennett, J.W., Rabe, K.M., and Vanderbilt, D. (2014) Pseudopotentials for high-  
1062 throughput DFT calculations. *Computational Materials Science*, 81, 446-452.
- 1063 Ghiorso, M. (1985) Chemical mass transfer in magmatic processes. *Contributions to Mineralogy  
1064 and Petrology*, 90, 107-120.
- 1065 Ghiorso, M.S., and Spera, F.J. (2010) Large scale simulations. *Reviews in Mineralogy and  
1066 Geochemistry*, 71, 437-463.
- 1067 Ghiorso, M.S., Hirschmann, M.M., Reiners, P.W., and Kress, V.C. (2002) The pMELTS: A  
1068 revision of MELTS for improved calculation of phase relations and major element  
1069 partitioning related to partial melting of the mantle to 3 GPa. *Geochemistry, Geophysics,  
1070 Geosystems*, 3, 1-35.
- 1071 Ghosh, D.B., and Karki, B.B. (2011) Diffusion and viscosity of Mg<sub>2</sub>SiO<sub>4</sub> liquid at high pressure  
1072 from first-principles simulations. *Geochimica et Cosmochimica Acta*, 75, 4591-4600.
- 1073 Gonze, X., Amadon, B., Anglade, P.M., Beuken, J.M., Bottin, F., Boulanger, P., Bruneval, F.,  
1074 Caliste, D., Caracas, R., Côté, M., and others (2009) ABINIT: First-principles approach to  
1075 material and nanosystem properties. *Computer Physics Communications*, 180, 2582-2615.
- 1076 Grimme, S., (2006) Semiempirical GGA-type density functional constructed with a long-range  
1077 dispersion correction. *Journal of Computational Chemistry*, 27, 1787-1799.
- 1078 Guillot, B., and Sator, N. (2007) A computer simulation study of natural silicate melts. Part I:  
1079 Low pressure properties. *Geochimica et Cosmochimica Acta*, 71, 1249-1265.
- 1080 Haas, P., Tran, F., and Blaha, P. (2009) Calculation of the lattice constant of solids with  
1081 semilocal functionals. *Physical Review B*, 79, 085104.



- 1082 Hageman, V.B.M., and Oonk, H.A.J. (1986) Liquid immiscibility in the  $\text{SiO}_2 + \text{MgO}$ ,  $\text{SiO}_2 +$   
1083  $\text{SrO}$ ,  $\text{SiO}_2 + \text{La}_2\text{O}_3$ , and  $\text{SiO}_2 + \text{Y}_2\text{O}_3$  systems. *Physics and Chemistry of Glasses*, 27, 194-  
1084 198.
- 1085 Hamann, D.R. (1996) Generalized gradient theory for silica phase transitions. *Physical Review*  
1086 *Letters*, 76, 660-663.
- 1087 Hautier, G., Ong, S.P., Jain, A., Moore, C.J., and Ceder, G. (2012) Accuracy of density  
1088 functional theory in predicting formation energies of ternary oxides from binary oxides and  
1089 its implication on phase stability. *Physical Review B*, 85, 155208.
- 1090 Hayoun, M., Pontikis, V., and Winter, C. (1998) Computer simulation study of surface  
1091 segregation on  $\text{Cu}_3\text{Au}$ . *Surface Science*, 398, 125-133.
- 1092 Hicks, D.G., Boehly, T.R., Eggert, J.H., Miller, J.E., Celliers, P.M., and Collins, G.W. (2006)  
1093 Dissociation of liquid silica at high pressures and temperatures. *Physical Review Letters*, 97,  
1094 025502.
- 1095 Hirschmann, M.M. (2006) Water, melting, and the deep earth  $\text{H}_2\text{O}$  cycle. *Annual Review of*  
1096 *Earth and Planetary Sciences*, 34, 629-653.
- 1097 Holm, J.L., Kleppa, O.J., and Westrum E.F.Jr. (1967) Thermodynamics of polymorphic  
1098 transformations in silica. Thermal properties from 5 to  $1070^\circ \text{K}$  and pressure-temperature  
1099 stability fields for coesite and stishovite. *Geochimica et Cosmochimica Acta*, 31, 2289-2307.
- 1100 Horbach, J., Kob, W., Binder, K., and Angell, C.A. (1996) Finite size effects in simulations of  
1101 glass dynamics. *Physical Review E*, 54, R5897-R5900.
- 1102 Hudon, P., Jung, I.-H., and Baker, D.R. (2004) Effect of pressure on liquid-liquid miscibility  
1103 gaps: A case study of the systems  $\text{CaO-SiO}_2$ ,  $\text{MgO-SiO}_2$ , and  $\text{CaMgSi}_2\text{O}_6\text{-SiO}_2$ . *Journal of*  
1104 *Geophysical Research: Solid Earth*, 109, B03207.

- 1105 Hudon, P., Jung, I.-H., and Baker, D.R. (2005) Experimental investigation and optimization of  
1106 thermodynamic properties and phase diagrams in the systems CaO-SiO<sub>2</sub>, MgO-SiO<sub>2</sub>,  
1107 CaMgSi<sub>2</sub>O<sub>6</sub>-SiO<sub>2</sub>, and CaMgSi<sub>2</sub>O<sub>6</sub>-Mg<sub>2</sub>SiO<sub>4</sub> to 1.0 GPa. *Journal of Petrology*, 46, 1859-  
1108 1880.
- 1109 Ising, E. (1925) Beitrag zur theorie des ferromagnetismus (in German). *Zeitschrift für Physik*,  
1110 31, 253-258.
- 1111 Jahn, S., and Madden, P.A. (2007) Modeling Earth materials from crustal to lower mantle  
1112 conditions: A transferable set of interaction potentials for the CMAS system. *Physics of the*  
1113 *Earth and Planetary Interiors*, 162, 129-139.
- 1114 Jollet, F., Torrent, M., and Holzwarth, N. (2014) Generation of Projector Augmented-Wave  
1115 atomic data: A 71 element validated table in the XML format. *Computer Physics*  
1116 *Communications*, 185, 1246-1254.
- 1117 Kambayashi, S., and Kato, E. (1983) A thermodynamic study of (magnesium oxide + silicon  
1118 dioxide) by mass spectrometry. *The Journal of Chemical Thermodynamics*, 15, 701-707.
- 1119 Kambayashi, S., and Kato, E. (1984) A thermodynamic study of (magnesium oxide + silicon  
1120 dioxide) by mass spectrometry at 1973 K. *The Journal of Chemical Thermodynamics*, 16,  
1121 241-248.
- 1122 Karato, S.-i., and Jung, H. (1998) Water, partial melting and the origin of the seismic low  
1123 velocity and high attenuation zone in the upper mantle. *Earth and Planetary Science Letters*,  
1124 157, 193-207.
- 1125 Karki, B.B., Stixrude, L., and Wentzcovitch, R.M. (2001) High-pressure elastic properties of  
1126 major materials of Earth's mantle from first principles. *Reviews of Geophysics*, 39, 507-534.

- 1127 Karki, B.B., Bhattarai, D., and Stixrude, L. (2006) First-principles calculations of the structural,  
1128 dynamical, and electronic properties of liquid MgO. *Physical Review B*, 73, 174208.
- 1129 Karki, B.B., Bhattarai, D., and Stixrude, L. (2007) First-principles simulations of liquid silica:  
1130 Structural and dynamical behavior at high pressure. *Physical Review B*, 76, 104205.
- 1131 Karki, B.B., Zhang, J., and Stixrude, L. (2013) First principles viscosity and derived models for  
1132 MgO-SiO<sub>2</sub> melt system at high temperature. *Geophysical Research Letters*, 40, 94-99.
- 1133 Kikuchi, R. (1951) A theory of cooperative phenomena. *Physical Review*, 81, 988-1003.
- 1134 Klein, E.M., and Langmuir, C.H. (1987) Global correlations of ocean ridge basalt chemistry with  
1135 axial depth and crustal thickness. *Journal of Geophysical Research: Solid Earth*, 92 (B8),  
1136 8089-8115.
- 1137 Kotomin, E.A., Zhukovskii, Y.F., Piskunov, S., and Ellis, D.E. (2008) Hybrid DFT calculations  
1138 of the F centers in cubic ABO<sub>3</sub> perovskites. *Journal of Physics: Conference Series*, 117,  
1139 012019.
- 1140 Kresse, G., and Furthmüller, J. (1996a) Efficiency of ab-initio total energy calculations for  
1141 metals and semiconductors using a plane-wave basis set. *Computational Materials Science*, 6,  
1142 15-50.
- 1143 Kresse, G., and Furthmüller, J. (1996b) Efficient iterative schemes for ab initio total-energy  
1144 calculations using a plane-wave basis set. *Physical Review B*, 54, 11169-11186.
- 1145
- 1146 Kresse, G., and Hafner, J. (1993) Ab initio molecular dynamics for liquid metals. *Physical*  
1147 *Review B*, 47, 558-561.
- 1148 Kresse, G., and Hafner, J. (1994) Ab initio molecular-dynamics simulation of the liquid-metal-  
1149 amorphous-semiconductor transition in germanium. *Physical Review B*, 49, 14251-14269.

- 1150 Kresse, G., and Joubert, D. (1999) From ultrasoft pseudopotentials to the projector augmented-  
1151 wave method. *Physical Review B*, 59, 1758-1775.
- 1152 Lacks, D.J., Rear, D.B., and Van Orman, J.A. (2007) Molecular dynamics investigation of  
1153 viscosity, chemical diffusivities and partial molar volumes of liquids along the MgO–SiO<sub>2</sub>  
1154 join as functions of pressure. *Geochimica et Cosmochimica Acta*, 71, 1312-1323.
- 1155 Lacks, D.J., Goel, G., Bopp, C.J.I.V., Van Orman, J.A., Leshner, C.E., and Lundstrom, C.C.  
1156 (2012) Isotope fractionation by thermal diffusion in silicate melts. *Physical Review Letters*,  
1157 108, 065901.
- 1158 Laio, A., Bernard, S., Chiarotti, G.L., Scandolo, S., and Tosatti, E. (2000) Physics of iron at  
1159 Earth's core conditions. *Science*, 287, 1027-1030.
- 1160 Lange, R.A., and Carmichael, I.S.E. (1987) Densities of Na<sub>2</sub>O-K<sub>2</sub>O-CaO-MgO-FeO-Fe<sub>2</sub>O<sub>3</sub>-  
1161 Al<sub>2</sub>O<sub>3</sub>-TiO<sub>2</sub>-SiO<sub>2</sub> liquids: New measurements and derived partial molar properties.  
1162 *Geochimica et Cosmochimica Acta*, 51, 2931-2946.
- 1163 Langreth, D.C., Dion, M., Rydberg, H., Schröder, E., Hyldgaard, P., and Lundqvist, B.I. (2005)  
1164 Van der Waals density functional theory with applications. *International Journal of Quantum*  
1165 *Chemistry*, 101, 599-610.
- 1166 Lay, T., Garnero, E.J., and Williams, Q. (2004) Partial melting in a thermo-chemical boundary  
1167 layer at the base of the mantle. *Physics of the Earth and Planetary Interiors*, 146, 441-467.
- 1168 Lee, B.-J., Shim, J.-H., and Baskes, M.I. (2003) Semiempirical atomic potentials for the fcc  
1169 metals Cu, Ag, Au, Ni, Pd, Pt, Al, and Pb based on first and second nearest-neighbor  
1170 modified embedded atom method. *Physical Review B*, 68, 144112.
- 1171 Lee, C., Yang, W., and Parr, R. G. (1988) Development of the Colle-Salvetti correlation-energy  
1172 formula into a functional of the electron density. *Physical Review B*, 37, 785-789.

- 1173 Lee, C.-T.A., Luffi, P., Plank, T., Dalton, H., and Leeman, W.P. (2009) Constraints on the depths  
1174 and temperatures of basaltic magma generation on Earth and other terrestrial planets using  
1175 new thermobarometers for mafic magmas. *Earth and Planetary Science Letters*, 279, 20-33.
- 1176 Lehnert, K., Su, Y., Langmuir, C.H., Sarbas, B., and Nohl, U. (2000) A global geochemical  
1177 database structure for rocks. *Geochemistry, Geophysics, Geosystems*, 1, 1012.
- 1178 Lejaeghere, K., Van Speybroeck, V., Van Oost, G., Cottenier, S. (2013) Error estimates for  
1179 solid-state density-functional theory predictions: An overview by means of the ground-state  
1180 elemental crystals. *Critical Reviews in Solid State and Materials Sciences*, 39, 1-24.
- 1181 Leshner, C.E. (2010) Self-diffusion in silicate melts: Theory, observations and applications to  
1182 magmatic systems. *Reviews in Mineralogy and Geochemistry*, 72, 269-309.
- 1183 Liang, Y. (2003) Kinetics of crystal-melt reaction in partially molten silicates: 1. Grain scale  
1184 processes. *Geochemistry, Geophysics, Geosystems*, 4, 1045.
- 1185 Lundstrom, C.C., Chaussidon, M., Hsui, A.T., Kelemen, P., and Zimmerman, M. (2005)  
1186 Observations of Li isotopic variations in the Trinity ophiolite: evidence for isotopic  
1187 fractionation by diffusion during mantle melting. *Geochimica et Cosmochimica Acta*, 69,  
1188 735-751.
- 1189 Martin, G.B., Ghiorso, M., and Spera, F.J. (2012) Transport properties and equation of state of 1-  
1190 bar eutectic melt in the system  $\text{CaAl}_2\text{Si}_2\text{O}_8\text{-CaMgSi}_2\text{O}_6$  by molecular dynamics simulation.  
1191 *American Mineralogist*, 97, 1155-1164.
- 1192 Matsui, M. (1998) Computational modeling of crystals and liquids in the system  $\text{Na}_2\text{O-CaO-}$   
1193  $\text{MgO-Al}_2\text{O}_3\text{-SiO}_2$ , *Properties of Earth and Planetary Materials at High Pressure and*  
1194 *Temperature*. AGU, Washington, DC, 145-151.

- 1195 Matsuno, T., Evans, R.L., Seama, N., and Chave, A.D. (2012) Electromagnetic constraints on a  
1196 melt region beneath the central Mariana back-arc spreading ridge. *Geochemistry,*  
1197 *Geophysics, Geosystems*, 13, Q10017.
- 1198 McDonough, W.F., and Sun, S.-s. (1995) The composition of the Earth. *Chemical Geology*, 120,  
1199 223-253.
- 1200 McKenzie, D. (1984) The generation and compaction of partially molten rock. *Journal of*  
1201 *Petrology*, 25, 713-765.
- 1202 McKenzie, D. (1985) The extraction of magma from the crust and mantle. *Earth and Planetary*  
1203 *Science Letters*, 74, 81-91.
- 1204 McQuarrie, D.A. (2000) *Statistical Mechanics*. University Science Books.
- 1205 McWilliams, R.S., Spaulding, D.K., Eggert, J.H., Celliers, P.M., Hicks, D.G., Smith, R.F.,  
1206 Collins, G.W., and Jeanloz, R. (2012) Phase transformations and metallization of magnesium  
1207 oxide at high pressure and temperature. *Science*, 338, 1330-1333.
- 1208 Metropolis, N., Rosenbluth, A.W., Rosenbluth, M.N., Teller, A.H., and Teller, E. (1953)  
1209 Equation of state calculations by fast computing machines. *The Journal of Chemical Physics*,  
1210 21, 1087-1092.
- 1211 Morgan, N.A., and Spera, F.J., (2001a) Glass transition, structural relaxation, and theories of  
1212 viscosity: a molecular dynamics study of amorphous  $\text{CaAl}_2\text{Si}_2\text{O}_8$ . *Geochimica et*  
1213 *Cosmochimica Acta*, 65, 4019-4041.
- 1214 Morgan, N.A., and Spera, F.J., (2001b) A molecular dynamics study of the glass transition in  
1215  $\text{CaAl}_2\text{Si}_2\text{O}_8$ : Thermodynamics and tracer diffusion. *American Mineralogist*, 86, 915-926.
- 1216 Neuville, D.R., and Richet, P. (1991) Viscosity and mixing in molten (Ca, Mg) pyroxenes and  
1217 garnets. *Geochimica et Cosmochimica Acta*, 55, 1011-1019.

- 1218 Niu, X., Lehnert, K.A., Williams, J., and Brantley, S.L. (2011) CZChemDB and EarthChem:  
1219 Advancing management and access of critical zone geochemical data. Applied  
1220 Geochemistry, 26 ( Supplement), S108-S111.
- 1221 Oates, W.A., and Wenzl, H. (1996) The cluster/site approximation for multicomponent solutions  
1222 -- A practical alternative to the cluster variation method. Scripta Materiala, 35, 623-627.
- 1223 Oates, W.A., Zhang, F., Chen, S.L., and Chang, Y.A. (1999) Improved cluster-site  
1224 approximation for the entropy of mixing in multicomponent solid solutions. Physical Review  
1225 B, 59, 11221-11225.
- 1226 Occelli, F., Loubeyre, P., and LeToullec, R. (2003) Properties of diamond under hydrostatic  
1227 pressures up to 140 GPa. Nature Materials, 2, 151-154.
- 1228 Okamoto, H., Chakrabarti, D.J., Laughlin, D.E., and Massalski, T.B. (1987) The Au-Cu (gold-  
1229 copper) system. Bulletin of Alloy Phase Diagrams, 8, 454-474.
- 1230 Ol'shanskii, Y.I. (1951) Equilibria of two immiscible liquids in systems of alkaline earth  
1231 silicates. Doklady Akademii Nauk SSSR, 76, 93-96.
- 1232 Pelton, A.D., Degterov, S.A., Eriksson, G., Robelin, C., and Dessureault, Y. (2000) The  
1233 modified quasichemical model I - binary solutions. Metallurgical and Materials Transactions  
1234 B, 31, 651-659.
- 1235 Perdew, J.P. (1985) Density functional theory and the band gap problem. International Journal of  
1236 Quantum Chemistry, 28, 497-523.
- 1237 Perdew, J.P., and Wang, Y. (1992) Accurate and simple analytic representation of the electron-  
1238 gas correlation energy. Physical Review B, 45, 13244-13249.
- 1239 Pickard, C.J., and Needs, R.J. (2009) Dense low-coordination phases of lithium. Physical Review  
1240 Letters, 102, 146401.

- 1241 Plank, T., and Langmuir, C.H. (1988) An evaluation of the global variations in the major element  
1242 chemistry of arc basalts. *Earth and Planetary Science Letters*, 90, 349-370.
- 1243 Poe, B.T., McMillan, P.F., rubie, D.C., Chakraborty, S., Yarger, J., and Diefenbacher, J. (1997)  
1244 Silicon and oxygen self-diffusivities in silicate liquids measured to 15 gigapascals and 2800  
1245 kelvin. *Science*, 276, 1245-1248.
- 1246 Putnis, A. (1996) Quantification of disorder in silicate melts, glasses and crystals using NMR  
1247 spectroscopy. *Physics and Chemistry of Minerals*, 23, 247.
- 1248 Rein, R.H., and Chipman, J. (1965) Activities in the liquid solution  $\text{SiO}_2\text{-CaO-MgO-Al}_2\text{O}_3$  at  
1249  $1600^\circ$ . *Transactions of the American Institute of Mining, Metallurgical and Petroleum*  
1250 *Engineers*, 233, 415-425.
- 1251 Revcolevschi, A. (1976) Cooling-rate determination in splat-cooling of oxides. *Journal of*  
1252 *Materials Science*, 11, 563-565.
- 1253 Richter, F.M., Davis, A.M., DePaolo, D.J., and Watson, E.B. (2003) Isotope fractionation by  
1254 chemical diffusion between molten basalt and rhyolite. *Geochimica et Cosmochimica Acta*,  
1255 67, 3905-3923.
- 1256 Rigden, S.M., Ahrens, T.J., and Stolper, E.M. (1988) Shock compression of molten silicate:  
1257 Results for a model basaltic composition. *Journal of Geophysical Research: Solid earth*, 93  
1258 (B1), 367-382.
- 1259 Ringwood, A.E. (1962) A model for the upper mantle. *Journal of Geophysical Research*, 67,  
1260 857-867.
- 1261 Sakamaki, T., Ohtani, E., Urakawa, S., Suzuki, A., and Katayama, Y. (2009) Measurement of  
1262 hydrous peridotite magma density at high pressure using the x-ray absorption method. *Earth*  
1263 *and Planetary Science Letters*, 287, 293-297.



- 1264 Sakamaki, T., Ohtani, E., Urakawa, S., Suzuki, A., and Katayama, Y. (2010a) Density of dry  
1265 peridotite magma at high pressure using an X-ray absorption method. American  
1266 Mineralogist, 95, 144-147.
- 1267 Sakamaki, T., Ohtani, E., Urakawa, S., Suzuki, A., Katayama, Y., and Zhao, D. (2010b) Density  
1268 of high-Ti basalt magma at high pressure and origin of heterogeneities in the lunar mantle.  
1269 Earth and Planetary Science Letters, 299, 285-289.
- 1270 Sakamaki, T., Ohtani, E., Urakawa, S., Terasaki, H., and Katayama, Y. (2011) Density of  
1271 carbonated peridotite magma at high pressure using an X-ray absorption method. American  
1272 Mineralogist, 96, 553-557.
- 1273 Saksengwitt, A., and Heuer, A. (2007) Finite-size effects in silica: a landscape perspective.  
1274 Journal of Physics: Condensed Matter, 19, 205143.
- 1275 Salanne, M., Rotenberg, B., Jahn, S., Vuilleumier, R., Simon, C., and Madden, P.A. (2012)  
1276 Including many-body effects in models for ionic liquids. Theoretical Chemistry Accounts,  
1277 131, 1143.
- 1278 Schön, J.C., Pentin, I.V., and Jansen, M. (2006) Ab initio computation of low-temperature phase  
1279 diagrams exhibiting miscibility gaps. Physical Chemistry Chemical Physics, 8, 1778-1784.
- 1280 Schubert, G., Turcotte, D.L., and Olson, P. (2001) Mantle Convection in the Earth and Planets.  
1281 Cambridge University Press.
- 1282 Simon, S., Wilke, M., Chernikov, R., Klemme, S., and Hennet, L. (2013) The influence of  
1283 composition on the local structure around yttrium in quenched silicate melts - Insights from  
1284 EXAFS. Chemical Geology, 346, 3-13.
- 1285 Sipp, A., and Richet, P. (2002) Equivalence of volume, enthalpy and viscosity relaxation kinetics  
1286 in glass-forming silicate liquids. Journal of Non-Crystalline Solids, 298, 202-212.

- 1287 Soler, J. M., Artacho, E., Gale, J. D., García, A., Junquera, J., Ordejón, P., and Sánchez-Portal,  
1288 D. (2002) The SIESTA method for ab initio order-N materials simulation. *Journal of Physics:*  
1289 *Condensed Matter*, 14, 2745-2779.
- 1290 Solomatov, V.S., and Stevenson, D.J. (1993) Suspension in convective layers and style of  
1291 differentiation of a terrestrial magma ocean. *Journal of Geophysical Research: Planets*, 98  
1292 (E3), 5375-5390.
- 1293 Soules, T.F., Gilmer, G.H., Matthews, M.J., Stolken, J.S., and Feit, M.D. (2011) Silica molecular  
1294 dynamic force fields—A practical assessment. *Journal of Non-Crystalline Solids*, 357, 1564-  
1295 1573.
- 1296 Sparks, D.W., and Parmentier, E.M. (1991) Melt extraction from the mantle beneath spreading  
1297 centers. *Earth and Planetary Science Letters*, 105, 368-377.
- 1298 Spaulding, D.K., McWilliams, R.S., Jeanloz, R., Eggert, J.H., Celliers, P.M., Hicks, D.G.,  
1299 Collins, G.W., and Smith, R.F. (2012) Evidence for a phase transition in silicate melt at  
1300 extreme pressure and temperature conditions. *Physical Review Letters*, 108, 065701.
- 1301 Spera, F. J., Ghiorso, M. S., and Nevins, D. (2011) Structure, thermodynamic and transport  
1302 properties of liquid MgSiO<sub>3</sub>: Comparison of molecular models and laboratory results.  
1303 *Geochimica et Cosmochimica Acta*, 75, 1272-1296.
- 1304 Stackhouse, S., Stixrude, L., and Karki, B.B. (2010) Determination of the high-pressure  
1305 properties of fayalite from first-principles calculations. *Earth and Planetary Science Letters*,  
1306 289, 449-456.
- 1307 Strassner, T., and Taige, M.A. (2005) Evaluation of functionals O3LYP, KMLYP, and MPW1K  
1308 in comparison to B3LYP for deleted transition-metal compounds. *Journal of Chemical*  
1309 *Theory and Computation*, 1, 848-855.

- 1310 Stephens, P.J., Devlin, F.J., Chabalowski, C.F., and Frisch, M.J. (1994) Ab initio calculation of  
1311 vibrational absorption and circular dichroism spectra using density functional force fields.  
1312 The Journal of Physical Chemistry, 98, 11623-11627.
- 1313 Sterten, A., and Maeland, I. (1985) Thermodynamics of molten mixtures of sodium  
1314 hexafluoroaluminate-aluminum oxide and sodium fluoride-aluminum fluoride. Acta Chemica  
1315 Scandinavica, A39, 241-257.
- 1316 Stixrude, L., and Karki, B. (2005) Structure and freezing of MgSiO<sub>3</sub> liquid in Earth's lower  
1317 mantle. Science, 310, 297-299.
- 1318 Stixrude, L., de Koker, N., Sun, N., Mookherjee, M., and Karki, B.B. (2009) Thermodynamics of  
1319 silicate liquids in the deep Earth. Earth and Planetary Science Letters, 278, 226-232.
- 1320 Stolper, E., Walker, D., Hager, B.H., and Hays, J.F. (1981) Melt segregation from partially  
1321 molten source regions: The importance of melt density and source region size. Journal of  
1322 Geophysical Research: Solid Earth, 86, 6261-6271.
- 1323 Sugiyama, K., Shinkai, T., and Waseda, Y. (1996) New ADXD/EDXD spectrometer for  
1324 determining the structure of melts at high temperature. Science Reports of the Research  
1325 Institutes, Tohoku University, A42, 231-237.
- 1326 Tikunoff, D., and Spera, F. (2014) Thermal conductivity of molten and glassy NaAlSi<sub>3</sub>O<sub>8</sub>,  
1327 CaMgSi<sub>2</sub>O<sub>6</sub> and Mg<sub>2</sub>SiO<sub>4</sub> by non equilibrium molecular dynamics at elevated  
1328 temperature and pressure: Methods and results. American Mineralogist, in press.
- 1329 Tkatchenko, A., and Scheffler, M. (2009) Accurate molecular van der Waals interactions from  
1330 ground-state electron density and free-atom reference data. Physical Review Letters, 102,  
1331 073005.

- 1332 Tran, F., and Blaha, P. (2009) Accurate band gaps of semiconductors and insulators with a  
1333 semilocal exchange-correlation potential. *Physical Review Letters*, 102, 226401.
- 1334 Urbain, G., Bottinga, Y., and Richet, P. (1982) Viscosity of liquid silica, silicates, and  
1335 aluminosilicates. *Geochimica et Cosmochimica Acta*, 46, 1061-1072.
- 1336 van de Walle, A., and Ceder, G. (1999) Correcting overbinding in local-density-approximation  
1337 calculations. *Physical Review B*, 59, 14992-15001.
- 1338 Vetere, F., Behrens, H., Schuessler, J.A., Holtz, F., Misiti, V., and Borchers, L. (2008) Viscosity  
1339 of andesite melts and its implication for magma mixing prior to Unzen 1991–1995 eruption.  
1340 *Journal of Volcanology and Geothermal Research*, 175, 208-217.
- 1341 Vinet, P., Ferrante, J., Rose, J.H., and Smith, J.R. (1987) Compressibility of solids. *Journal of*  
1342 *Geophysical Research: Solid Earth*, 92, 9319-9325.
- 1343 Vollmayr, K., Kob, W., and Binder, K. (1996) Cooling-rate effects in amorphous silica: A  
1344 computer-simulation study. *Physical Review B*, 54, 15808-15827.
- 1345 von Bargen, N., and Waff, H.S. (1986) Permeabilities, interfacial areas and curvatures of  
1346 partially molten systems: Results of numerical computations of equilibrium microstructures.  
1347 *Journal of Geophysical Research: Solid Earth*, 91, 9261-9276.
- 1348 Vosko, S.H., Wilk, L., and Nusair, M., (1980) Accurate spin-dependent electron liquid  
1349 correlation energies for local spin density calculations: A critical analysis. *Canadian Journal*  
1350 *of Physics*, 58, 1200-11.
- 1351 Vuilleumier, R., Sator, N., and Guillot, B. (2009) Computer modeling of natural silicate melts:  
1352 What can we learn from ab initio simulations. *Geochimica et Cosmochimica Acta*, 73, 6313-  
1353 6339.

- 1354 Wan, J.T.K., Duffy, T.S., Scandolo, S., and Car, R. (2007) First-principles study of density,  
1355 viscosity, and diffusion coefficients of liquid  $\text{MgSiO}_3$  at conditions of the Earth's deep  
1356 mantle. *Journal of Geophysical Research, Solid Earth*, 112, B03208.
- 1357 Wentzcovitch, R.M., Yu, Y.G., and Wu, Z. (2010) Thermodynamic properties and phase  
1358 relations in mantle minerals investigated by first principles quasiharmonic theory. *Reviews in*  
1359 *Mineralogy and Geochemistry*, 71, 59-98.
- 1360 Whittington, A.G., Bouhifd, M.A., and Richet, P. (2009) Amorphous materials: properties,  
1361 structure, and durability: The viscosity of hydrous  $\text{NaAlSi}_3\text{O}_8$  and granitic melts:  
1362 Configurational entropy models. *American Mineralogist*, 94, 1-16.
- 1363 Williams, Q., and Garnero, E.J. (1996) Seismic evidence for partial melt at the base of Earth's  
1364 mantle. *Science*, 273, 1528-1530.
- 1365 Wu, P., Eriksson, G., Pelton, A.D., and Blander, M. (1993) Prediction of the thermodynamic  
1366 properties and phase diagrams of silicate systems -- Evaluation of the  $\text{FeO-MgO-SiO}_2$   
1367 system. *ISIJ International*, 33, 26-35.
- 1368 Yeh, I.-C., and Hummer, G. (2004) System-size dependence of diffusion coefficients and  
1369 viscosities from molecular dynamics simulations with periodic boundary conditions. *The*  
1370 *Journal of Physical Chemistry B*, 108, 15873-15879.
- 1371 Zaitsev, A.I., Shelkova, N.E., and Mogutnov, B.M. (2000) Thermodynamics of  $\text{Na}_2\text{O-SiO}_2$   
1372 melts. *Inorganic Materials*, 36, 529-543.
- 1373 Zhang, L. (2011) Thermodynamic properties calculation for  $\text{MgO-SiO}_2$  liquids using both  
1374 empirical and first-principles molecular simulations. *Physical Chemistry Chemical Physics*,  
1375 13, 21009-21015.

- 1376 Zhang, Y., and Ni, H. (2010) Diffusion of H, C, and O components in silicate melts. Reviews in  
1377 Mineralogy and Geochemistry, 72, 171-225.
- 1378 Zhang, Y., Ni, H., and Chen, Y. (2010) Diffusion data in silicate melts. Reviews in Mineralogy  
1379 and Geochemistry, 72, 311-408.
- 1380 Zhang, Y., Guo, G., Refson, K., and Zhao, Y. (2004) Finite-size effect at both high and low  
1381 temperatures in molecular dynamics calculations of the self-diffusion coefficient and  
1382 viscosity of liquid silica. Journal of Physics: Condensed Matter, 16, 9127-9135.
- 1383

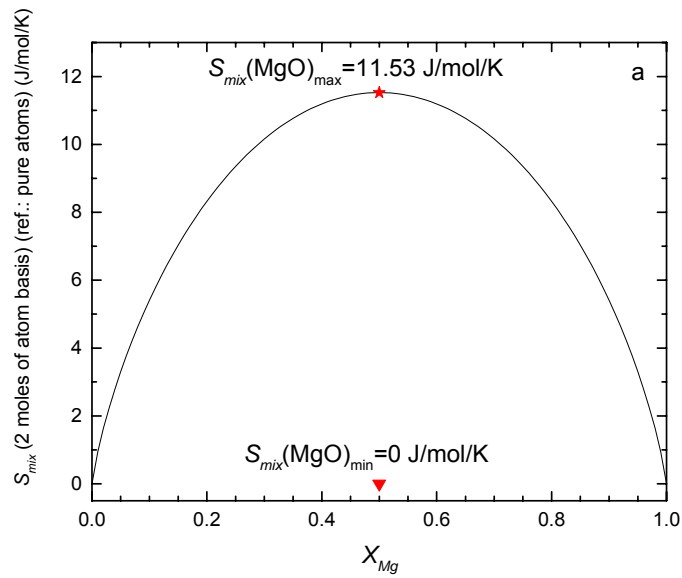
1384 Table 1: Evaluation of corrections to internal energy  $\Delta U_{emp} = -P_{emp} (V_0^{FST} - V_0)$  and enthalpy  
 1385  $\Delta H_{emp} = \Delta U_{emp} + (P_{emp} V_0^{FST} - P_0 V_0^{FST})$  induced by correction for LDA overbinding using an  
 1386 empirical pressure shift  $P_{emp}$ .

Compo.	$V_0$ (cm <sup>3</sup> /mol) <sup>a</sup>	$P_{emp}$ (GPa) <sup>a</sup>	$V_0^{FST}$ (cm <sup>3</sup> /mol)	$\Delta U_{emp}$ (J/mol)	$\Delta H_{emp}$ (J/mol)
MgO	16.46	1.23	15.91	677	20246
$Mg_{5/6}Si_{1/6}O_{7/6}$	17.92	1.54	16.97	1457	27572
$Mg_{2/3}Si_{1/3}O_{4/3}$	19.47	1.74	18.11	2369	33915
$Mg_{3/5}Si_{2/5}O_{7/5}$	20.10	1.79	18.60	2691	36059
$Mg_{1/2}Si_{1/2}O_{3/2}$	20.90	1.84	19.24	3054	38456
$Mg_{1/3}Si_{2/3}O_{5/3}$	22.50	1.83	20.34	3958	41226
$Mg_{1/4}Si_{3/4}O_{7/4}$	23.38	1.79	21.11	4052	41812
$Mg_{1/6}Si_{5/6}O_{11/6}$	24.33	1.72	21.99	4022	41821
		2.34 <sup>b</sup>		5476	56932
SiO <sub>2</sub>	27.80	1.50	24.63	4755	41700

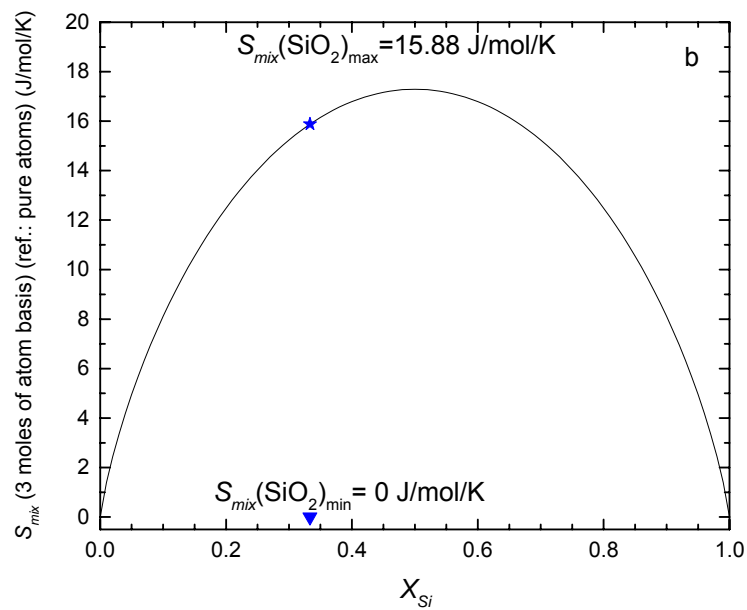
1387 Note(s): The Helmholtz energy function parameterized by de Koker et al. (2013) based on the  
 1388 finite strain theory is applied to each MgO-SiO<sub>2</sub> melts composition at 3000 K and 0 GPa in order  
 1389 to evaluate uncorrected volumes  $V_0^{FST}$  presented in this table.

1390 <sup>a</sup> Data provided by de Koker et al. (2013)

1391 <sup>b</sup> Predicted thermo-physical properties based on a new  $P_{emp}$  estimate (see Figure 5).



1392

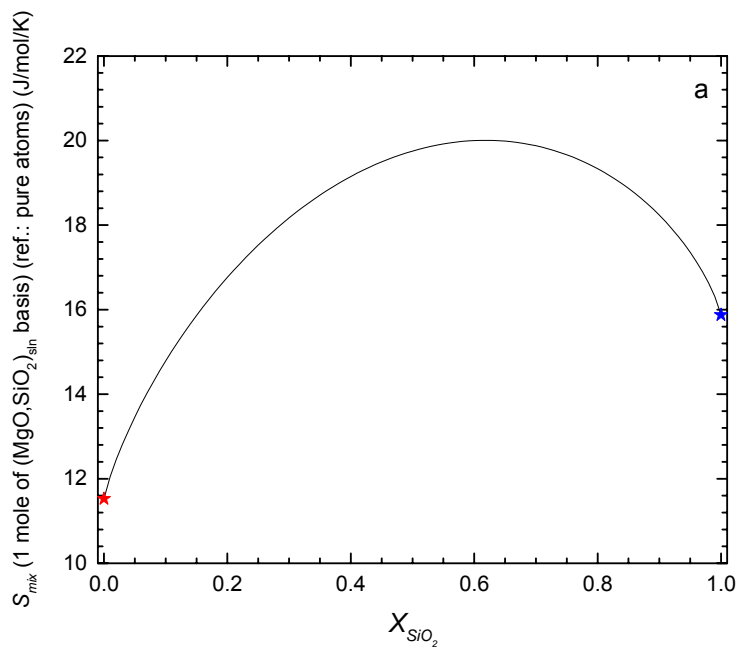


1393

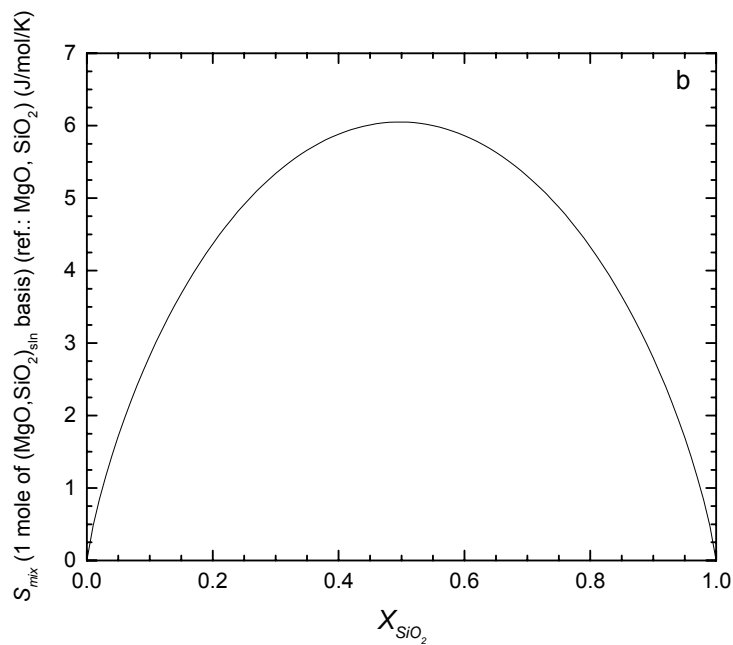
1394 Figure 1: Configurational entropy of mixing,  $S_{mix}$ , for the (a) Mg-O (2 atom basis) and (b) Si-O  
1395 (3 atom basis) systems

1396





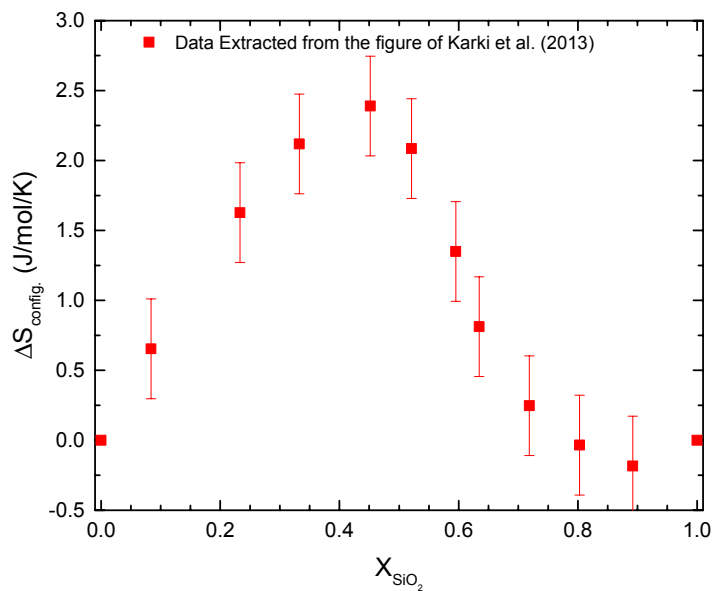
1397



1398

1399 Figure 2: Maximum configurational entropy of mixing,  $S_{mix}$ , for the  $\text{MgO-SiO}_2$  system  
1400 referenced to (a) atoms and (b) to  $\text{MgO}$  and  $\text{SiO}_2$  species

1401



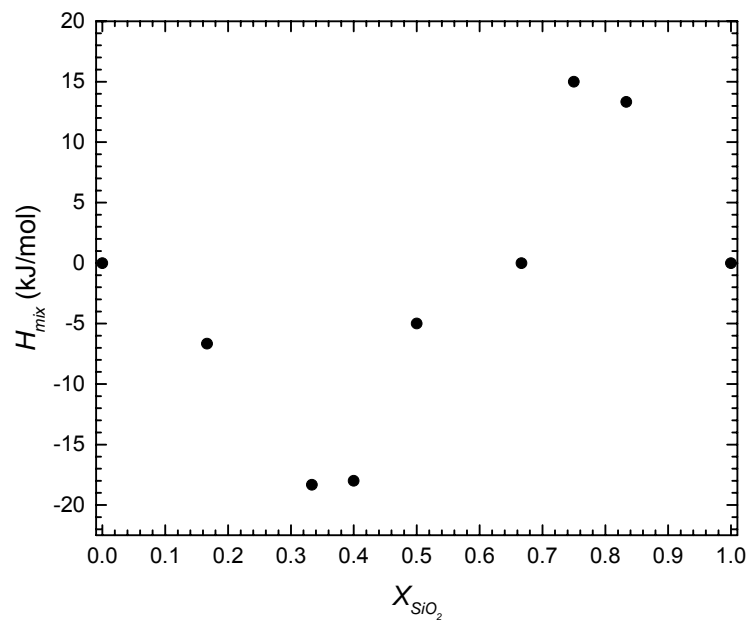
1402

1403 Figure 3: Configurational entropy of mixing,  $S_{\text{mix}}$ , at 0 GPa for the liquid MgO-SiO<sub>2</sub> system  
1404 obtained by applying the Adam-Gibbs theory to represent AIMD viscosity data as presented by

1405

Karki et al. (2013)

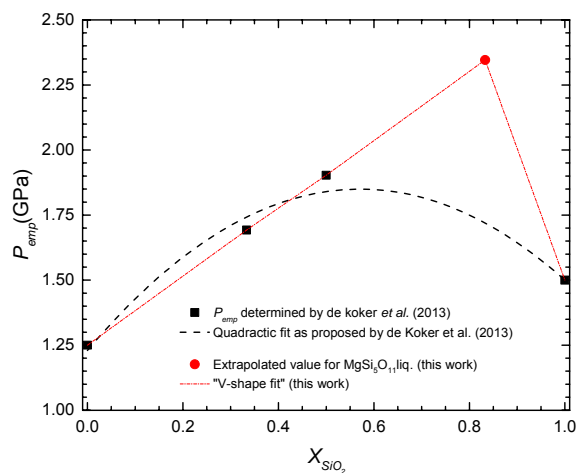
1406



1407

1408 Figure 4: Enthalpy of mixing,  $H_{mix}$ , vs  $X_{SiO_2}$  at 2000 K and 0 GPa obtained from the raw  
1409 appendix data of de Koker et al. (2013).

1410



1411

1412 Figure 5: Evaluation of the pressure correction function for systematic overbinding of LDA for

1413 the MgO-SiO<sub>2</sub> system based on computed anchor points where solid phases of known zero-

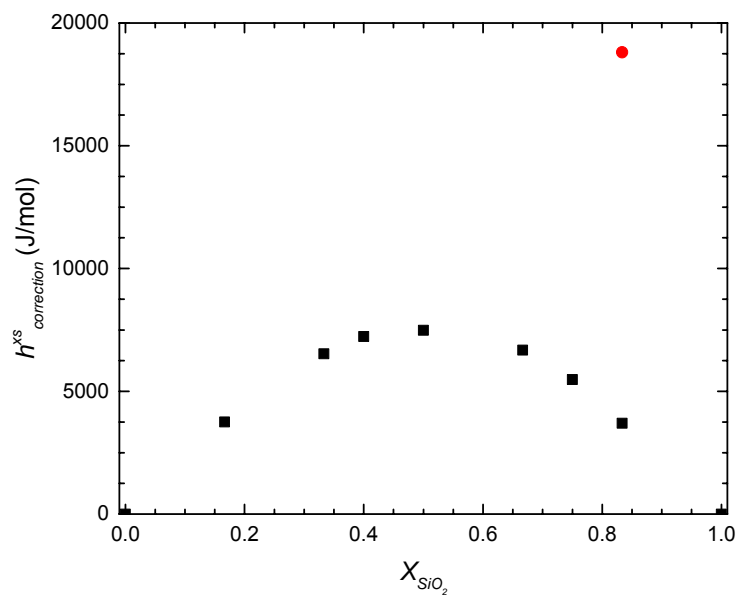
1414 pressure volume are available (square symbols), from de Koker et al. (2013). The dashed line

1415 represents the quadratic representation of de Koker et al. (2013) while the dashed and dotted line

1416 represents a “V-shape” pressure correction function based on a linear extrapolation of all the

1417 MgO-rich anchor points up to the MgSi<sub>5</sub>O<sub>11</sub> liquid composition as proposed in this work.

1418



1419

1420 Figure 6: Excess enthalpy correction  $h_{\text{correction}}^{\text{xs}}$  induced by the systematic pressure correction of  
1421 LDA calculations as proposed by de Koker et al. (2013) for the MgO-SiO<sub>2</sub> system at 0 GPa and  
1422 3000 K. Symbols presented in this figure show the  $h_{\text{correction}}^{\text{xs}}$  correction to be applied using  $P_{\text{emp.}}$  of  
1423 de Koker et al. (2013) (squares) and as proposed in the present work (filled circle).

Table 1: Evaluation of corrections to internal energy  $\Delta U_{emp} = -P_{emp} (V_0^{FST} - V_0)$  and enthalpy

$\Delta H_{emp} = \Delta U_{emp} + (P_{emp} V_0^{FST} - P_0 V_0^{FST})$  induced by correction for LDA overbinding using an empirical pressure shift  $P_{emp}$ .

Compo.	$V_0$ (cm <sup>3</sup> /mol) <sup>a</sup>	$P_{emp}$ (GPa) <sup>a</sup>	$V_0^{FST}$ (cm <sup>3</sup> /mol)	$\Delta U_{emp}$ (J/mol)	$\Delta H_{emp}$ (J/mol)
MgO	16.46	1.23	15.91	677	20246
$Mg_{5/6}Si_{1/6}O_{7/6}$	17.92	1.54	16.97	1457	27572
$Mg_{2/3}Si_{1/3}O_{4/3}$	19.47	1.74	18.11	2369	33915
$Mg_{3/5}Si_{2/5}O_{7/5}$	20.10	1.79	18.60	2691	36059
$Mg_{1/2}Si_{1/2}O_{3/2}$	20.90	1.84	19.24	3054	38456
$Mg_{1/3}Si_{2/3}O_{5/3}$	22.50	1.83	20.34	3958	41226
$Mg_{1/4}Si_{3/4}O_{7/4}$	23.38	1.79	21.11	4052	41812
$Mg_{1/6}Si_{5/6}O_{11/6}$	24.33	1.72	21.99	4022	41821
		2.34 <sup>b</sup>		5476	56932
SiO <sub>2</sub>	27.80	1.50	24.63	4755	41700

Note(s): The Helmholtz energy function parameterized by de Koker et al. (2013) based on the finite strain theory is applied to each MgO-SiO<sub>2</sub> melts composition at 3000 K and 0 GPa in order to evaluate uncorrected volumes  $V_0^{FST}$  presented in this table.

<sup>a</sup> Data provided by de Koker et al. (2013)

<sup>b</sup> Predicted thermo-physical properties based on a new  $P_{emp}$  estimate (see Fig. 5).