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Temperature dependence of crystal structure of CaGeO₃ high-pressure perovskite phase and experimental determination of its Debye temperatures studied by low- and high-temperature single crystal X-ray diffraction

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ABSTRACT

24 Single crystal X-ray diffraction study of CaGeO_3 perovskite has been conducted over the
25 temperature range of 98 to 1048 K. The crystal begins to deteriorate at a temperature above about
26 900 K and completely amorphizes by 980 K. The diffraction-intensity distribution and the structure
27 refinements indicate that the $Pbnm$ structure is kept until the occurrence of amorphization. The
28 obtained unit-cell parameters, unit-cell volumes, bond lengths and displacement parameters increase
29 monotonously with increasing temperature. Thus, no evidence for the existence of the $Cmcm$
30 high-temperature phase, previously suggested above 520 K, is observed. The Ge–O bond lengths
31 show much smaller thermal expansions than the Ca–O bond lengths; the former ranges between
32 $0.42(2) \times 10^{-5} \text{ K}^{-1}$ and $0.57(2) \times 10^{-5} \text{ K}^{-1}$, and the latter between $1.58(4) \times 10^{-5} \text{ K}^{-1}$ and $3.96(6) \times$
33 10^{-5} K^{-1} . The Debye temperatures and static disorder components for each constituent atom were
34 determined by applying the Debye model to the temperature dependence of mean square
35 displacements (MSDs) of the atoms. Consequently, no significant static disorder components can be
36 detected in each atom. The Debye temperatures averaged over all directions, obtained from the
37 Debye model fitting to U_{eq} , yield the harmonic one particle potential coefficients of $4.76(2) \text{ eV}\text{\AA}^{-2}$
38 for Ca, $11.0(1) \text{ eV}\text{\AA}^{-2}$ for Ge, $5.02(2) \text{ eV}\text{\AA}^{-2}$ for O1 and $5.33(5) \text{ eV}\text{\AA}^{-2}$ for O2. These values become
39 larger in order of $\text{Ca} < \text{O1} < \text{O2} \ll \text{Ge}$, which shows that the one particle potential of Ge is much
40 narrower than that of Ca. This relationship between Ca and Ge is consistent reasonably with bonding
41 stiffness expected from the thermal expansion coefficients of the bond lengths. The anisotropies of
42 MSDs are remarkable in O1 and O2 atoms as a consequence of the strong interaction with adjacent
43 Ge atoms, forming the rigid bonds with these O atoms.

44 In comparison of the three $Pbnm$ orthorhombic perovskites of CaGeO_3 , CaTiO_3 and MgSiO_3 , all
45 of these have the BO_6 octahedra more rigid than the AO_{12} polyhedra ($A = \text{Ca}$ or Mg ; $B = \text{Ge}$, Ti or
46 Si) and the tilt angles of BO_6 octahedra are the largest in MgSiO_3 perovskite. These observations
47 indicate that if MgSiO_3 perovskite under high pressures undergoes the same sequence of the
48 high-temperature phase transitions as CaTiO_3 perovskite, the phase boundaries have positive
49 Clapeyron slopes and the phase transition temperatures should become further much higher than
50 those (1512 K for the $Pbnm$ to $I4/mcm$ transition and 1635 K for the $I4/mcm$ to $Pm\bar{3}m$ transition)
51 observed in CaTiO_3 perovskite at ambient pressure. This leads to the conclusion that the
52 high-temperature phase transition to a perovskite phase with different symmetry under high
53 pressures previously suggested in MgSiO_3 perovskite is unlikely.

54 **Keywords:** CaGeO₃, perovskite, single crystal X-ray diffraction, phase transition, thermal expansion,
55 Debye temperature, one particle potential

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INTRODUCTION

59 MgSiO₃ perovskite is the most dominant constituent in the Earth's lower mantle. Because of such
60 importance, studies on physical properties and crystal structure of MgSiO₃ perovskite have
61 extensively been performed from both sides of experiments (e.g., Horiuchi et al. 1987; Kudoh et al.
62 1987; Ross and Hazen 1989) and theoretical simulations (e.g. Dekura et al. 2013; Tsuchiya et al.
63 2004; Wentzcovitch et al. 2004). However, the quenched samples of MgSiO₃ perovskite recovered at
64 ambient condition are well known to rapidly amorphize under electron beam in transmission
65 electron microscope, and such low thermal stability has severely limited the experimental
66 investigations of its structural and physical properties at high temperatures (Madon et al. 1989).
67 Because of this, various analogues with higher thermal stability, such as CaGeO₃ (McMillan and
68 Ross 1988; Durben et al. 1991; Liu et al. 1991), CaTiO₃ (McMillan and Ross 1988; Guyot et al.
69 1993; Liu and Liebermann 1993; Ali and Yashima 2005; Yashima and Ali 2009) and NaMgF₃
70 (Yoshiasa et al. 2005) have been investigated to help the understanding for high-temperature
71 properties of MgSiO₃ perovskite.

72 In particular CaGeO₃ is an excellent analogue of MgSiO₃, because both compounds not only are
73 of the A²⁺B⁴⁺O₃-type but also exhibit the similar sequence of phase transitions at high pressures and
74 high temperatures of pyroxenoid (wollastonite) type → tetragonal garnet type → orthorhombic
75 perovskite type (Prewitt and Sleight 1969; Susaki et al. 1985). The perovskite phase of CaGeO₃ was
76 confirmed to have the *Pbnm* structure (orthorhombic), isostructural with MgSiO₃ perovskite, at
77 ambient condition by single crystal X-ray diffraction (Sasaki et al. 1983), although it had previously
78 been indexed as a cubic perovskite (Ringwood and Major 1967). The crystal structure consists of a
79 network of corner-linked GeO₆ octahedra with the larger Ca atoms located at the centers of cavities
80 in the network and is distorted largely from the ideal cubic structure with *Pm* $\bar{3}$ *m* symmetry owing to
81 the tilting of GeO₆ octahedra (Figs. 1a–1c). Later, Liu et al. (1991) suggested from the
82 high-temperature powder X-ray diffraction that CaGeO₃ perovskite undergoes the phase transition to
83 the *Cmcm* structure (orthorhombic) near 520 K at ambient pressure. This suggestion was based on
84 (1) the appearance of the reflections that should be forbidden in *Pbnm* but allowed in *Cmcm* and (2)

85 the convergence of the *a*- and *b*-axis lengths above 520 K. However, in the observation (1), the very
86 weak peaks assigned as the forbidden reflections in *Pbnm* seem to be invisible as far as we see the
87 powder X-ray diffraction pattern shown by Liu et al. (1991). In the observation (2), the
88 determination of the unit-cell parameters from their Rietveld refinements was performed by fixing
89 the atomic positions of O atoms to the values given by the single-crystal structure refinement at
90 room temperature (Sasaki et al. 1983). No heat capacity anomaly at 520 K was also detected in their
91 calorimetric measurements (Liu et al. 1991), which is consistent with the implications of the
92 high-temperature Raman scattering study (Durben et al. 1991) that no symmetry breaking transition
93 occurs in CaGeO₃ perovskite at high temperatures. There is thus some doubt as for the existence of
94 the *Cmcm* high-temperature phase, and the sufficient structural knowledge at high temperatures has
95 not been provided even in the *Pbnm* phase.

96 We here report the single crystal X-ray diffraction study of CaGeO₃ perovskite in the range of 98
97 to 1048 K, to investigate its structural behavior at high temperatures. Together with the examination
98 as for whether or not the phase transition to the *Cmcm* phase is present, temperature dependence of
99 the crystal structure of the *Pbnm* phase is discussed with attention to thermal expansions and tilting
100 of GeO₆ octahedra. Moreover, the harmonic one particle potentials of each atom are evaluated from
101 the Debye temperatures determined from temperature dependence of mean square displacements
102 (MSDs) of atoms to gain knowledge of bonding stiffness.

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EXPERIMENTS AND ANALYSIS

105 **Single-crystal growth under high pressure**

106 Single crystals of CaGeO₃ perovskite were grown at 12 GPa and 1253 K using a Kawai-type
107 high-pressure apparatus installed at the ISEI of Okayama University. A 14 mm regular octahedron of
108 a sintered MgO containing 5% of Cr₂O₃ was employed as a pressure-transmitting-medium. The
109 starting material was powdered CaGeO₃ wollastonite prepared by solid-state reaction of special
110 grade reagents of CaCO₃ and GeO₂, and was mixed with a 5 mol% of PbO flux. The mixture was
111 enclosed in a gold capsule and then set in the center of a MgO sleeve by inserting ZrO₂ plugs into
112 the upper- and lower-ends of the sleeve. After that, the MgO sleeve was inserted into a cylindrical
113 graphite heater and then put into a ZrO₂ sleeve embedded in the MgO octahedron; the ZrO₂ sleeve
114 was used as a thermal insulator. This cell assembly was set in the anvil assembly of tungsten carbide
115 cubes with truncated edge lengths of 7 mm, and then was compressed with the aid of the high

116 pressure apparatus. The sample temperature was monitored by the W25%Re-W3%Re thermocouple
117 with 0.05 mm in diameter. The junction of the thermocouple was put at the midpoint of the outer
118 surface of the MgO sleeve. No correction was made for the pressure effect on emf. After being kept
119 under a desired condition (12 GPa, 1253 K) for 1 hour, the product was quenched by shutting off the
120 electric power supply. The pressure was released slowly and the product was recovered at ambient
121 condition. Numerical single crystals of CaGeO₃ perovskite with the size of about 100–200 μm were
122 found in the recovered sample.

123

124 **X-ray diffraction intensity measurements and structure refinements**

125 The single crystal with the size of 0.15 × 0.14 × 0.14 mm³ was selected and put into a silica-glass
126 capillary for X-ray diffraction experiments. The experiments were conducted in the range of 98 to
127 1048 K using a Rigaku AFC-7R four-circle diffractometer with a graphite-monochromatized Mo
128 K α radiation ($\lambda = 0.71069 \text{ \AA}$) at the operating condition of 60 kV and 250 mA. The experiments
129 below room temperature were conducted by directly cooling the crystal through a continuous cold
130 N₂ gas flow generated using a Rigaku GN2 low temperature apparatus. In the experiments above
131 room temperature, high temperature was achieved by a continuous flow of N₂ gas heated by electric
132 resistance heater and thereby the crystal was directly heated. Prior to the data collections, sample
133 temperatures were calibrated using a K-type thermocouple set exactly at the sample position.
134 Temperature fluctuation during the data collections was kept within ± 0.2 K. The unit-cell parameters
135 at 33 temperature-points in the range of 98 to 898 K were determined by the least-squares method
136 from a set of 25 reflections within the range of $46^\circ \leq 2\theta \leq 50^\circ$. X-ray diffraction intensity data within
137 $2^\circ \leq 2\theta \leq 100^\circ$ were collected at 18 temperature-points in this temperature range using the
138 continuous ω - 2θ scan mode. Between 2446 and 2522 reflections were measured for each
139 temperature. The intensity data were corrected for Lorentz-polarization factors and absorption effects
140 (ψ -scan method). After these corrections, the intensity data were averaged in Laue symmetry *mmm*
141 to give between 1123 and 1156 independent reflections for each temperature. Of these, independent
142 reflections with $F_o \leq 3\sigma(F_o)$ were eliminated. Even if independent reflections had intensities of $F_o >$
143 $3\sigma(F_o)$ after averaging, those averaged from data set of equivalent reflections including reflection(s)
144 with $F_o \leq 3\sigma(F_o)$ were also discarded since these reflections were potentially affected by multiple
145 scattering. Moreover, independent reflections with $(\sin\theta)/\lambda < 0.329 \text{ \AA}^{-1}$ were eliminated to reduce
146 secondary extinction effects and to avoid dependence on atomic charge as far as possible in the

147 choice of atomic scattering factors. Finally, between 653 and 844 independent reflections were used
148 in the present refinements at each temperature. Internal residuals of the equivalent reflections (R_{int})
149 varied between 0.0092 and 0.0203 for each temperature.

150 The structure refinements were carried out by minimizing the function $\sum w(F_o - F_c)^2$ using a full
151 matrix least-squares program RADY (Sasaki 1987). Scattering factors of Ca^{2+} , Ge^{4+} (*International*
152 *Tables for Crystallography*, Table 6.1.1.3; Wilson 1992) and O^{2-} (Tokonami 1965) were used.
153 Anomalous dispersion coefficients for each scattering factor were taken from *International Tables*
154 *for Crystallography* (Table 4.2.6.8; Wilson 1992). Several correction models for the secondary
155 extinction effects were attempted during the refinements, and the isotropic correction of Type I
156 (Becker and Coppens 1974a, 1974b) with a Lorentzian mosaic spread distribution model yielded the
157 best fits. The structure refinements at each temperature converged smoothly to $R = 0.0142\text{--}0.0229$
158 and $wR = 0.0117\text{--}0.0184$. The summary of data collection and refinement parameters is given in
159 Table 1. The positional parameters and equivalent isotropic displacement parameters (U_{eq}) are given
160 in Table 2. CIF is available on deposit.

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RESULTS AND DISCUSSION

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Evidence for absence of the phase transition to the *Cmcm* structure

164 As shown in Figure 1c, the *Pbnm* orthorhombic and *Cmcm* orthorhombic structures have the
165 approximate unit-cell dimensions of $\sqrt{2} \times \sqrt{2} \times 2$ and $2 \times 2 \times 2$ to the pseudo-cubic lattice,
166 respectively. The relationship among their unit-cell edge lengths is expressed as $a_p \approx \sqrt{2}a_c/2 \approx$
167 $\sqrt{2}a_0$, $b_p \approx \sqrt{2}b_c/2 \approx \sqrt{2}a_0$ and $c_p \approx c_c \approx 2a_0$, where the subscripts “p”, “c” and “0”
168 represent *Pbnm*, *Cmcm* and pseudo-cubic structures, respectively. The temperature dependence of
169 the present unit-cell parameters (a_p , b_p , c_p , α_p , β_p , γ_p) and volume (V_p) measured by assuming the
170 $\sqrt{2} \times \sqrt{2} \times 2$ lattice is shown in Figures 2 and 3, respectively. The a_p , b_p , c_p and V_p increase
171 monotonously with increasing temperature. The a_p approaches to b_p while keeping $a_p < b_p$ up to
172 about 680 K, and then a_p gets ahead of b_p above this temperature after both agreed (Fig. 2a). During
173 such variations of the unit-cell edge lengths, the α_p , β_p and γ_p angles are independent of temperature
174 and exhibit no deviation from 90° (Fig. 2b). This situation continues up to about 900 K, but the
175 diffraction peaks begin to broaden above this temperature and completely disappear by 980 K (Fig.
176 4). This agrees well with the Raman scattering study by Durben et al. (1991), which reported that
177 CaGeO_3 perovskite, metastable at ambient pressure, amorphizes near 923 K. They also reported that

178 the amorphous phase is an intermediate phase and recrystallizes to the thermodynamically stable
179 wollastonite phase near 1023 K. However, no diffraction peak could be detected in our single crystal
180 X-ray diffraction experiments up to 1048 K after the amorphization. This is probably because the
181 present sample recrystallized as a fine-grained polycrystalline assemblage.

182 The *Cmcm* structure requires the $2 \times 2 \times 2$ lattice to be orthorhombic. This shows that if the
183 phase transition of *Pbnm* to *Cmcm* occurs, the unit-cell parameters measured assuming the
184 $\sqrt{2} \times \sqrt{2} \times 2$ lattice must satisfy the condition of " $a_p = b_p$ and $\gamma_p \neq 90^\circ$ " at the phase transition
185 point and above, as illustrated in Figure 5. The symmetry constraints also require that this phase
186 transition must necessarily be first order (Durben et al. 1991; Liu et al. 1991). However, the present
187 data shows that the condition of " $a_p = b_p$ and $\gamma_p \neq 90^\circ$ " is not satisfied over the investigated
188 temperature range and no resolvable discontinuity is observed in the temperature dependence of the
189 unit-cell edge lengths and volume (Figs. 2 and 3).

190 To gain further evidence for the absence of the *Cmcm* high-temperature phase, we measured
191 ω -scan profiles at positions of reflections that should be forbidden in the *Pbnm* structure but allowed
192 in the *Cmcm* structure, at the 9 temperature-points in the range between 98 and 898 K. As examples,
193 the profiles measured at the positions of 104, 300 and 302 reflections in the $\sqrt{2} \times \sqrt{2} \times 2$ lattice
194 (corresponding to 114, 330 and 332 reflections in the $2 \times 2 \times 2$ lattice, respectively) are shown in
195 Figure 6. As shown in this figure, none of these reflections could be detected at all over the
196 investigated temperature range. The coincidence of equivalent-reflection intensities assuming the
197 *Pbnm* structure is much better than that assuming the *Cmcm* structure over the investigated
198 temperature range (e.g., $R_{\text{int}} = 0.0125$ for *Pbnm* and $R_{\text{int}} = 0.0495$ for *Cmcm* at 723 K). Indeed, the
199 crystal structures were successfully refined in the *Pbnm* structure over the investigated temperature
200 range, and the reliability indices reached $R = 0.0142$ – 0.0229 and $wR = 0.0117$ – 0.0184 for each
201 temperature. Meanwhile, the refinements assuming *Cmcm* resulted in much higher reliability indices
202 (e.g., $R = 0.0836$ and $wR = 0.0487$ at 723 K) with non-positive-definite anisotropic displacement
203 parameters. It is evident from these observations that the phase transition of *Pbnm* to *Cmcm*,
204 suggested by Liu et al. (1991), is absent and that the crystal structure keeps *Pbnm* orthorhombic
205 symmetry over the investigated temperature range although the temperature dependence of a_p and b_p
206 intersects at about 680 K. This conclusion agrees with the high-temperature Raman scattering study
207 (Durben et al. 1991) that showed no evidence for the phase transition in CaGeO_3 perovskite.

208 The high-temperature powder X-ray diffraction study of Liu et al. (1991) reported that a_p and b_p

209 converge at about 520 K. This convergence is a base of their argument that the phase transition of
210 *Pbnm* to *Cmcm* occurs at this temperature. However, in the present data, such convergence is not
211 observed at this temperature (Fig. 2a). To examine the reason for this discrepancy, we show in Figure
212 7 the temperature dependence of $\sqrt{2}a_p/2$, $\sqrt{2}b_p/2$ and $c_p/2$, corresponding to the unit-cell
213 edge lengths in the pseudo-cubic lattice. These lengths are very close (e.g. $\sqrt{2}a_p/2 = 3.7299(3)$ Å,
214 $\sqrt{2}b_p/2 = 3.7323(4)$ Å, $c_p/2 = 3.7323(3)$ Å at 523 K). In particular, it is noteworthy that the
215 temperature dependence of the present $\sqrt{2}b_p/2$ and $c_p/2$ (Fig. 7) intersects at about 520 K, the
216 same temperature as the convergence point of a_p and b_p suggested by Liu et al. (1991). Such
217 similarity of the unit-cell edge lengths makes it difficult to index the powder X-ray diffraction pattern
218 especially around this temperature. The discrepancy in temperature dependence of a_p and b_p between
219 our and Liu's studies is thus likely to originate in mis-indexing of powder X-ray diffraction pattern
220 by the latter study.

221

222 Thermal expansion

223 **Unit-cell volume.** The unit-cell volume data (Fig. 3) were fitted to the following equation (Suzuki
224 et al. 1979) derived from the Grüneisen theory of volumetric thermal expansion, using a non-linear
225 weighted least-squares method:

$$V(T) = \frac{V_t}{2k_0 a_v} \left(1 + 2k_0 - \sqrt{1 - \frac{4k_0 U(T)}{Q_0}} \right) \quad \dots (1)$$

$$U(T) = 9nRT \left(\frac{T}{\Theta_D} \right)^3 \int_0^{\Theta_D/T} \frac{x^3}{\exp(x) - 1} dx \quad \dots (2)$$

226 where $Q_0 = K_0 V_0 / \gamma_G$, $k_0 = (dK_0/dP - 1)/2$ and $a_v = V_t/V_0$; V_0 and K_0 are the volume and the bulk
227 modulus at 0 K, respectively, and V_t is the volume at a reference temperature t ; γ_G is the Grüneisen
228 parameter, $U(T)$ the internal energy, n the number of atoms in the formula unit, R the gas constant,
229 Θ_D the Debye temperature and T the absolute temperature. The fitting parameters resulted in $\Theta_D =$
230 $666(16)$ K, $Q_0 = 3.92(2) \times 10^6$ J/mol and $a_v (= V_{298}/V_0) = 1.00397(7)$ by fixing k_0 to 2.7. This fixed
231 k_0 value was approximated by substituting the pressure derivative of isothermal bulk modulus
232 $\partial K_T / \partial P (= 6.4)$ at 300 K (Liu and Li 2007; Liu et al. 2008) for dK_0/dP .

233 Liu et al. (2008) approximated the temperature dependence of isothermal bulk modulus K_T of
234 CaGeO_3 perovskite in the temperature range of their experiments ($300 \leq T \leq 1100$ K) by the

235 following linear equation:

$$\begin{aligned} K_T(T) &= K_T(300 \text{ K}) + (\partial K_T / \partial T)_P (T - 300 \text{ K}) \\ &= 193 - 0.025(T - 300) \text{ GPa} \quad \dots (3) \end{aligned}$$

236 Alternatively, its temperature dependence might be represented by the following theoretical equation
237 that was originally derived for the adiabatic bulk modulus $K_S(T)$ (Anderson 1966):

$$K_T(T) = K_0 - (g/V_0)U(T) \quad \dots (4)$$

238 where $g = \gamma_G \delta$; δ is the Anderson-Grüneisen parameter. To evaluate the K_0 value, the data within the
239 temperature range of Liu's experiments ($300 \leq T \leq 1100 \text{ K}$) calculated from Equation (3) were fitted
240 to Equation (4). The fit converged to $K_0 = 196 \text{ GPa}$ and $g = 6.57$ by fixing at $\Theta_D = 666 \text{ K}$ obtained
241 above, yielding $\gamma_G (= K_0 V_0 / Q_0) = 1.55$ and hence $\delta (= g / \gamma_G) = 4.24$. This γ_G is in fair agreement with
242 the value (1.35) from the high-pressure experiments (Liu and Li 2007; Liu et al. 2008).

243 The temperature dependence of the volumetric thermal expansion coefficients
244 $\alpha_V(T) [= 1/V(T) \cdot dV(T)/dT]$ obtained by differentiation of Equation (1) is also given in Figure
245 3. The $\alpha_V(T)$ increases steeply with heating up to about 350 K, after which it increases almost
246 linearly with a gentle slope above about 650 K.

247 **Unit-cell edge lengths and bond lengths.** Figures 8 and 9 show the temperature dependence of
248 the bond lengths and their thermal expansion coefficients $\alpha_L(T) [= 1/L(T) \cdot dL(T)/dT]$ together
249 with those of the unit-cell edge lengths, respectively, where $L(T)$ is geometric dimensions such as
250 unit-cell edge lengths and bond lengths as a function of temperature and functions obtained from the
251 parabolic fits were employed for $L(T)$. The resulting $\alpha_L(T)$ data was fitted to the approximation
252 $\alpha_L(T) = \varepsilon_0 + \varepsilon_1 T + \varepsilon_2 T^{-2}$ ($\varepsilon_2 \leq 0$) (Fei 1995), and the coefficients ε_0 , ε_1 and ε_2 determined for
253 each geometric observation are tabulated in Table 3. For comparison with other compounds, the
254 conventional mean thermal expansion coefficients $\langle \alpha_L \rangle$, determined from the fits to the linear
255 equation $L(T) = L_r \{1 + \langle \alpha_L \rangle (T - T_r)\}$, are also listed in Table 3, where L_r is geometric
256 dimensions at a reference temperature T_r , and 298 K was adopted as T_r in the present study.

257 The thermal expansivities of the unit-cell edges increase in order of $b_p < c_p < a_p$. Such anisotropy
258 in the thermal expansion is consistent with those of other *Pbnm* perovskites such as CaTiO_3 (Liu and
259 Liebermann 1993) and MgSiO_3 (Ross and Hazen 1989). In ABO_3 perovskites with the *Pbnm*
260 structure, the great structural distortion due to the tilting of BO_6 octahedra yields much longer
261 separations between an A atom and four of twelve O atoms surrounding its A atom. In CaGeO_3
262 perovskite, the four longer $\text{Ca}\cdots\text{O}$ separations [$\text{Ca}\cdots\text{O1}^{\text{i}}$, $\text{Ca}\cdots\text{O1}^{\text{ii}}$, $\text{Ca}\cdots\text{O2}^{\text{iii}}$ and $\text{Ca}\cdots\text{O2}^{\text{iv}}$] range

263 between 2.856(2) and 3.072(1) Å at 298 K, and the remaining eight shorter ones [Ca···O1^v, Ca···O1,
264 Ca···O2^{vi}, Ca···O2^v, Ca···O2^{vii}, Ca···O2^{viii}, Ca···O2^{ix} and Ca···O2^x] range between 2.343(2) and
265 2.599(1) Å at 298 K (Table 4). The bond valence sums assuming the twelve-fold coordinated Ca
266 atom deviate largely from the valences of Ca and O atoms, whereas those assuming the eight-fold
267 coordinated Ca atom agree well with their valences (Table 4). Thus, the four longer Ca···O
268 separations are not involved in chemical bonding. This can also be understood from the observations
269 that these separations exhibit largely negative thermal expansions (Fig. 9 and Table 3).

270 The symmetrically non-equivalent three Ge–O bond lengths are very close as shown in Fig. 8 and
271 Table 4. They do not depend largely on temperature and have much smaller thermal expansion
272 coefficients than Ca–O bond lengths; the former ranges between $0.42(2) \times 10^{-5} \text{ K}^{-1}$ and $0.57(2) \times$
273 10^{-5} K^{-1} and the latter varies between $1.58(4) \times 10^{-5} \text{ K}^{-1}$ and $3.96(6) \times 10^{-5} \text{ K}^{-1}$ (Table 3). The
274 symmetrical constraints always request 180° for O1–Ge–O1^{xi}, O2^{vii}–Ge–O2^{xiii} and O2^{ix}–Ge–O2^{xii}
275 angles, and the remaining twelve O–Ge–O angles in a GeO₆ octahedron vary between 89.22(1)° and
276 90.78(1)° at 98 K. These variable bond-angles only show the fluctuation of 0.3° at maximum in the
277 investigated temperature range and hardly depend on temperature. Thus, individual GeO₆ is rigid
278 and keeps a nearly regular octahedron independent of temperature. This shows that the response of
279 CaGeO₃ perovskite structure to temperature is dominated mainly by the tilting between
280 corner-linked GeO₆ octahedra as will be described in the next section.

281

282 Temperature dependence of GeO₆ octahedral tilting

283 The octahedral tilting can be characterized by the three tilt angles (Glazer 1972, 1975). We here
284 use ϕ_i^+ , ϕ_i^- and ϕ_i^0 ($i = x, y$ or z) after Yashima and Ali (2009) as a notation of the tilt angles. The
285 ϕ_i^+ , ϕ_i^- and ϕ_i^0 indicate the in-phase tilt angle of octahedra, the out-of-phase tilt angle and no
286 octahedral tilting about i -axis ($i = x, y$ or z), respectively. The x -, y - and z -axes represent [100]₀,
287 [010]₀ and [001]₀, respectively, where the subscript “0” represents the pseudo-cubic lattice. The
288 tilting system of $Pbnm$ orthorhombic perovskites is described by two identical out-of-phase tilting
289 about the [100]₀ and [010]₀ axes ($\phi_x^- = \phi_y^-$) and an in-phase tilting about the [001]₀ axis (ϕ_z^+),
290 being expressed as $\phi_x^- \phi_y^- \phi_z^+$ ($\phi_x^- = \phi_y^-$). This tilting system is also described as $a^- a^- c^+$ using
291 well-known Glazer’s notation (Glazer 1972, 1975). The tilt angles have often been defined only
292 from the fractional coordinates of O atoms as in Kennedy et al. (1999). However, the tilt angles
293 calculated in this way will more or less be influenced by the variation of distortion of octahedra

294 themselves with temperature. We here calculated them via the symmetry-adapted mode approach,
295 which can completely separate the tilts and distortions of octahedra. In terms of this approach, the tilt
296 angle ϕ is given by $\phi = \tan^{-1}(2d')$ (Wang and Angel 2011), where d' is the amplitude of
297 octahedral tilt mode. The d' values are converted from the standard supercell-normalized
298 amplitude “ As ” and “ $normfactor$ ” calculated using the program *ISODISTORT* (or the earlier
299 *ISODISPLACE*; Campbell et al. 2006). The details are described in Wang and Angel (2011).

300 Figure 10 shows the temperature dependence of the tilt angles $\phi_x^- (= \phi_y^-)$ and ϕ_z^+ of GeO_6
301 octahedra in CaGeO_3 perovskite calculated in this way, together with that of the positional
302 parameters x and y of Ca atoms, located at the coordinates $(x, y, 0.25)$. For comparison, that of
303 CaTiO_3 perovskite (Yashima and Ali 2009) with the same $A^{2+}B^{4+}O_3$ -type $Pbnm$ structure as CaGeO_3
304 perovskite is also shown in this figure. The ϕ_x^- and ϕ_z^+ in CaGeO_3 perovskite decrease with
305 increasing temperature (Fig. 10a). The Ca atom approaches its ideal position $(0, 0, 0.25)$,
306 corresponding to the Ca position in the $Pm\bar{3}m$ cubic structure, with increasing temperature (Fig.
307 10b). These tendencies are consistent with the case of CaTiO_3 perovskite. The negative thermal
308 expansions of the four non-bonding $\text{Ca}\cdots\text{O}$ separations (Fig. 9 and Table 3) are a consequence of
309 such decrease in structural distortion and a sign of the approach to the $Pm\bar{3}m$ cubic structure with
310 twelve equivalent Ca–O bond lengths and without any octahedral tilting ($\phi_x^0 \phi_y^0 \phi_z^0$).

311 The existence of the intermediate orthorhombic phase $Cmcm$ at high temperatures had previously
312 been suggested in CaTiO_3 perovskite (Guyot et al. 1993; Kennedy et al. 1999) as well. However, the
313 recent high-temperature powder neutron diffraction studies of CaTiO_3 perovskite (Ali and Yashima
314 2005; Yashima and Ali 2009) proved the absence of $Cmcm$ phase, as well as in the present CaGeO_3
315 perovskite, and instead reported the phase transitions of orthorhombic $Pbnm \rightarrow$ tetragonal $I4/mcm$ at
316 1512 K and tetragonal $I4/mcm \rightarrow$ cubic $Pm\bar{3}m$ at 1635 K. Unfortunately, the present sample of
317 metastable CaGeO_3 perovskite, recovered at ambient condition, amorphized at the temperature
318 beyond 900 K. However, CaGeO_3 perovskite may undergo the same sequence of the
319 high-temperature phase transitions as CaTiO_3 perovskite under high pressures, from the similarity in
320 the temperature dependence of crystal structure between both compounds described above.

321

322 **Temperature dependence of MSDs of atoms**

323 Figures 11 and 12 show the temperature dependence of equivalent isotropic displacement
324 parameter U_{eq} and MSD_{hkl} of each atom, respectively, where U_{eq} corresponds to mean square

325 displacement (MSD) of atom averaged over all directions; MSD_{hkl} is MSD of atom in the $[hkl]_p$
326 direction and the subscript “p” represents the *Pbnm* orthorhombic lattice. The MSD determined by
327 diffraction method includes the contributions of both static and dynamic disorders, and can be
328 described on the basis of the Debye model as follows (Willis and Pryor 1975):

$$MSD = \langle u^2 \rangle_s + \langle u^2 \rangle_d = \langle u^2 \rangle_s + \frac{3\hbar^2 T}{mk_B \Theta_D^2} \left[\frac{T}{\Theta_D} \int_0^{\frac{\Theta_D}{T}} \frac{x}{\exp(x) - 1} dx + \frac{\Theta_D}{4T} \right] \dots (5)$$

329 where $\langle u^2 \rangle_s$ is the temperature-independent static disorder component, $\langle u^2 \rangle_d$ the
330 temperature-dependent dynamic disorder component, m the mass of atoms, k_B the Boltzmann
331 constant, \hbar the reduced Planck constant, Θ_D the Debye temperature and T the absolute temperature.
332 A non-linear least squares fitting of the U_{eq} or MSD_{hkl} data to Equation (5) (Debye model fitting)
333 allows us to evaluate $\langle u_{eq}^2 \rangle_s$ and $\Theta_{D_{eq}}$ or, $\langle u_{hkl}^2 \rangle_s$ and $\Theta_{D_{hkl}}$ where $\langle u_{eq}^2 \rangle_s$ and $\Theta_{D_{eq}}$, determined from
334 the U_{eq} data, are the static disorder component and the Debye temperature averaged over all
335 directions, respectively; $\langle u_{hkl}^2 \rangle_s$ and $\Theta_{D_{hkl}}$ determined from the MSD_{hkl} data, are the static disorder
336 component and the Debye temperature in the $[hkl]_p$ direction, respectively.

337 The temperature dependence of U_{eq} and MSD_{hkl} for each atom is well represented by the Debye
338 model over the investigated temperature range (Figs.11 and 12). Their variations are approximately
339 linear in the high-temperature range and begin to flatten below about 200–300 K. This behavior at
340 low temperatures is an obvious indication of the zero-point energy contribution. The $\langle u_{eq}^2 \rangle_s$ values
341 resulted in $-0.0003(1) \text{ \AA}^2$ for Ca, $-0.0004(1) \text{ \AA}^2$ for Ge, $0.0000(1) \text{ \AA}^2$ for O1 and $-0.0007(2) \text{ \AA}^2$ for O2,
342 and can be regarded as zero within the error of about 3σ although some of them converged to the
343 slightly negative values. Therefore, we can consider that the static disorder is not present in each
344 atom, and the Debye model fitting was finally performed by fixing at $\langle u_{eq}^2 \rangle_s = 0$ or $\langle u_{hkl}^2 \rangle_s = 0$. The
345 final values of $\Theta_{D_{eq}}$ and $\Theta_{D_{hkl}}$ obtained in this way are provided in Table 5. The average of the
346 resulting $\Theta_{D_{eq}}$ values for each atom [$= \{\Theta_{D_{eq}}(\text{Ca}) + \Theta_{D_{eq}}(\text{Ge}) + \Theta_{D_{eq}}(\text{O1}) + 2\Theta_{D_{eq}}(\text{O2})\}/5$] is 636(2)
347 K, being in agreement with the Debye temperature Θ_D of 666(16) K determined from the volumetric
348 thermal expansion.

349 At high temperatures of $T > \Theta_D$, where the contribution of zero-point energy is negligible, the
350 dynamic disorder component, i.e. thermal vibration component, in Equation (5) reduces to
351 $\langle u^2 \rangle_d = \{3\hbar^2/(mk_B \Theta_D^2)\}T$. The slope [$= 3\hbar^2/(mk_B \Theta_D^2)$] in this linear equation is equal to k_B/q
352 when we assume the harmonic one particle potential $V_{OPP}(u) = (q/2)\langle u^2 \rangle_d$ (Willis and Pryor

353 1975), where q is the one particle potential (OPP) coefficient. Therefore, the OPP coefficient q is
354 related to the Debye temperature Θ_D by

$$q = \frac{mk_B^2\Theta_D^2}{3\hbar^2} \dots (6)$$

355 The increment of q value, corresponding to a force constant, indicates the enhancement of bonding
356 stiffness. The substitution of $\Theta_{D_{eq}}$ for Θ_D in Equation (6) can provide the OPP coefficient, q_{eq} ,
357 averaged over all directions. The present q_{eq} values are calculated as 4.76(2) eVÅ⁻² for Ca, 11.0(1)
358 eVÅ⁻² for Ge, 5.02(2) eVÅ⁻² for O1 and 5.33(5) eVÅ⁻² for O2, and become larger in order of Ca <
359 O1 < O2 << Ge. This relationship between Ca and Ge is reasonably consistent with bonding stiffness
360 expected from the thermal expansion coefficients of the bond lengths (Fig. 9 and Table 3). The q_{eq}
361 values of O1 and O2 atoms are significantly smaller than those of O atoms observed usually in
362 silicates (≈ 6 eVÅ⁻²). For comparison with silicates, as an example, we show the q_{eq} values
363 calculated using the $\Theta_{D_{eq}}$ data of pyrope (Mg₃Al₂Si₃O₁₂), a silicate garnet, reported by our previous
364 study (Nakatsuka et al. 2011): 3.34(2) eVÅ⁻² for ^{VIII}Mg, 8.36(4) eVÅ⁻² for ^{VI}Al, 9.45(7) eVÅ⁻² for
365 ^{IV}Si, 5.89(2) eVÅ⁻² for O, where the superscripts IV, VI and VIII represent the four-, six- and
366 eight-fold coordination, respectively. It is noteworthy that the q_{eq} value of ^{VI}Ge in CaGeO₃
367 perovskite is significantly larger than that of ^{IV}Si in pyrope although the former atom has the higher
368 coordination number and the longer distances to ligands than the latter atom. This large stiffness of
369 ^{VI}Ge–O bonds can also be understood from the above observation that the distortion of GeO₆
370 octahedron is almost temperature-independent.

371 As shown in example at 298 K (Fig. 13), the atomic thermal vibration is almost isotropic in Ge
372 atom over the investigated temperature range, but shows obvious anisotropies in Ca, O1 and O2
373 atoms. Such anisotropies are remarkable especially in O1 and O2 atoms, and their displacement
374 ellipsoids elongate near perpendicularly to Ge–O1 and Ge–O2 bonds, respectively. When we focus
375 on the O2–Ge–O2 linkage formed by an O2 atom and its adjacent O2 atom in a GeO₆ octahedron
376 (such as O2^{vii}–Ge–O2^{xii}, O2^{xii}–Ge–O2^{xiii}, O2^{ix}–Ge–O2^{xiii} or O2^{vii}–Ge–O2^{ix} linkage in Fig. 13), the
377 largest ellipsoid-axes of the corresponding two O2 atoms run out of the plane made by this
378 O2–Ge–O2 linkage. The largest ellipsoid-axes of an O1 atom and its adjacent O2 atom in a GeO₆
379 octahedron are also under the similar situation. Such out-of-plane bending vibrations of Ge–O bonds
380 correspond to the twisting vibrations of perovskite framework centered on Ge atoms, often discussed
381 in spectroscopic studies of perovskite compounds. The displacement ellipsoids of O2 atoms elongate

382 also toward Ca atoms, which is indicative of the stretching vibrations of Ca–O2 bonds. The same
383 vibrational manners as these are observed over the investigated temperature range.

384 Meanwhile, the shortest ellipsoid-axes of O atoms run completely along $[001]_p$ for O1 and nearly
385 along $[110]_p$ for O2 over the investigated temperature range. These directions are consistent nearly
386 with the directions of Ge–O1 and Ge–O2 bonds, respectively (Fig. 13). Thus, the large anisotropies
387 of MSDs of O1 and O2 atoms will originate in the strong interaction with adjacent Ge atoms,
388 forming the rigid bonds with these O atoms. For a closer estimation of the bonding stiffness, we here
389 evaluate the anisotropic OPP coefficients q_{hkl} by the substitution of $\Theta_{D_{hkl}}$ for Θ_D in Equation (6),
390 where q_{hkl} is the OPP coefficient in the $[hkl]_p$ direction. The OPP coefficients of Ge and O1 atoms in
391 the direction of Ge–O1 bond can be approximated by $q_{001}(\text{Ge}) = 9.90(10) \text{ eV}\text{\AA}^{-2}$ evaluated from
392 $\Theta_{D_{001}}(\text{Ge})$ and $q_{001}(\text{O1}) = 10.3(2) \text{ eV}\text{\AA}^{-2}$ evaluated from $\Theta_{D_{001}}(\text{O1})$, respectively. Both values are
393 comparable. The OPP coefficient of O2 atom in the direction of Ge–O2 bond can be approximated
394 by $q_{110}(\text{O2}) = 12.5(3) \text{ eV}\text{\AA}^{-2}$ evaluated from $\Theta_{D_{110}}(\text{O2})$. These q_{hkl} values show that Ge–O2 bond is
395 more rigid than Ge–O1 bond, as also expected from the thermal expansivities of both bonds (Fig. 9
396 and Table 3).

397

398

IMPLICATIONS

399 The present study has revealed that GeO_6 octahedra in CaGeO_3 perovskite are considerably rigid.
400 Such knowledge of rigidities of polyhedra in perovskites is quite important because the relative
401 compressibility of AO_{12} and BO_6 polyhedra in ABO_3 perovskites is closely associated with their
402 phase transition behaviors under high pressures and high temperatures (Zhao et al. 2004; Angel et al.
403 2005). The compressibility ratio (β_B/β_A) of the two polyhedra is given by $\beta_B/\beta_A = M_A/M_B$
404 (Zhao et al. 2004), where the subscripts “A” and “B” represent the AO_{12} and BO_6 polyhedra,
405 respectively. The parameter M_i ($i = \text{A or B}$) is usually defined as

$$M_i = (R_i N_i / B) \exp[(R_0 - R_i) / B] \quad \dots (7)$$

406 where N_i and R_i are the coordination number of cations and the average bond length at ambient
407 condition, respectively; R_0 and B the bond valence parameters. In most $Pbnm$ orthorhombic
408 perovskites, Equation (7) is a good approximation for M_B , but it is invalid for M_A because of
409 considerable distortions of the AO_{12} polyhedra and the following approximation was proposed for its
410 calculation (Zhao et al. 2004):

$$M_A = (8 R_{A8} / B) \exp[(R_0 - R_{A8}) / B] + (4 R_{A4} / B) \exp[(R_0 - R_{A4}) / B] \quad \dots (8)$$

411 where R_{A8} and R_{A4} are the average distances of eight shorter A–O bonds and of four longer A–O
412 separations, respectively. According to Angel et al. (2005), when the BO_6 octahedra are more rigid
413 than the AO_{12} polyhedra (i.e., $M_A/M_B < 1$), the phase transition temperature T_c rises with
414 increasing pressure as a consequence of the increase in the octahedral tilting; thus, the phase
415 boundary has a positive Clapeyron slope ($dP/dT_c > 0$). Conversely, when the BO_6 octahedra are
416 less rigid than the AO_{12} polyhedra (i.e., $M_A/M_B > 1$), T_c reduces with increasing pressure as a
417 consequence of the decrease in the tilting; thus, the phase boundary has a negative Clapeyron slope
418 ($dP/dT_c < 0$). When we here consider the three *Pbnm* orthorhombic perovskites of $CaGeO_3$ (the
419 present study), $CaTiO_3$ (Yashima and Ali 2009) and $MgSiO_3$ (Sugahara et al. 2006), the M_A/M_B
420 ratios at ambient condition calculated using Equation (7) for M_B and Equation (8) for M_A are 0.66,
421 0.56 and 0.67, respectively, where the bond valence parameters R_0 and B used here are the values
422 quoted for the bond valence sum calculations in Table 4. It follows therefore that if these three
423 perovskites undergo phase transitions to perovskite phases with different symmetries under high
424 pressures and high temperatures, their phase boundaries have positive Clapeyron slopes. This result
425 is quite significant for the understanding of phase transitions in the Earth's interior, especially in
426 $MgSiO_3$ perovskite, the most dominant constituent in the Earth's lower mantle.

427 $MgSiO_3$ perovskite is now believed to undergo the phase transition to post-perovskite structure
428 with $CaIrO_3$ structure, associated with the D'' seismic discontinuity, at 125 GPa and 2500 K
429 (Murakami et al. 2004; Tsuchiya et al. 2004). Some high pressure studies (Wang et al. 1991; Shim et
430 al. 2001) suggested the possibility for the existence of a perovskite phase with different symmetry
431 intervening between the *Pbnm* perovskite and the post-perovskite phases. In particular, Wang et al.
432 (1991) reported from X-ray diffraction experiments up to 1253 K at a constant pressure of 7.3 GPa
433 that such a phase transition occurred near 600 K under this pressure. However, the tilt angles at
434 ambient condition of $MgSiO_3$ perovskite calculated using the structural parameters in the published
435 literature (Sugahara et al. 2006) are $\phi_x^- = 11.7^\circ$ and $\phi_z^+ = 11.6^\circ$, much larger than those of $CaGeO_3$
436 ($\phi_x^- = 7.2^\circ$, $\phi_z^+ = 7.7^\circ$) and $CaTiO_3$ ($\phi_x^- = 8.3^\circ$, $\phi_z^+ = 8.8^\circ$) shown in Figure 10. $MgSiO_3$ should have
437 the phase transition temperature T_c much higher than those of the other two perovskites because the
438 larger tilting yields the higher T_c when $M_A/M_B < 1$ as mentioned above. The increase in T_c is
439 further promoted by the increase in pressure. If the same sequence of the high-temperature phase
440 transitions as $CaTiO_3$ perovskite also appears in $MgSiO_3$ perovskite under high pressures, thus, T_c
441 should become further much higher than those (1512 K for the *Pbnm* to *I4/mcm* transition and 1635

442 K for the $I4/mcm$ to $Pm\bar{3}m$ transition; Yashima and Ali 2009) observed in CaTiO_3 perovskite at
443 ambient pressure. The phase transition near 600 K at 7.3 GPa suggested in MgSiO_3 perovskite
444 (Wang et al. 1991) is therefore unlikely.

445 Finally, we emphasize the Earth-scientific significance for the experimental determination of the
446 Debye temperatures from MSDs and for its application to the Earth's interior materials and their
447 related materials. The Debye temperature is generally evaluated from macroscopic properties such as
448 heat capacity and volumetric thermal expansion; these approaches only provide the values averaged
449 for the whole constituent atoms in crystals and over all directions. In contrast, the Debye model
450 fitting to temperature dependence of MSDs determined from X-ray diffraction can not only
451 determine the Debye temperatures individually for each constituent atom in crystals but also provide
452 their anisotropies. The determination of anisotropies in the Debye temperatures of each atom enables
453 the quantitative estimation of anisotropies in the magnitudes of interatomic interactions as shown in
454 the present discussion on OPP. Moreover, the Debye model fitting to temperature dependence of
455 MSDs is a good approach to detect the static disorders of atoms and evaluate the directions and
456 quantities of atomic displacements due to the static disorders, as shown in our previous study on
457 pyrope garnet (Nakatsuka et al. 2011). Consequently, we can separately discuss the dynamic nature
458 and the static nature of the atomic disorders in crystals including the static disorder. Since the static
459 disorders of atoms influence entropy and hence free energy, the separation of dynamic disorder
460 components and static disorder components from MSDs is important to discuss energetic structural
461 stabilities of crystals from microscopic viewpoints. Application of the Debye model fitting to
462 temperature dependence of MSDs to various Earth's interior materials and their related materials can
463 thus provide significant insights into their structural stabilities, being promising for further
464 understanding of the mechanisms of phase transformations occurring in the Earth's interior.

465

466

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575

Figure captions

576 Figure 1. Crystal structure of CaGeO₃ perovskite: (a) a crystallographic viewing, (b) the projection
577 along [010]_p and (c) the projection along [001]_p. Symmetry codes for equivalent atoms are as in
578 Table 3.

579

580 Figure 2. Temperature dependence of the unit-cell parameters (a) a_p , b_p , c_p and (b) α_p , β_p , γ_p .

581

582 Figure 3. Temperature dependence of the unit-cell volume (V_p) and the volumetric thermal
583 expansion coefficient $\alpha_V(T)$. The fitted curve of the V_p data is based on Equation (1).

584

585 Figure 4. Temperature dependence of the ω -scan profile of 200 reflection.

586

587 Figure 5. Relationship between $\sqrt{2} \times \sqrt{2} \times 2$ and $2 \times 2 \times 2$ lattices.

588

589 Figure 6. Temperature dependence of the ω -scan profiles measured at the positions of 104, 300 and
590 302 reflections. These correspond to 114, 330 and 332 reflections in the $2 \times 2 \times 2$ lattice,
591 respectively.

592

593 Figure 7. Temperature dependence of the unit-cell edge lengths in pseudo-cubic lattice ($\sqrt{2}a_p/2$,
594 $\sqrt{2}b_p/2$ and $c_p/2$).

595

596 Figure 8. Temperature dependence of the interatomic distances.

597

598 Figure 9. Temperature dependence of the thermal expansion coefficients $\alpha_L(T)$ of unit-cell edge
599 lengths and interatomic distances.

600

601 Figure 10. Temperature dependence of (a) the octahedral tilt angles ϕ_x^- and ϕ_z^+ calculated from
602 the symmetry-adapted mode approach (Wang and Angel, 2011) and (b) the Ca positional parameters.
603 Solid and open symbols represent CaGeO₃ perovskite (the present study) and CaTiO₃ perovskite
604 (Yashima and Ali, 2009), respectively. The sharp decrease in ϕ_x^- of CaTiO₃ perovskite is observed
605 at the temperature beyond 1423 K owing to the approach of the phase transition to the *I4/mcm*

606 tetragonal phase at 1512 K.

607

608 Figure 11. Temperature dependence of U_{eq} . Solid curves represent the Debye model fitting.

609

610 Figure 12. Temperature dependence of MSD_{hkl} ($hkl = 100, 010, 001, 110$) in the $[hkl]_p$ directions. In
611 the O2 atom, MSD in the direction of the shortest ellipsoid-axis is plotted as MSD_{110} because the
612 axis is parallel nearly to $[110]_p$ within $\pm 2^\circ$ over the investigated temperature range. Solid curves
613 represent the Debye model fitting.

614

615 Figure 13. Displacement ellipsoids at 298 K projected along $[001]_p$. Atoms are drawn at 80%
616 probability level.

617

618

619 Table 1. Summary of data collection and refinement parameters

| | | | | | | |
|--|--------------|--------------|--------------|--------------|--------------|--------------|
| <i>T</i> (K) | 98 | 123 | 173 | 223 | 273 | 298 |
| Cell setting | orthorhombic | orthorhombic | orthorhombic | orthorhombic | orthorhombic | orthorhombic |
| Space group | <i>Pbnm</i> | <i>Pbnm</i> | <i>Pbnm</i> | <i>Pbnm</i> | <i>Pbnm</i> | <i>Pbnm</i> |
| <i>a</i> (Å) | 5.2539(3) | 5.2544(4) | 5.2564(3) | 5.2585(3) | 5.2609(2) | 5.2631(3) |
| <i>b</i> (Å) | 5.2672(4) | 5.2680(5) | 5.2687(4) | 5.2694(4) | 5.2705(3) | 5.2709(3) |
| <i>c</i> (Å) | 7.4354(5) | 7.4376(6) | 7.4402(5) | 7.4433(5) | 7.4466(3) | 7.4485(4) |
| <i>V</i> (Å ³) | 205.76(2) | 205.88(3) | 206.06(2) | 206.25(2) | 206.49(2) | 206.64(2) |
| 2 θ_{\max} (°) | 100 | 100 | 100 | 100 | 100 | 100 |
| No. of measured reflections | 2446 | 2446 | 2448 | 2450 | 2450 | 2454 |
| No. of independent reflections | | | | | | |
| No. of observed reflections [$F_o > 3\sigma(F_o)$] | | | | | | |
| R_{int} | 0.0101 | 0.0097 | 0.0099 | 0.0092 | 0.0094 | 0.0093 |
| No. of reflections using refinements | 844 | 833 | 827 | 821 | 799 | 798 |
| No. of parameters | 29 | 29 | 29 | 29 | 29 | 29 |
| <i>R</i> | 0.0152 | 0.0144 | 0.0151 | 0.0156 | 0.0159 | 0.0152 |
| <i>wR</i> | 0.0141 | 0.0125 | 0.0143 | 0.0145 | 0.0148 | 0.0143 |
| <i>T</i> (K) | 323 | 373 | 423 | 473 | 523 | 573 |
| Cell setting | orthorhombic | orthorhombic | orthorhombic | orthorhombic | orthorhombic | orthorhombic |
| Space group | <i>Pbnm</i> | <i>Pbnm</i> | <i>Pbnm</i> | <i>Pbnm</i> | <i>Pbnm</i> | <i>Pbnm</i> |
| <i>a</i> (Å) | 5.2644(2) | 5.2673(3) | 5.2697(3) | 5.2722(4) | 5.2748(4) | 5.2779(4) |
| <i>b</i> (Å) | 5.2719(2) | 5.2733(3) | 5.2750(4) | 5.2768(4) | 5.2782(5) | 5.2801(5) |
| <i>c</i> (Å) | 7.4507(3) | 7.4548(4) | 7.4581(4) | 7.4615(5) | 7.4646(6) | 7.4689(6) |
| <i>V</i> (Å ³) | 206.78(1) | 207.07(2) | 207.33(2) | 207.60(2) | 207.83(3) | 208.15(3) |
| 2 θ_{\max} (°) | 100 | 100 | 100 | 100 | 100 | 100 |
| No. of measured reflections | 2455 | 2458 | 2458 | 2459 | 2474 | 2482 |
| No. of independent reflections | | | | | | |
| No. of observed reflections [$F_o > 3\sigma(F_o)$] | | | | | | |
| R_{int} | 0.0102 | 0.0096 | 0.0096 | 0.0123 | 0.0116 | 0.0116 |
| No. of reflections using refinements | 788 | 769 | 747 | 745 | 742 | 727 |
| No. of parameters | 29 | 29 | 29 | 29 | 29 | 29 |
| <i>R</i> | 0.0154 | 0.0161 | 0.0144 | 0.0212 | 0.0142 | 0.0148 |
| <i>wR</i> | 0.0143 | 0.0143 | 0.0122 | 0.0158 | 0.0127 | 0.0125 |
| <i>T</i> (K) | 623 | 673 | 723 | 773 | 823 | 873 |
| Cell setting | orthorhombic | orthorhombic | orthorhombic | orthorhombic | orthorhombic | orthorhombic |
| Space group | <i>Pbnm</i> | <i>Pbnm</i> | <i>Pbnm</i> | <i>Pbnm</i> | <i>Pbnm</i> | <i>Pbnm</i> |
| <i>a</i> (Å) | 5.2815(3) | 5.2845(4) | 5.2880(4) | 5.2919(4) | 5.2956(4) | 5.2992(3) |
| <i>b</i> (Å) | 5.2825(3) | 5.2847(4) | 5.2875(4) | 5.2904(4) | 5.2928(4) | 5.2958(4) |
| <i>c</i> (Å) | 7.4741(4) | 7.4782(5) | 7.4829(5) | 7.4884(5) | 7.4929(5) | 7.4980(4) |
| <i>V</i> (Å ³) | 208.54(2) | 208.85(2) | 209.23(3) | 209.66(3) | 210.03(3) | 210.43(2) |
| 2 θ_{\max} (°) | 100 | 100 | 100 | 100 | 100 | 100 |
| No. of measured reflections | 2482 | 2486 | 2506 | 2506 | 2508 | 2522 |
| No. of independent reflections | | | | | | |
| No. of observed reflections [$F_o > 3\sigma(F_o)$] | | | | | | |
| R_{int} | 0.0130 | 0.0113 | 0.0125 | 0.0130 | 0.0138 | 0.0203 |
| No. of reflections using refinements | 714 | 687 | 689 | 677 | 659 | 653 |
| No. of parameters | 29 | 29 | 29 | 29 | 29 | 29 |
| <i>R</i> | 0.0147 | 0.0157 | 0.0156 | 0.0154 | 0.0147 | 0.0229 |
| <i>wR</i> | 0.0118 | 0.0126 | 0.0126 | 0.0128 | 0.0117 | 0.0184 |

620 Table 2. Refined positional parameters and equivalent isotropic displacement parameters U_{eq} (\AA^2)

| | T (K) | 98 | 123 | 173 | 223 | 273 | 298 |
|---------|-----------------|--------------|--------------|--------------|--------------|--------------|--------------|
| Ca (4c) | x | -0.00571(9) | -0.00565(9) | -0.00553(10) | -0.00533(10) | -0.00517(11) | -0.00509(11) |
| | y | 0.03055(5) | 0.03033(5) | 0.02979(5) | 0.02920(6) | 0.02850(6) | 0.02819(6) |
| | z | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 |
| | U_{eq} | 0.00255(5) | 0.00285(5) | 0.00345(5) | 0.00425(5) | 0.00518(6) | 0.00554(5) |
| Ge (4b) | x | 0 | 0 | 0 | 0 | 0 | 0 |
| | y | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| | z | 0 | 0 | 0 | 0 | 0 | 0 |
| | U_{eq} | 0.00095(3) | 0.00112(3) | 0.00134(3) | 0.00169(4) | 0.00215(4) | 0.00226(3) |
| O1 (4c) | x | 0.0639(3) | 0.0635(2) | 0.0627(3) | 0.0627(3) | 0.0618(3) | 0.0616(3) |
| | y | 0.4898(3) | 0.4898(2) | 0.4901(3) | 0.4902(3) | 0.4904(3) | 0.4904(3) |
| | z | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 |
| | U_{eq} | 0.0036(2) | 0.0039(2) | 0.0043(2) | 0.0049(2) | 0.0056(3) | 0.0059(2) |
| O2 (8d) | x | 0.7151(2) | 0.7150(2) | 0.7152(2) | 0.7155(2) | 0.7155(2) | 0.7158(2) |
| | y | 0.2841(2) | 0.2839(2) | 0.2837(2) | 0.2838(2) | 0.2836(2) | 0.2835(2) |
| | z | 0.0326(1) | 0.0327(1) | 0.0326(1) | 0.0324(1) | 0.0321(1) | 0.0322(1) |
| | U_{eq} | 0.0030(2) | 0.0031(2) | 0.0035(2) | 0.0041(2) | 0.0050(2) | 0.0052(2) |
| | T (K) | 323 | 373 | 423 | 473 | 523 | 573 |
| Ca (4c) | x | -0.00494(11) | -0.00484(11) | -0.00463(10) | -0.00448(14) | -0.00448(11) | -0.00431(12) |
| | y | 0.02786(6) | 0.02710(6) | 0.02641(5) | 0.02584(7) | 0.02534(6) | 0.02465(6) |
| | z | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 |
| | U_{eq} | 0.00601(6) | 0.00710(6) | 0.00794(5) | 0.00884(8) | 0.00920(6) | 0.01003(6) |
| Ge (4b) | x | 0 | 0 | 0 | 0 | 0 | 0 |
| | y | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| | z | 0 | 0 | 0 | 0 | 0 | 0 |
| | U_{eq} | 0.00254(4) | 0.00313(4) | 0.00350(3) | 0.00392(4) | 0.00395(3) | 0.00433(3) |
| O1 (4c) | x | 0.0612(3) | 0.0606(3) | 0.0596(2) | 0.0598(3) | 0.0597(3) | 0.0594(3) |
| | y | 0.4908(3) | 0.4908(3) | 0.4911(3) | 0.4913(4) | 0.4913(3) | 0.4914(3) |
| | z | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 |
| | U_{eq} | 0.0062(2) | 0.0072(3) | 0.0080(2) | 0.0087(3) | 0.0091(3) | 0.0097(3) |
| O2 (8d) | x | 0.7159(2) | 0.7163(2) | 0.7163(2) | 0.7163(2) | 0.7168(2) | 0.7171(2) |
| | y | 0.2833(2) | 0.2830(2) | 0.2829(2) | 0.2827(2) | 0.2826(2) | 0.2822(2) |
| | z | 0.0320(1) | 0.0319(1) | 0.0315(1) | 0.0313(2) | 0.0311(1) | 0.0310(1) |
| | U_{eq} | 0.0057(2) | 0.0066(2) | 0.0073(2) | 0.0080(2) | 0.0083(2) | 0.0092(2) |
| | T (K) | 623 | 673 | 723 | 773 | 823 | 873 |
| Ca (4c) | x | -0.00413(11) | -0.00405(13) | -0.00381(13) | -0.00374(15) | -0.00350(13) | -0.00337(22) |
| | y | 0.02395(6) | 0.02307(7) | 0.02234(7) | 0.02152(8) | 0.02085(8) | 0.02002(13) |
| | z | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 |
| | U_{eq} | 0.01109(6) | 0.01223(6) | 0.01330(8) | 0.01439(8) | 0.01524(8) | 0.01646(11) |
| Ge (4b) | x | 0 | 0 | 0 | 0 | 0 | 0 |
| | y | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| | z | 0 | 0 | 0 | 0 | 0 | 0 |
| | U_{eq} | 0.00482(3) | 0.00532(4) | 0.00586(4) | 0.00634(4) | 0.00666(3) | 0.00738(5) |
| O1 (4c) | x | 0.0589(3) | 0.0575(3) | 0.0574(3) | 0.0568(3) | 0.0563(3) | 0.0556(4) |
| | y | 0.4916(3) | 0.4918(4) | 0.4920(4) | 0.4921(4) | 0.4925(4) | 0.4926(6) |
| | z | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 |
| | U_{eq} | 0.0107(3) | 0.0118(3) | 0.0129(3) | 0.0141(3) | 0.0145(3) | 0.0154(5) |
| O2 (8d) | x | 0.7174(2) | 0.7176(2) | 0.7179(2) | 0.7184(2) | 0.7186(2) | 0.7189(3) |
| | y | 0.2819(2) | 0.2817(2) | 0.2814(2) | 0.2808(2) | 0.2807(2) | 0.2802(3) |
| | z | 0.0306(1) | 0.0303(1) | 0.0300(1) | 0.0298(1) | 0.0294(1) | 0.0291(2) |
| | U_{eq} | 0.0102(2) | 0.0111(2) | 0.0122(2) | 0.0133(2) | 0.0141(2) | 0.0153(4) |

622 Table 3. Temperature-dependent thermal expansion coefficients $\alpha_L(T)$ (K^{-1}) of unit-cell edge lengths
 623 and interatomic distances expressed by the polynomial approximation and their mean thermal
 624 expansion coefficients $\langle\alpha_L\rangle$ (K^{-1})
 625

| | $\alpha_L(T) = \varepsilon_0 + \varepsilon_1 T + \varepsilon_2 T^{-2} (\varepsilon_2 \leq 0)$ | | | $\langle\alpha_L\rangle \times 10^5$ |
|---|---|-----------------------------|-----------------------------|--------------------------------------|
| | $\varepsilon_0 \times 10^5$ | $\varepsilon_1 \times 10^8$ | $\varepsilon_2 \times 10^3$ | |
| Unit-cell edge lengths | | | | |
| a_p | 0.7547 | 0.7478 | -0.2982 | 1.13(1) |
| b_p | 0.1140 | 1.1951 | -0.2829 | 0.71(2) |
| c_p | 0.7029 | 0.7519 | -0.2859 | 1.08(1) |
| Non-bonding Ca...O | | | | |
| Ca...O1 ⁱ | -2.8606 | 0.1958 | 0 | -2.75(3) |
| Ca...O1 ⁱⁱ | -0.7143 | 0.7297 | 0 | -0.36(5) |
| Ca...O2 ⁱⁱⁱ (= Ca...O2 ^{iv}) | -1.3913 | -1.6781 | -1.0745 | -2.19(4) |
| Ca-O bonds | | | | |
| Ca-O1 ^v | 1.9162 | 1.0322 | -0.7695 | 2.42(6) |
| Ca-O1 | 2.8784 | 2.1918 | -2.6707 | 3.96(6) |
| Ca-O2 ^{vi} (= Ca-O2 ^v) | 0.3767 | 2.9298 | -1.4931 | 1.79(6) |
| Ca-O2 ^{vii} (= Ca-O2 ^{viii}) | 2.0241 | 1.0590 | -0.8309 | 2.54(3) |
| Ca-O2 ^{ix} (= Ca-O2 ^x) | 0.9943 | 1.2121 | -0.5723 | 1.58(4) |
| <Ca-O> | 1.6406 | 1.6767 | -1.2430 | 2.46(4) |
| Ge-O bonds | | | | |
| Ge-O1 (= Ge-O1 ^{xi}) | 0.1520 | 0.8782 | -0.1440 | 0.57(2) |
| Ge-O2 ^{ix} (= Ge-O2 ^{xii}) | 0.3326 | 0.1859 | -0.0237 | 0.42(2) |
| Ge-O2 ^{vii} (= Ge-O2 ^{xiii}) | 0.0643 | 0.7506 | -0.0900 | 0.43(3) |
| <Ge-O> | 0.1828 | 0.6051 | -0.0829 | 0.47(1) |

Symmetry codes for equivalent atoms:

- (i) $x, y-1, z$; (ii) $-x-\frac{1}{2}, y-\frac{1}{2}, z$; (iii) $-x+1, -y, z+\frac{1}{2}$; (iv) $-x+1, -y, -z$;
 (v) $-x+\frac{1}{2}, y-\frac{1}{2}, z$; (vi) $-x+\frac{1}{2}, y-\frac{1}{2}, -z+\frac{1}{2}$; (vii) $x-1, y, z$; (viii) $x-1, y, -z+\frac{1}{2}$;
 (ix) $x-\frac{1}{2}, -y+\frac{1}{2}, -z$; (x) $x-\frac{1}{2}, -y+\frac{1}{2}, z+\frac{1}{2}$; (xi) $-x, -y+1, -z$; (xii) $-x+\frac{1}{2}, y+\frac{1}{2}, z$;
 (xiii) $-x+1, -y+1, -z$.

627 Table 4. Interatomic distances (Å) and bond valence sums (BVS) at 298 K
628

| | Distances | *Bond valences | |
|---|------------|------------------|-----------------|
| | | †C.N. (Ca) 12 | †C.N. (Ca) 8 |
| Ca | | | |
| Ca...O1 ⁱ | 2.8562(16) | 0.0961 | |
| Ca...O1 ⁱⁱ | 2.9358(15) | 0.0792 | |
| Ca...O2 ⁱⁱⁱ (= Ca...O2 ^{iv}) | 3.0719(10) | 0.0568 × 2 | |
| Ca-O1 ^v | 2.3426(15) | 0.3365 | 0.3365 |
| Ca-O1 | 2.4616(16) | 0.2517 | 0.2517 |
| Ca-O2 ^{vi} (= Ca-O2 ^v) | 2.3504(10) | 0.3301 × 2 | 0.3301 × 2 |
| Ca-O2 ^{vii} (= Ca-O2 ^{viii}) | 2.5692(10) | 0.1936 × 2 | 0.1936 × 2 |
| Ca-O2 ^{ix} (= Ca-O2 ^x) | 2.5991(10) | 0.1800 × 2 | 0.1800 × 2 |
| | BVS | 2.28 | 2.00 |
| Ge | | | |
| Ge-O1 (= Ge-O1 ^{xi}) | 1.8908(3) | 0.6798 × 2 | 0.6798 × 2 |
| Ge-O2 ^{ix} (= Ge-O2 ^{xii}) | 1.8921(10) | 0.6774 × 2 | 0.6774 × 2 |
| Ge-O2 ^{vii} (= Ge-O2 ^{xiii}) | 1.8968(10) | 0.6689 × 2 | 0.6689 × 2 |
| | BVS | 4.05 | 4.05 |
| O1 | | | |
| Ca ^{xiv} ...O1 | 2.8562(16) | 0.0961 | |
| Ca ^{xv} ...O1 | 2.9358(15) | 0.0792 | |
| Ca ^{xii} -O1 | 2.3426(15) | 0.3365 | 0.3365 |
| Ca-O1 | 2.4616(16) | 0.2517 | 0.2517 |
| Ge-O1 (= Ge ^{xvi} -O1) | 1.8908(3) | 0.6798 × 2 | 0.6798 × 2 |
| | BVS | 2.12 | 1.95 |
| O2 | | | |
| Ca ^{iv} ...O2 | 3.0719(10) | 0.0568 | |
| Ca ^{xii} -O2 | 2.3504(10) | 0.3301 | 0.3301 |
| Ca ^{xvii} -O2 | 2.5692(10) | 0.1936 | 0.1936 |
| Ca ^{xviii} -O2 | 2.5991(10) | 0.1800 | 0.1800 |
| Ge ^v -O2 | 1.8921(10) | 0.6774 | 0.6774 |
| Ge ^{xiii} -O2 | 1.8968(10) | 0.6689 | 0.6689 |
| | BVS | 2.11 | 2.05 |

Symmetry codes for equivalent atoms:

(xiv) $x, y + 1, z$; (xv) $-x - \frac{1}{2}, y + \frac{1}{2}, z$; (xvi) $x, y, -z + \frac{1}{2}$; (xvii) $x + 1, y, z$;

(xviii) $x + \frac{1}{2}, -y + \frac{1}{2}, -z$. The symmetry codes (i)-(xiii) are as in Table 3.

* The bond valence parameters are taken from Allmann (1975) for Ca; from Brown and Altermatt (1985) for Ge.

† C.N.: coordination number

630 Table 5. Debye temperatures (K) determined from the Debye model fitting
 631

| | Ca | Ge | O1 | O2 |
|--------------------|--------|--------|----------|----------|
| $\Theta_{D_{eq}}$ | 448(2) | 505(3) | 728(2) | 750(5) |
| $\Theta_{D_{100}}$ | 467(3) | 521(6) | 719(4) | 820(9) |
| $\Theta_{D_{010}}$ | 436(2) | 517(6) | 597(3) | 831(6) |
| $\Theta_{D_{001}}$ | 442(2) | 480(3) | 1041(13) | 646(4) |
| $\Theta_{D_{110}}$ | — | — | — | 1149(23) |

632

633

























