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## REVISION 2

# Mid-infrared optical constants of clinopyroxene and orthoclase derived from oriented single-crystal reflectance spectra

Jessica A. Arnold<sup>1</sup>, Timothy D. Glotch<sup>1</sup> and Anna M. Plonka<sup>1</sup>

<sup>1</sup>Department of Geosciences, Stony Brook University, Stony Brook, New York 11794, U.S.A.

### Abstract

We have determined the mid-IR optical constants of one alkali feldspar and four pyroxene compositions in the range of 250–4000  $\text{cm}^{-1}$ . Measured reflectance spectra of oriented single crystals were iteratively fit to modeled spectra derived from classical dispersion analysis. We present the real and imaginary indices of refraction ( $n$  and  $k$ ) along with the oscillator parameters with which they were modeled. While materials of orthorhombic symmetry and higher are well-covered by the current literature, optical constants have been derived for only a handful of geologically relevant monoclinic materials, including gypsum and orthoclase. Two input parameters that go into radiative transfer models, the scattering phase function and the single scattering albedo, are functions of a material's optical constants. Pyroxene is a common rock-forming mineral group in terrestrial bodies as well as meteorites and is also detected in cosmic dust. Hence, having a set of pyroxene optical constants will provide additional details about the composition of solar system bodies and circumstellar materials. We follow the method of Mayerhofer et al. (2010), which is based on the Berreman 4x4 matrix formulation. This approach provides a consistent way to calculate the reflectance coefficients in low-symmetry cases. Additionally, while many models assume normal incidence to simplify the dispersion relations, this more general model applies to reflectance spectra collected at non-normal incidence.

### 1. Introduction

23 Pyroxenes are common rock-forming minerals in terrestrial planets and asteroids. They are also  
24 abundant in meteorites and interplanetary and cosmic dust particles. The pyroxene crystal structure  
25 allows for a variety of cations to occupy its M1 and M2 sites, with ordering and preference between the  
26 two sites being controlled by temperature, pressure, and cooling rate. As a result, pyroxene composition  
27 is a good indicator of the thermal history of the source magma (Kilma et al. 2008) and can be used to  
28 compare the evolution of different planetary bodies (Karner et al. 2006). Pyroxene is readily detected in  
29 the near-infrared (NIR) via two strong  $\text{Fe}^{2+}$  crystal field absorption bands near 1 and 2  $\mu\text{m}$  whose  
30 positions and strengths are functions of composition. As the mid-IR (MIR, around 3-15  $\mu\text{m}$ , though  
31 definitions vary) is sensitive to Si–O vibrational modes (Salisbury 1972), it can be used to provide  
32 additional details about composition as well as to estimate abundance relative to more felsic  
33 components (Ramsey and Christensen 1998).

34 The wavelength-dependent complex index of refraction ( $\tilde{n}=n+ik$ ), where  $n$  and  $k$  are the optical  
35 constants, is an essential input into scattering models of planetary surfaces. Commonly used radiative  
36 transfer models of airless bodies include Lumme and Bowell (1981), Hapke (1993a, 2012), Shkuratov  
37 et al. (1999) and Mishchenko et al. (1999). Hapke's theory uses optical constants to calculate the single  
38 scattering albedo and the phase function. In Shkuratov's model of lunar regolith scattering, the  
39 reflectance of the surface ( $A$ ) is dependent on only four parameters  $A(n,k,S,q)$ , where  $n$  and  $k$  are the  
40 optical constants,  $S$  is the scattering path-length and  $q$  is the packing density (Shkuratov et al. 1999).  
41 Hence, accurate determination of optical constants for a variety of minerals is necessary for using  
42 remote sensing data to make quantitative estimates of mineralogical composition. At MIR wavelengths,  
43 materials of orthorhombic symmetry and higher are well-covered by the current literature; these  
44 include quartz, calcite, olivine, orthopyroxene, kaolinite, serpentine, palagonite, and iron oxides  
45 (Spitzer and Kleinman 1961; Wenrich and Christensen 1996; Lucy 1998; Lane 1999; Suto et al. 2002;  
46 Glotch et al. 2006; Sogawa et al. 2006; Dyar et al. 2009; Roush et al. 1991; Glotch and Rossman 2009).

47 Monoclinic and triclinic minerals have largely been ignored in optical constant research,  
48 although in some cases orientation-averaged effective optical constants have been calculated and  
49 presented (e.g., Roush et al. 1991; Glotch et al. 2007). This is due to the additional complexity of  
50 obtaining values for low-symmetry materials, despite the fact that these crystal systems contain  
51 important rock-forming mineral groups such as clinopyroxene and feldspar. In this work, we use  
52 dispersion analysis to calculate the optical constants of four distinct clinopyroxene compositions in the  
53 range of 250–4000  $\text{cm}^{-1}$ , following the method of Mayerhofer et al. (2010) and present a procedure that  
54 can be applied to derive the optical constants of monoclinic single crystals. We also calculate the  
55 optical constants of orthoclase and compare to the previously computed optical constants of Aronson  
56 (1986) as a test of our model.

## 57 **2. Background**

### 58 *2.1 The Moon*

59 The Clementine mission provided a global lunar map of pyroxene abundance, broadly  
60 highlighting that it is a major component of the mare basalts and less prevalent in highlands. Modeled  
61 clinopyroxene concentrations are especially high in fresh mare craters due the short exposure ages of  
62 these surfaces (Shkuratov et al. 2004). While pyroxene is only a minor component in the lunar  
63 highlands it is found in the following highlands rock types: ferroan anorthosites (FAN's), alkali- and  
64 Mg-suite rocks (Tompkins and Pieters 1999). Attempting to put these in a global context, Klima et al.  
65 (2011) used NIR data from the Moon Mineralogy Mapper ( $M^3$ ) to detail the distribution of high- vs.  
66 low-Ca pyroxene and estimate Mg #. Low-Ca pyroxene is limited to regional exposures within the  
67 South Pole-Aitken Basin and in the highlands north and south of Mare Frigoris, approximately half of  
68 which have Mg # ( $\text{Mg}/(\text{Mg}+\text{Fe})$ ) commensurate with FAN rocks ( $\sim$ Mg # 55–75).

69 MIR emission data from the Diviner Lunar Radiometer Experiment complements visible near-  
70 IR (VNIR) measurements from Clementine and  $M^3$ , as they are able to detect Fe-poor lithologies in the

71 presence of mafic materials such as pyroxene and olivine. Three of Diviner's channels are centered near  
72 the silicate Christiansen feature (CF) (Paige et al. 2010), which is an emissivity maximum whose  
73 position moves to shorter wavelengths with increasing SiO<sub>2</sub> polymerization (Conel 1969; Logan et al.  
74 1973; Salisbury and Walter 1989), allowing Diviner to be used to map silicate compositions across the  
75 Moon (Greenhagen et al. 2010; Glotch et al. 2010, 2011; Song et al. 2013; Allen et al. 2011; Arnold et  
76 al. 2013). A global map of CF position clearly demonstrates the difference between the relatively  
77 pyroxene-rich mare and feldspar-rich highlands (Greenhagen et al. 2010). Although Diviner has only  
78 three spectral bands in the 8 μm range, laboratory measurements in a simulated lunar environment  
79 show the estimated CF position combined with spectral shape is adequate to distinguish pyroxene from  
80 a mixture of plagioclase and olivine (Donaldson Hanna et al. 2012).

## 81 *2.2 Mars*

82 The low-albedo regions of Mars were initially categorized into two surface compositions based  
83 on Mars Global Surveyor Thermal Emission Spectrometer (MGS-TES) data, where Surface Type 1  
84 was determined to be basalt containing ~25% clinopyroxene (Bandfield et al 2000, Hamilton et al.  
85 2001). Rogers et al. (2007) defined 4 distinct spectral types within low-albedo terrains. Subsequently,  
86 Rogers and Christensen (2007) determined the mineralogy of these four groups, with clinopyroxene  
87 abundance being one of the major distinguishing features. While low-Ca pyroxenes were mostly  
88 detected in Noachian terrain, high-Ca pyroxene was found in a wider age-range of units. Some of the  
89 mineralogical differences between the four groups, especially high silica phases, are the result of  
90 weathering. However, the differences in pyroxene abundance likely represent differences in the magma  
91 from which the crust was derived. These differences were also identified with data from the OMEGA  
92 (Observatoire pour la Minéralogie, l'Eau, les Glaces et l'Activité) NIR imaging spectrometer. Using  
93 this instrument, Ody et al. (2012) presented global maps of anhydrous minerals, including pyroxene.  
94 The highest pyroxene concentration occurs in the Syrtis Major volcanic province, which is coincident  
95 with Group 2 of Rogers and Christensen (2007). Glotch and Rogers (2013) suggested that the unusually

96 high concentration of high-Ca pyroxene at this site was the result of subsurface interactions of basaltic  
97 magma and a Ca-carbonate-bearing layer.

### 98 *2.3 Asteroids*

99 Pyroxene has been identified in several S-type asteroid families (Chapman et al. 1975).  
100 Sunshine et al. (2004) applied the Modified Gaussian Model (MGM) to VNIR asteroid spectra  
101 collected by the NASA Infrared Telescope Facility (IRTF), to determine the relative amounts of low-  
102 and high calcium pyroxene. They determined that the Vestoids and well as the S-type asteroid families  
103 Merxia and Agnia have a high ratio of high calcium pyroxene to total pyroxene, indicating that they are  
104 derived from differentiated bodies.

105 Pyroxene absorptions are the most common VNIR spectral feature on the surface of the asteroid  
106 Vesta, which was first linked to the HED (howardite-eucrite-diogenite) meteorites due to the similarity  
107 of their 0.9  $\mu\text{m}$  band in VNIR spectra (McCord et al. 1970). The findings of Dawn's Visible InfraRed  
108 (VIR) mapping spectrometer support this connection. Much of the surface shows a howardite-like  
109 spectrum intermixed with smaller-scale regions resembling eucrites and diogenites. The south polar  
110 region (Rheasilvia), a large impact basin, is consistent with more Mg-rich pyroxene characteristic of  
111 diogenites (De Sanctis et al. 2012). These spectral variations are indicative of a differentiated crust  
112 where the deeper diogenitic materials have been exposed through impact.

### 113 *2.4 Interplanetary and Interstellar Dust*

114 The presence of crystalline silicates in interplanetary dust particles (IDPs) was established  
115 through micrometeorite samples collected from Earth's stratosphere (Bradley et al. 1983, Mackinnon  
116 and Rietmeijer 1987). Additionally, ground-based MIR spectra from three different instruments  
117 provided evidence for crystalline silicates in comets (Hanner et al. 1994). Prior to the Infrared Space  
118 Observatory (ISO), whose instruments covered a range of  $\sim 2.5 - 240 \mu\text{m}$ , silicates in cosmic dust were  
119 thought to be primarily amorphous (Roush et al. 1991; Molster and Kemper 2004). This assumption

120 guided most previous laboratory studies of pyroxene optical constants (Jäger et al. 1994; Dorschner et  
121 al. 1995; Henning and Mutschke 1997). Using ISO, crystalline pyroxene was detected in dust produced  
122 by evolved stars (Molster et al. 2002), the circumstellar disks of young stellar objects (YSO's)  
123 (Bouwman et al. 2001) and planetary nebula (Beintema 1997). As a result of the detection of crystalline  
124 pyroxene around planetary nebula, Jäger et al. (1998) estimated the MIR optical constants for a natural  
125 enstatite (low-Ca orthopyroxene) sample along the crystallographic axes using Kramers-Kronig  
126 analysis. Better characterization of the materials in circumstellar environments would improve our  
127 understanding of dust formation and processing and enable comparison to our own solar system.

### 128 **3. Methods**

#### 129 *3.1 Sample description and preparation*

130 All samples are large single crystals (roughly 0.5–1 cm on a side) and their chemical  
131 compositions are summarized in Table 1. The major element abundances were determined by scanning  
132 electron microscope energy dispersive x-ray spectroscopy (SEM-EDS). To produce the chemical  
133 formulas in Table 1, atomic abundances of oxygen were assumed by stoichiometry to be  $O_6$ , which  
134 may introduce error if any non-pyroxene phases are present in the samples. We prepared thin sections  
135 of each sample in order to optically examine the samples for exsolution and twinning. One augite  
136 (Aug1) sample was loaned from the Stony Brook Geosciences Department mineral collection and  
137 originates from Indian Well, AZ. This sample does not show signs of exsolution or twinning. The  
138 diopside (Diop1) sample was purchased from Kelly's Rocks and is from Magog, QC, Canada. While  
139 twinning is present over ~30–40% of a section perpendicular to the  $a$ - $c$  plane in the diopside sample,  
140 the IR spectra match previously published oriented reflectance spectra (Johnson et al. 2002). The  
141 hedenbergite (Hed1) and second augite (Aug2) sample were provided by D. Lindsley at Stony Brook  
142 University. The hedenbergite sample does not exhibit exsolution or twinning, while Aug2 appears to  
143 contain lamellae that remain dark under crossed polar illumination at all orientations. Micro-FTIR

144 spectra of these lamellae are quite noisy compared to the rest of the augite sample, and show a broad  
145 reflectance peak at around  $3900\text{ cm}^{-1}$  and a peak at  $1015\text{ cm}^{-1}$ . These spectral features indicate some  
146 type of clay, however the exact composition is difficult to discern. The FTIR spectrum of the whole  
147 sample does not show a  $3900\text{ cm}^{-1}$  peak, indicating this is a relatively minor component. The diopside,  
148 hedenbergite, and augite compositions are shown on the ternary diagram in Figure 1. We cut each  
149 sample with a diamond saw blade so that one surface is parallel to the  $a$ - $c$  plane (010) and the other  
150 perpendicular to it -- planes (001) or (100). We polished these surfaces to  $0.25\text{ }\mu\text{m}$  surface roughness,  
151 and confirmed the orientations of the cut surfaces by single crystal XRD. XRD analysis involved  
152 collection of the preliminary set of frames (pre-experiment) for the determination of unit cell and the  
153 orientation matrix. Using this information, we indexed the faces of the crystal with the CrysAlisPRO  
154 software. We collected reflections for determination of the orientation matrix using a four-circle kappa  
155 Oxford Gemini diffractometer equipped with an Atlas detector ( $\lambda = 0.71073\text{ \AA}$ ).

### 156 *3.2 Collection of mid and far-IR reflectance spectra*

157 We acquired specular reflectance spectra on the Stony Brook University Vibrational  
158 Spectroscopy Laboratory's Nicolet 6700 Fourier Transform Infrared (FTIR) spectrometer using a  
159 FT-30 specular reflectance accessory with incidence and reflection angles of 30 degrees at a spectral  
160 sampling of  $2\text{ cm}^{-1}$ . An InfraSpecs wire grid IR polarizer was placed in the beam path for collection of  
161 polarized reflectance spectra. For each sample, we measured reflectance spectra for four orientations of  
162 the crystal with respect to the incident polarized beam. Depending on whether the  $a$ - or  $c$ -axis was  
163 more readily identifiable, these orientations were  $0^\circ$ ,  $45^\circ$ ,  $90^\circ$  and  $110^\circ$  from the identified  
164 crystallographic axis. MIR spectra were collected over a range of  $400\text{--}4000\text{ cm}^{-1}$  using a KBr  
165 beamsplitter and DTGS detector with a CsI window. Each spectrum is an average of 256 scans. Far-IR  
166 (FIR) spectra were acquired from  $250\text{--}600\text{ cm}^{-1}$  using a Nicolet Solid Substrate beamsplitter and a  
167 DTGS detector with a polyethylene window. Each of these spectra is an average of 512 scans. The two

168 wavelength ranges are joined at  $600\text{ cm}^{-1}$  using the overlapping region ( $600\text{ cm}^{-1} - 400\text{ cm}^{-1}$ ) to scale  
169 the spectral contrast of the FIR spectra. MIR and FIR spectral contrast varies by up to 10% due to slight  
170 differences in measurement conditions and/or a few degree error in crystal orientation. Samples were  
171 placed on a holder that acts as a mask, controlling the spot size. We chose the largest spot size that can  
172 be completely covered by the polished surface of the sample. The bottom of the sample holders is  
173 coated with paraffin soot to reduce stray reflectance off of the holder into the sample chamber. For the  
174 smaller spot-sizes ( $<0.5\text{ cm}$  diameter) we subtracted a spectrum of the sample holder without a sample  
175 or standard placed on top. While the sample holder does not affect the mid-IR reflectance, there is a  
176 steep upward slope starting at around  $400\text{ cm}^{-1}$  which rapidly increases from 0.01 to several percent  
177 reflectance at  $\sim 100\text{ cm}^{-1}$ .

### 178 *3.3 Modeling of optical constants*

179 The optical constants ( $n, k$ ) are related to the wavelength-dependent dielectric function ( $\epsilon(\lambda)$ ) of  
180 the material ( $\tilde{n} = \epsilon^{1/2}$ ). Electron states in non-conducting materials can be modeled as damped harmonic  
181 oscillators, which exhibit spring-like behavior (Griffiths 1999). The E-field of the incoming beam acts  
182 as a driving force on the oscillators, which have resonant frequencies  $\nu_j$  ( $\text{cm}^{-1}$ ). The parameters  $s_j$  ( $\text{cm}^{-2}$ ),  $\gamma_j$  ( $\text{cm}^{-1}$ ) and  $\epsilon_\infty$  are the oscillator strength, damping coefficient (proportional to the oscillator  
183 velocity) and infinite frequency dielectric constant respectively. These parameters are explained in  
184 more detail in section 5.1.

186 For minerals of orthorhombic and higher symmetry, we can assume that these oscillations occur  
187 parallel to the crystallographic axes. In this case, optical constants can be determined using the  
188 approach of Spitzer and Kleinman (1961). Reflectance spectra are acquired for each principal axis with  
189 the polarization parallel to each axis. Each spectrum is iteratively fit with estimated values of the  
190 oscillator parameters as input.



191 When a mineral is biaxial and the axes are not orthogonal, the oscillators cannot be assumed to  
 192 be parallel to the crystallographic axes (Belousov and Pavinich 1978). In a monoclinic material, optical  
 193 constants with E parallel to the *b*-axis (where the oscillators are parallel to the axis) can be determined  
 194 in the same manner as the orthorhombic case. In the *a*-*c* plane, oscillators are coplanar with, but not  
 195 parallel to the crystallographic axes. To determine the two remaining principle refractive indices, it is  
 196 necessary to make measurements at three different angles ( $\Omega$ ) with respect to the crystallographic axes  
 197 in the *a*-*c* plane (Aronson et al. 1983), with measurements at more angles providing more robust fits.  
 198 This results in an additional oscillator parameters  $\theta_j$  and  $\varphi_j$ . A useful diagram of such an experimental  
 199 setup is given in Fig. 1 of Ivanovski et al. (2007). The principle complex indices of refraction are the  
 200 eigenvalues of the complex dielectric-permittivity tensor, which can be calculated as follows, where N  
 201 is the number of oscillators:

$$\begin{aligned}
 202 \quad E &= (n + ik)^2 = \begin{pmatrix} \epsilon_{\infty xx} & \epsilon_{\infty xy} & \epsilon_{\infty xz} \\ \epsilon_{\infty yx} & \epsilon_{\infty yy} & \epsilon_{\infty yz} \\ \epsilon_{\infty zx} & \epsilon_{\infty zy} & \epsilon_{\infty zz} \end{pmatrix} + \sum_{j=1}^N \frac{S_j}{v_j^2 - i\gamma_j v - v^2} \times \\
 203 \quad &\begin{pmatrix} \sin^2\theta_j \cos^2\varphi_j & \sin^2\theta_j \sin\varphi_j \cos\varphi_j & \sin\theta_j \cos\theta_j \cos\varphi_j \\ \sin^2\theta_j \sin\varphi_j \cos\varphi_j & \sin^2\theta_j \sin^2\varphi_j & \sin\theta_j \cos\theta_j \sin\varphi_j \\ \sin\theta_j \cos\theta_j \cos\varphi_j & \sin\theta_j \cos\theta_j \sin\varphi_j & \cos^2\theta_j \end{pmatrix} \quad (1)
 \end{aligned}$$

205 The reflectance spectra for all  $\Omega$  are fit simultaneously. Fixing the *b*-axis as the *z*-axis of the  
 206 dielectric tensor,  $\epsilon$  needs to be rotated with  $\Omega$ , according to:

$$207 \quad \epsilon = \begin{pmatrix} \cos\Omega & \sin\Omega \\ -\sin\Omega & \cos\Omega \end{pmatrix} \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} \\ \epsilon_{xy} & \epsilon_{zz} \end{pmatrix} \begin{pmatrix} \cos\Omega & -\sin\Omega \\ \sin\Omega & \cos\Omega \end{pmatrix} \quad (2)$$

208 Because of the additional complexity in measurement and calculation, optical constants have  
 209 been derived from single crystal samples for only a few monoclinic materials, including gypsum and  
 210 orthoclase (Long et al.1992; Aronson et al. 1983, Aronson 1986). A similar method for triclinic  
 211 materials is outlined by Aronson et al. (1985). However, substitution of the relevant variables in their  
 212 equations for the reflection coefficients results in a singularity for the monoclinic case (Mayerhofer et  
 213

214 al. 2010) instead of reducing to the formulas given in Aronson et al. (1983). In addition, Aronson et  
 215 al.'s formulation applies only to the case of normal incidence. The Berreman 4x4 matrix formulation  
 216 (Berreman 1972) gives a more consistent approach for calculating the reflectance coefficients in low-  
 217 symmetry cases (Mayerhofer and Popp 2007).

218 Mayerhofer and Popp (2007) define a matrix  $\tilde{M}$  that can be used to calculate the reflection  
 219 coefficients as a function of incidence angle ( $\alpha$ ):

$$220 \quad \tilde{M} = D_{\psi}^{-1}(0)D_{\psi}(1) \quad (3)$$

$$221 \quad r_{xx} = \frac{\tilde{M}_{21}\tilde{M}_{33} - \tilde{M}_{23}\tilde{M}_{31}}{\tilde{M}_{11}\tilde{M}_{33} - \tilde{M}_{13}\tilde{M}_{31}} \quad (4a)$$

$$222 \quad r_{xy} = \frac{\tilde{M}_{41}\tilde{M}_{33} - \tilde{M}_{43}\tilde{M}_{31}}{\tilde{M}_{11}\tilde{M}_{33} - \tilde{M}_{13}\tilde{M}_{31}} \quad (4b)$$

223  $D\Psi(0)$  and  $D\Psi(1)$  are the dynamical matrices of the incident and refracted waves (Yeh 1979):

$$224 \quad D_{\psi}^{-1}(0) = \frac{1}{2} \begin{pmatrix} 1 & 1/n\cos\alpha & 0 & 0 \\ 1 & -1/n\cos\alpha & 0 & 0 \\ 0 & 0 & 1/\cos\alpha & 1/n \\ 0 & 0 & -1/\cos\alpha & 1/n \end{pmatrix} \quad (5)$$

$$225 \quad D_{\psi}(1) = \begin{pmatrix} \epsilon_{xy}(1 - \frac{k_y^2}{\epsilon_{zz}}) & 0 & \epsilon_{xy}(1 - \frac{k_y^2}{\epsilon_{zz}}) & 0 \\ \hat{a}_{xy}(1 - \frac{k_y^2}{\epsilon_{zz}})\gamma_1 & 0 & \epsilon_{xy}(1 - \frac{k_y^2}{\epsilon_{zz}})\gamma_3 & 0 \\ (1 - \frac{k_y^2}{\epsilon_{zz}})[\gamma_1^2 - (\epsilon_{xx} - k_y^2)] & 0 & (1 - \frac{k_y^2}{\epsilon_{zz}})[\gamma_3^2 - (\epsilon_{xx} - k_y^2)] & 0 \\ [\gamma_1^2 - (\epsilon_{xx} - k_y^2)]\gamma_1 & 0 & [\gamma_3^2 - (\epsilon_{xx} - k_y^2)]\gamma_3 & 0 \end{pmatrix} \quad (6)$$

227 where  $n$  is the index of refraction of the incident medium,  $k_y$  is the y-component of the wave-vector (the  
 228 magnitude and direction of the incoming beam), and

$$229 \quad \gamma_1 = \sqrt{-\frac{1}{2\epsilon_{zz}}(K_1 + \sqrt{K_1^2 + K_2})} \quad (7a)$$

$$230 \quad \gamma_3 = \sqrt{\frac{1}{\epsilon_{zz}}(-K_1 + \sqrt{K_1^2 + K_2})} \quad (7b)$$

$$231 \quad K_1 = -\epsilon_{zz}(\epsilon_{xx} + \epsilon_{yy}) + k_y^2(\epsilon_{yy} + \epsilon_{zz}) \quad (8a)$$

232 
$$K_2 = -4 \left[ \varepsilon_{xy}^2 + \varepsilon_{yy} \left( k_y^2 - \varepsilon_{xx} \right) \right] \left( k_y^2 - \varepsilon_{zz} \right) \varepsilon_{zz}. \quad (8b)$$

233 To perform the required calculations, we use the Matlab non-linear fitting routine `lsqcurvefit`,  
234 which allows us to set lower and upper bounds, ensuring that the final values for the oscillator  
235 parameters are positive. To produce the initial parameters for this model, we first fit the spectrum for  
236 each  $\Omega$  as if it is orthorhombic and at normal incidence. The parameters  $v_j$  and  $\gamma_j$  can be estimated  
237 reasonably by visual inspection, while  $s_j$  is estimated as in Pavinich and Belousov (1978). Reflectance  
238 spectra for all orientations are then fit simultaneously, using the  $\Omega$  where  $s_j$  is the largest for a given  $v_j$   
239 as an initial guess for  $\varphi_j$ .

240 We attempt to fit the reflectance spectrum with the minimum number of oscillators necessary.  
241 Starting with the number of main peaks, modes are added until a reasonable fit is obtained for any  
242 overlapping peaks or shoulders (a difference between the measured and modeled reflectance of less  
243 than 10%). Sometimes modes can be at the same frequency or very close in frequency, with different  
244  $\theta_j$ . In this case, although there is only one peak, the fit will be poor without an additional oscillator.

### 245 *3.4 Oscillator parameter error estimates*

246 The model fit is commonly assessed using the standard error of the oscillator parameters. We  
247 report the standard error ( $\sigma(v)$ ,  $\sigma(\gamma)$  etc.) for each parameter in Tables 2-6. This does not reflect the  
248 impact of the initial guesses of the oscillator parameters on the final reflectance fit. Poor initial  
249 estimates of starting values may result in slow convergence, non-convergence or convergence on a  
250 local rather than global minimum in our model. For each sample, we used five different sets of starting  
251 values centered around those produced by the fit to the orthorhombic model to qualitatively evaluate  
252 how much the final  $n$  and  $k$  values depended on the initial estimates. Those that gave the best  
253 reflectance fit were used to calculate the optical constants. As an additional test, we independently  
254 varied the sets of  $v_j$ ,  $\gamma_j$ ,  $s_j$ , and  $\varphi_j$  parameters for a gem-quality orthoclase sample to determine which  
255 had the largest impact on the final  $n$  and  $k$  values. Large ( $\pm 30 \text{ cm}^{-1}$  for  $v_j$  and  $\pm 50\%$  for all others)  
256 variations in the other parameters do not appear to have a major effect on the fit (see Figures 2-5).

257 Figure 6 shows the final fits when all parameters are varied by these amounts (top) as well the 95%  
258 confidence bounds based of the standard error of the parameters (bottom). The two measures of  
259 uncertainty appear to diverge from the calculated optical constants at different wavenumbers.

260

## 261 **4. Results**

262 For each sample we present the measured and modeled reflectance and the plots of  $n$  and  $k$  as a  
263 function of wavelength derived from the reflectance fit. The measured and modeled spectra, as well as  
264 the derived optical constants and oscillator parameters are available at  
265 [http://aram.ess.sunysb.edu/tglotch/optical\\_constants.html](http://aram.ess.sunysb.edu/tglotch/optical_constants.html).

### 266 *4.1 Augite 1*

267 Measured reflectance and model are shown in Figure 7, with incident E-field polarization parallel to the  
268  $a$ - $c$  plane (top three) and parallel to  $b$  (bottom). Derived values of  $n$  and  $k$  are shown in Figure 8. In  
269 Figure 8  $n_1$ ,  $n_2$  and  $k_1$ ,  $k_2$  are the optical constants for the rays whose polarization direction is parallel  
270 to the  $a$ - $c$  plane, while  $n_b$  and  $k_b$  are the optical constants for the rays with polarization direction along  
271 the  $b$  crystallographic axis. Oscillator parameters for this sample are given in Table 2.

### 272 *4.2 Augite 2*

273 Measured reflectance and model are shown in Figure 9, with incident E-field polarization parallel to the  
274  $a$ - $c$  plane (top three) and parallel to  $b$  (bottom). Derived values of  $n$  and  $k$  are given in Figure 10. In  
275 Figure 10  $n_1$ ,  $n_2$  and  $k_1$ ,  $k_2$  are the optical constants for the rays whose polarization direction is parallel  
276 to the  $a$ - $c$  plane, while  $n_b$  and  $k_b$  are the optical constants for the rays with polarization direction along  
277 the  $b$  crystallographic axis. Oscillator parameters for this sample are given in Table 3.

### 278 *4.3 Diopside*

279 Measured reflectance and model are shown in Figure 11, with incident E-field polarization parallel to  
280 the  $a$ - $c$  plane (top three) and parallel to  $b$  (bottom). Derived values of  $n$  and  $k$  are shown in Figure 12.  
281 In Figure 12  $n_1$ ,  $n_2$  and  $k_1$ ,  $k_2$  are the optical constants for the rays whose polarization direction is  
282 parallel to the  $a$ - $c$  plane, while  $n_b$  and  $k_b$  are the optical constants for the rays with polarization  
283 direction along the  $b$  crystallographic axis. Oscillator parameters for this sample are given in Table 4.

### 284 *4.4 Hedenbergite*

285 Measured reflectance and model are shown in Figure 13, with incident E-field polarization parallel to  
286 the *a-c* plane (top three) and parallel to *b* (bottom). Derived values of *n* and *k* are shown in Figure 14.  
287 In Figure 14 *n*<sub>1</sub>, *n*<sub>2</sub> and *k*<sub>1</sub>, *k*<sub>2</sub> are the optical constants for the rays whose polarization direction is  
288 parallel to the *a-c* plane, while *n*<sub>*b*</sub> and *k*<sub>*b*</sub> are the optical constants for the rays with polarization  
289 direction along the *b* crystallographic axis. Oscillator parameters for this sample are given in Table 5.

#### 290 *4.5 Orthoclase*

291 Measured reflectance and model are shown in Figure 15, with incident E-field polarization parallel to  
292 the *a-c* plane (top three) and parallel to *b* (bottom). Derived values of *n* and *k* are shown in Figure 16 as  
293 well as those from Aronson et al. (1986). In Figure 16 *n*<sub>1</sub>, *n*<sub>2</sub> and *k*<sub>1</sub>, *k*<sub>2</sub> are the optical constants for  
294 the rays whose polarization direction is parallel to the *a-c* plane, while *n*<sub>*b*</sub> and *k*<sub>*b*</sub> are the optical  
295 constants for the rays with polarization direction along the *b* crystallographic axis. Oscillator  
296 parameters for this sample are given in Table 6.

297

## 298 **5. Discussion**

### 299 *5.1 Interpretation of dispersion parameters*

300 The optical constants *n* and *k* are related to the polarizability ( $\alpha$ ), the dependence of the  
301 magnitude of the dipole moment on an applied electric field, and conductivity ( $\sigma$ ), the ability of a  
302 material to carry a current, which are both properties of the material. The dependence of the  
303 polarizability and conductivity and hence the optical constants on wavelength are given by dispersion  
304 relations. To derive these dispersion relations, Lorentz postulated that insulating materials contain  
305 electrons that are bound to atoms or molecules by Hooke's law forces (Seitz 1940), so that the binding  
306 force is proportional to the charge displacement. The motion of the bound electron is described as a  
307 harmonic oscillator subject to a periodic electric field ( $\mathbf{E}=\mathbf{E}_0e^{-i\omega t}$ ) and damped by a force proportional  
308 to the electron's velocity (*\*note to editor the following should be in line\**)

$$\frac{dx}{dt}$$

309 ) giving: (\*end line\*)

$$310 \quad \frac{d^2x}{dt^2} + 2\pi\gamma \frac{dx}{dt} + 4\pi^2\nu_0^2x = \frac{-q_e}{m_e} E_0 e^{2\pi i\nu t} \quad (9)$$

$$311 \quad x = \frac{q_e}{4\pi^2 m_e} \frac{E_0 e^{-2\pi i\nu t}}{\nu_0^2 - \nu^2 - i\gamma\nu} \quad (10)$$

312 The resonant frequency of the oscillator is  $\nu_0$  and the damping constant is  $2\pi m_e \gamma$ . As  $\nu$  passes through  
313 the value  $\nu_0$ , there is a peak in the absorption coefficient with a half-width of roughly  $\gamma$ . The order of  
314 magnitude of  $\gamma$  can be estimated by the following (Seitz 1940):

$$315 \quad \gamma \sim \frac{4\pi\nu^2 q_e^2}{3m_e c} \quad (11)$$

316 Where  $q_e$  is in statcoulombs,  $\nu$  is in wavenumbers,  $m_e$  is in grams and  $c$  is in cm/s. Looking at Eq (10)  
317 and assuming the dipole moment has a linear response to the electric field ( $\mathbf{p}=\epsilon_0\alpha\mathbf{E}$ ), results in:

$$318 \quad \alpha = \frac{q_e^2}{4\pi^2 m_e \epsilon_0} \frac{1}{\nu_0^2 - \nu^2 - i\gamma\nu} \quad (12)$$

319 Using a form of the Clausius-Mosotti relation  $\epsilon = \epsilon_\infty + 4\pi\chi = \epsilon_\infty + 4\pi N\alpha$  (Spitzer et al. 1959) gives the  
320 dispersion relation for a single resonance.

321 Of course, for a given material, there will not be just one resonant frequency, but several  $\nu_j$  each  
322 with a different damping constant  $\gamma_j$ . The quantum mechanical approach to this problem deals with this  
323 and also gives one additional parameter oscillator “strength” ( $f_j$ ), a weighting for each oscillator that  
324 relates to the absorption or transition probability (Seitz 1940; Spitzer and Kleinman 1961; Born and  
325 Wolf 1970). The parameter  $s_j$  used in Eq (1) can be defined in terms of  $f_j$  as shown in Spitzer and  
326 Kleinman (1961).

$$327 \quad s_j = 4\pi\rho_j\nu_j^2 = \frac{Nq_e^2}{\pi m_e} f_j \quad (13)$$

328 While educated guesses can be made, the oscillator parameters ( $\nu_j$ ,  $\gamma_j$ , and  $s_j$ ) are difficult to  
329 derive exactly from first principles, which is why an iterative technique is normally used. The infinite  
330 frequency dielectric constant  $\epsilon_\infty$  can be estimated from the square of the visible-wavelength index of  
331 refraction (Roush et al. 1991).

## 332 *5.2 Comparison with previously derived Orthoclase optical constants*

333 Optical constants for orthoclase have been derived from oriented single crystal reflectance  
334 spectra by previous workers (Aronson et al. 1986). Figure 16 shows the values found in this study  
335 along with those from Aronson et al. (1986). The optical constants for the E-field polarization parallel  
336 to b orientation ( $n_b$  and  $k_b$ ) are very similar, while the optical constants ( $n_1, n_2$  and  $k_1, k_2$ ) for the E-  
337 field parallel to the  $a$ - $c$  plane have slight differences. Variations between the two data sets may be due  
338 to either compositional differences in the samples or uncertainties in orientation. Our orthoclase sample  
339 had a small amount of  $\text{Fe}^{3+}$  substituting for  $\text{Al}^{3+}$  (see Table 1). However, without any compositional

340 information on the sample used in the Aronson et al. (1986) study, it is difficult to make inferences  
341 about specific bands.

### 342 *5.3 Dependence of optical constants on crystal structure and chemistry*

343 Isometric minerals will have one set of optical constants for each wavelength. Because most  
344 minerals are anisotropic, the polarizability and conductivity will be directionally as well as wavelength  
345 dependent and therefore so will the optical constants. Dispersion in uniaxial and biaxial minerals is  
346 described by a dielectric tensor rather than just a dielectric constant. The number of oscillator  
347 parameters necessary to fit the reflectance spectrum generally increases with decreasing crystal  
348 symmetry. Figure 17 shows averaged optical constants in the MIR for several minerals that belong to  
349 different crystal systems. It is also important to measure optical constants for several different chemical  
350 compositions within the same mineral group. As shown in this work, optical constants vary  
351 considerably with chemistry. Simply using one composition of olivine, for example, may produce poor  
352 fits for mineralogical abundance as shown in Liu (2012) at VNIR wavelengths.

353 For silicates, there is a steep absorption edge in the UV-range after which  $k$ -values drop by  
354 orders of magnitude and remain low through the VNIR. As a result, after the UV-edge, dispersion is  
355 low until the MIR, where the fundamental vibrational bands occur. There is a steep rise in  $k$  between 8  
356 and 10  $\mu\text{m}$  ( $1250\text{cm}^{-1}$ – $1000\text{cm}^{-1}$ ), with the maximum  $k$  occurring where  $n=1$  or slightly lower than 1.  
357 This is known as the Christiansen feature and is associated with a minimum in reflectance and a  
358 maximum in emission. This feature is the means by which Diviner data are used to identify lunar  
359 silicates (Greenhagen et al. 2010). Following the Christiansen feature, there is a steep drop-off in  $k$  and  
360 rise in  $n$ , causing a drop-off in emission followed by Reststrahlen features. For pyroxenes, spectral  
361 features in the  $\sim 8$ – $22 \mu\text{m}$  ( $\sim 1250\text{cm}^{-1}$ – $450\text{cm}^{-1}$ ) range are mainly due to Si–O stretching, Si–O–Si  
362 bending and modes of the cation- $\text{O}_6$  octahedra. The positions of absorptions at these wavelengths vary  
363 with Mg # (defined as  $\text{Mg}/(\text{Mg}+\text{Fe})$ ) (Jäger et al. 1998; Hamilton 2000; Bowey et al. 2007; Lane et al.  
364 2011), likely due to the effect of the substitution on the octahedral layer, which in turn alters the Si–O  
365 vibrations (Hamilton 2000; Klima 2007). To demonstrate these effects, Figure 18 shows the reflectance  
366 and optical constants for olivine (from Zeidler et al. 2011) ranging from the UV through the MIR, with

367 the major spectroscopic features labeled. This figure also demonstrates that  $n$  and  $k$  values of silicates  
368 vary considerably in the mid-IR as compared with other wavelengths.

369 Amorphous pyroxenes produced by vapor phase condensation and synthetic glasses of pyroxene  
370 composition will lack many of the spectroscopic features of a crystalline pyroxene. As a result, the  
371 optical constants of these materials will have broader and fewer peaks than crystalline pyroxene. To  
372 date, little work has been done on oriented crystalline samples (Jäger et al. 1998), and none has been  
373 done to date on clinopyroxene. Figure 19 demonstrates the difference between optical constants of  
374 crystalline vs. amorphous phases. Additionally, oriented single crystal spectra are preferable to  
375 orientation-averaged data acquired from pellets pressed from powders, as the lower maxima of the  
376 specular reflectance peaks of pellets results in lower maxima of derived optical constants when  
377 compared to orientation-averaged single crystal optical constants (Roush 1991).

378

## 379 **6. Implications**

380 Radiative transfer models of planetary surfaces require optical constants for each component  
381 that may be present. Applying these models is difficult in the MIR due to the absence of optical  
382 constants for many important minerals in the monoclinic and triclinic crystal systems. We have applied  
383 the Berreman 4x4 matrix method to derive the MIR optical constants of 5 different pyroxenes for  
384 which oriented single-crystal data were not previously reported in the literature. Four spectra with a  
385 polarized incident beam at different orientations were acquired for each sample, resulting in three sets  
386 of optical constants that can be used to estimate  $n$  and  $k$  for a randomly oriented powdered sample.  
387 These data are necessary to improve compositional estimates of planetary regolith based on radiative  
388 transfer models.

389 The data presented here, in conjunction with a coupled optical-thermal model, such as that of  
390 Milan et al. (2011), can be used to model MIR emissivity spectra in a vacuum environment relevant to  
391 airless solar system bodies. Modeled emission spectra can be compared to emissivity of the lunar  
392 surface derived from Diviner Lunar Radiometer data. These optical constants are also applicable to  
393 any MIR data from Mars or any airless solar system objects including: existing spectra of asteroids  
394 from ISO, Spitzer and any future data from James Webb Space Telescope (JWST) or the OSIRIS-REx



395 mission. Future work will include expanding the current version of the code to include the triclinic  
396 case. This code for triclinic minerals will be applied to obtain feldspar optical constants. The Matlab  
397 scripts developed for this work will be made publicly available to other researchers at  
398 <http://aram.ess.sunysb.edu/tglotch/>

399

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687

## 688 **Figure and Table Captions**

689

690 Figure 1. Ternary of pyroxene compositions included in this study.

691

692 Figure 2. Changes in modeled optical constants ( $n, k$ ) of orthoclase with initial estimates of  $\nu_j$  shifted by  
693  $30 \text{ cm}^{-1}$ .

694

695 Figure 3. Changes in modeled optical constants ( $n, k$ ) of orthoclase with initial estimates of  $\gamma_j$  shifted by  
696  $\pm 50\%$ .

697

698 Figure 4. Changes in modeled optical constants ( $n, k$ ) of orthoclase with initial estimates of  $s_j$  shifted by  
699  $\pm 50\%$ .

700

701 Figure 5. Changes in modeled optical constants ( $n, k$ ) of orthoclase with initial estimates of  $\varphi_j$  shifted  
702 by  $\pm 50\%$ .

703

704 Figure 6. Changes in modeled optical constants ( $n, k$ ) of orthoclase with initial estimates of  $\nu_j$  shifted by  
705  $30 \text{ cm}^{-1}$  and  $\nu_j, \gamma_j, s_j$  and  $\varphi_j$  shifted by  $\pm 50\%$  (top).

706

707 Figure 7. Measured and modeled reflectance spectra of augite (Aug1) with incident E-field polarization  
708 parallel to  $a$ - $c$  plane at different polarization angles  $\phi$  (top three) and parallel to  $b$  (bottom).

709

710 Figure 8. Optical constants ( $n, k$ ) of augite (Aug1). In this figure,  $n_1, n_2$  and  $k_1, k_2$  are the optical  
711 constants for the rays whose polarization direction is parallel to the  $a$ - $c$  plane, while  $n_b$  and  $k_b$  are the optical  
712 constants for the rays with polarization direction along the  $b$  crystallographic axis.

713

714 Figure 9. Measured and modeled reflectance spectra of augite (Aug2) with incident E-field polarization  
715 parallel to  $a$ - $c$  plane at different polarization angles  $\phi$  (top three) and parallel to  $b$  (bottom).

716

717 Figure 10. Optical constants ( $n, k$ ) of augite (Aug2). In this figure,  $n_1, n_2$  and  $k_1, k_2$  are the optical  
718 constants for the rays whose polarization direction is parallel to the  $a$ - $c$  plane, while  $n_b$  and  $k_b$  are the optical  
719 constants for the rays with polarization direction along the  $b$  crystallographic axis.

720

721

722 Figure 11. Measured and modeled reflectance spectra of diopside (Diop1) with incident E-field  
723 polarization parallel to  $a$ - $c$  plane at different polarization angles  $\phi$  (top three) and parallel to  $b$  (bottom).

724

725 Figure 12. Optical constants ( $n, k$ ) of diopside (Diop1). In this figure,  $n_1, n_2$  and  $k_1, k_2$  are the optical  
726 constants for the rays whose polarization direction is parallel to the  $a$ - $c$  plane, while  $n_b$  and  $k_b$  are the optical  
727 constants for the rays with polarization direction along the  $b$  crystallographic axis.

728

729

730 Figure 13. Measured and modeled reflectance spectra of hedenbergite (Hed1) with incident E-field  
731 polarization parallel to  $a$ - $c$  plane at different polarization angles  $\phi$  (top three) and parallel to  $b$  (bottom).

732

733 Figure 14. Optical constants ( $n, k$ ) of hedenbergite (Hed1). In this figure,  $n_1, n_2$  and  $k_1, k_2$  are the optical  
734 constants for the rays whose polarization direction is parallel to the  $a$ - $c$  plane, while  $n_b$  and  $k_b$  are the optical  
735 constants for the rays with polarization direction along the  $b$  crystallographic axis.

736

737

738 Figure 15. Measured and modeled reflectance spectra of orthoclase (Orth1) with incident E-field  
739 polarization parallel to  $a$ - $c$  plane at different polarization angles  $\phi$  (top three) and parallel to  $b$  (bottom).

740

741 Figure 16. Optical constants ( $n, k$ ) of orthoclase (Orth1) calculated in this work are in the solid lines  
742 ( $n_1, k_1, n_2, k_2, n_b, k_b$ ), while the optical constants derived in Aronson et al. (1986) are in the dashed lines  
743 ( $n_{a1}, k_{a1}, n_{a2}, k_{a2}, n_{ab}, k_{ab}$ ).

744

745 Figure 17. Orientation-averaged optical constants in the MIR for several minerals that belong to  
746 various crystal systems. Nano-diamonds from a Murchison meteorite (cubic), quartz (hexagonal),  
747 olivine (orthorhombic), and diopside (monoclinic).

748

749 Figure 18. Orientation-averaged optical constants ( $n, k$ ) of high-Mg olivine ( $Mg_{1.9}Fe_{0.1}SiO_4$ ) from UV  
750 to FIR wavelengths (Zeidler et al. 2011). Reflectance spectrum calculated from  $n, k$  values. Labels:  
751 Christiansen feature (CF), Reststrahlen bands (RB), highlighted in grey.

752

753 Figure 19. A comparison of single-crystal (enstatite and diopside) vs. amorphous (enstatite and a glass  
754 of pyroxene composition) pyroxene optical constants.

755

756 Table 1. Information for pyroxenes and orthoclase included in this study.

757



758 Table 2. Oscillator parameters for Augite (Aug1). The parameters  $\nu$ ,  $\gamma$ ,  $s$ ,  $\phi$  and  $\theta$  are those described in  
 759 section 5.1. The standard error,  $\sigma$ , is given for each parameter.

760  
 761 Table 3. Oscillator parameters for Augite (Aug 2). The parameters  $\nu$ ,  $\gamma$ ,  $s$ ,  $\phi$  and  $\theta$  are those described  
 762 in section 5.1. The standard error,  $\sigma$ , is given for each parameter.

763  
 764 Table 4. Oscillator parameters for Diopside (Diop 1). The parameters  $\nu$ ,  $\gamma$ ,  $s$ ,  $\phi$  and  $\theta$  are those  
 765 described in section 5.1. The standard error,  $\sigma$ , is given for each parameter.

766  
 767 Table 5. Oscillator parameters for Hedenbergite (Hed 1). The parameters  $\nu$ ,  $\gamma$ ,  $s$ ,  $\phi$  and  $\theta$  are those  
 768 described in section 5.1. The standard error,  $\sigma$ , is given for each parameter.

769  
 770 Table 6. Oscillator parameters for Orthoclase (Orth 1). The parameters  $\nu$ ,  $\gamma$ ,  $s$ ,  $\phi$  and  $\theta$  are those  
 771 described in section 5.1. The standard error,  $\sigma$ , is given for each parameter.

772  
 773 **Tables**

774 Table 1.

775

Sample	Source (location)	Chemical formula
augite(Aug1)	SBU (Indian Well, AZ)	$\text{Na}_{0.11}\text{K}_{0.10}\text{Mg}_{0.54}\text{Fe}_{0.46}\text{Ti}_{0.13}\text{Ca}_{0.46}\text{Al}_{0.48}\text{Si}_{1.79}\text{O}_6$
diopside(Diop1)	Kelly's Rocks (Magog, QC)	$\text{Na}_{0.04}\text{Mg}_{0.64}\text{Fe}_{0.18}\text{Mn}_{0.01}\text{Ca}_{0.92}\text{Al}_{0.03}\text{Si}_{2.09}\text{O}_6$
clinopyrox(Aug2)	SBU	$\text{Na}_{0.02}\text{Mg}_{0.57}\text{Fe}_{0.40}\text{Ti}_{0.02}\text{Ca}_{0.69}\text{Al}_{0.13}\text{Si}_{2.06}\text{O}_6$
hedenbergite(Hed1)	SBU	$\text{Na}_{0.03}\text{Mg}_{0.29}\text{Fe}_{0.51}\text{Mn}_{0.07}\text{Ca}_{0.83}\text{Al}_{0.08}\text{Si}_{2.08}\text{O}_6$
orthoclase	GGGems (Itrongay, Madagascar)	$\text{K}_{0.96}\text{Na}_{0.04}\text{Al}_{0.96}\text{Fe}_{0.04}\text{Si}_3\text{O}_8$

776

777 Table 2.

j	$\nu_j$	$\sigma(\nu_j)$	$\gamma_j$	$\sigma(\gamma_j)$	$s_j$	$\sigma(s_j)$	$\phi_j$	$\sigma(\phi_j)$	$\theta_j$
1	1106.3575	0.4039	13.6440	1.4160	4710.7187	519.6075	33.7390	2.2315	90
2	1082.0787	0.7336	22.4311	2.8987	8993.4843	1526.8844	57.9436	2.2866	90
3	1049.0371	0.1930	34.2430	0.6058	179324.6400	2855.2145	71.6003	0.2040	90
4	1010.1750	0.5134	20.6827	1.4914	15743.7790	1164.9098	33.9720	1.4204	90
5	973.9422	1.2439	39.3002	2.5816	61647.1074	7198.6643	100.6033	1.9936	90
6	952.8605	0.6238	33.8723	1.1586	115080.9250	3657.2602	76.5908	1.6294	90

7	911.9700	1.9049	60.9354	5.4555	84647.1159	6815.1152	69.0827	1.9607	90	
8	944.8770	0.6580	48.8271	2.4135	96156.4057	6739.0871	130.0419	2.5202	90	
9	864.4122	0.4977	43.8169	1.0250	236251.8770	364.8184	1.5422	0.5618	90	
10	855.2310	0.9153	31.8268	2.0963	73587.6460	1012.5941	148.5916	2.1071	90	
11	768.1230	1.8893	89.4754	7.2442	55510.9508	4229.1492	106.3017	2.6079	90	
12	628.8353	0.2212	18.0435	0.7495	33689.7859	1649.9417	101.0689	0.8103	90	
13	558.8928	0.5426	20.5635	1.7036	3907.3075	383.2433	-1.2976	4.3107	90	
14	527.2604	0.5724	24.0277	2.0407	10163.0101	1027.2230	2.9381	3.4734	90	
15	511.0346	0.6194	42.7838	2.2268	48307.6346	3153.4361	113.9736	1.4942	90	
16	588.4716	5.8446	86.0548	18.4170	24961.4935	5574.7554	89.6034	3.3377	90	
17	487.9342	0.3804	27.1740	1.2809	59799.8695	3570.4805	-7.2227	0.8682	90	
18	452.2373	0.2418	29.8619	0.7627	256324.4190	4869.1466	-7.2227	0.1871	90	
19	378.5617	0.2780	36.6437	1.4695	53084.6470	8107.2438	84.8518	6.4041	90	
20	380.5842	1.0044	53.7977	4.0509	72332.4613	6006.1467	25.3526	6.0986	90	
21	316.6041	0.1706	22.8939	0.3635	121807.2190	1236.6558	76.0592	0.3385	90	
22	285.8952	0.3018	26.1164	0.7252	78823.7304	1650.5374	141.5585	0.5527	90	
23	1066.2639	0.1892	30.6764	0.2559	222090.2320	1434.7913	0	--	0	
24	963.3500	0.2452	24.6545	0.6687	89492.5772	2397.4128	0	--	0	
25	900.8600	0.5216	61.1955	1.6569	210326.1180	4518.3363	0	--	0	
26	673.6817	1.2406	34.7883	3.7145	19978.7527	1653.7995	0	--	0	
27	547.9000	0.4589	18.9619	1.7624	6433.4126	786.4962	0	--	0	
28	503.9100	0.2723	37.6983	0.8500	148239.2660	4230.4820	0	--	0	
29	473.3900	0.2018	18.3338	0.6366	104023.2710	2914.9693	0	--	0	
30	408.0200	0.4402	21.1392	1.4232	26911.5056	1472.1208	0	--	0	
31	367.1695	1.0108	22.6618	4.1379	12240.3510	2468.8277	0	--	0	
32	333.8303	0.7396	37.0955	2.4841	44673.8703	2939.0821	0	--	0	
$\epsilon_{xx}=2.3418(0.0040)$		$\epsilon_{xy}=0.0263(0.0036)$			$\epsilon_{yy}=2.3934(0.0030)$			$\epsilon_{zz}=2.6441(0.0043)$		

778

779 Table 3.

j	$v_i$	$\sigma(v_i)$	$\gamma_i$	$\sigma(\gamma_i)$	$s_i$	$\sigma(s_i)$	$\varphi_i$	$\sigma(\varphi_i)$	$\theta_i$
1	1061.3863	1.6775	16.7945	3.5355	24107.2736	9108.7181	109.9541	3.0586	90
2	1045.6266	1.0695	17.4601	4.5891	49146.8143	15451.7954	118.8724	1.4748	90
3	1021.4581	1.8514	25.7043	6.2705	46430.9850	12385.4430	125.3247	2.1043	90
4	992.3639	1.1743	36.2713	3.4181	26819.2312	2845.9399	4.0808	7.8728	90
5	950.6749	1.8544	56.3535	3.6006	350600.2530	198.6376	92.4733	1.0478	90
6	873.6918	3.5412	73.3247	7.0740	198501.9250	4841.0644	103.9087	5.7859	90
7	852.6279	0.9302	56.1304	2.4576	384449.5580	21164.2827	-0.0526	2.7969	90
8	768.7863	3.5090	100.1498	11.8050	351227.4630	80.4223	-35.0603	1.7465	90
9	600.0011	3.0444	103.2869	9.5882	345195.2700	27533.1404	-49.3688	1.6754	90
10	686.4430	3.5464	112.2921	13.4456	157635.0660	22704.7234	15.7006	3.6531	90
11	625.9819	1.2538	25.3481	4.3195	31759.5593	5445.8591	73.3052	5.5194	90
12	520.0000	1.4388	20.2592	3.8818	55527.2990	8895.0274	82.9539	8.5934	90
13	533.0903	2.2436	17.2663	6.5846	26237.0233	1283.3818	179.8287	8.5146	90
14	501.7328	1.5229	22.9771	4.8641	168521.7630	27.0692	8.2681	4.3965	90
15	479.9509	0.7686	16.6904	2.6651	294597.8250	25.7761	4.6831	2.0602	90
16	461.8681	0.8220	14.9005	1.5448	258672.0850	1733.0123	5.0887	1.8157	90
17	403.8875	1.6247	34.3784	4.1294	86703.4089	6799.8357	99.5010	4.3104	90
18	360.1886	6.2701	27.2829	19.1020	17351.0619	11466.8151	-7.5571	14.6287	90
19	332.0028	1.0169	21.7865	3.0345	38951.2369	4767.2072	72.8595	5.0749	90
20	281.8601	1.4334	41.1500	3.7859	226289.1590	41.8564	32.8478	1.1554	90
21	250.0777	4.5801	15.2753	5.7124	41555.6458	23085.6849	74.3621	10.3421	90
22	1077.2920	0.4198	22.8731	0.6367	149503.0950	2402.3553	0	--	0
23	964.0978	1.2006	48.8749	4.5997	138155.7890	15182.2000	0	--	0
24	899.9383	3.5474	106.4168	7.3552	286332.2180	24390.6181	0	--	0

25	684.1713	5.8182	108.5547	25.4532	66276.4418	26720.9365	0	--	0
26	557.1254	1047.5670	15.2229	666.7554	3104.1971	177321.3230	0	--	0
27	541.0487	3.4361	17.7469	31.3527	9273.8041	161727.1810	0	--	0
28	524.2434	1.6431	18.0151	10.7360	19617.9937	16815.8158	0	--	0
29	507.6945	0.5276	18.0932	2.3374	67758.0486	12997.2329	0	--	0
30	480.6085	0.8391	31.0358	2.9332	173741.2560	16787.0322	0	--	0
31	411.3853	5.8307	45.1586	20.6277	61591.7484	47708.6593	0	--	0
32	370.1525	19.3001	44.4246	135.0416	23586.7622	91495.5373	0	--	0
33	325.5302	12.2621	65.4127	22.8392	72943.9747	51540.5930	0	--	0
34	257.0514	0.9801	12.4295	2.7168	16853.4601	3824.3130	0	--	0
$\epsilon_{xx}=3.1888(0.0147)$		$\epsilon_{xy}=0.0050(0.0168)$			$\epsilon_{yy}=2.8116(0.0157)$			$\epsilon_{zz}=2.6090(0.0109)$	

780

781 Table 4.

j	$v_j$	$\sigma(v_j)$	$\gamma_j$	$\sigma(\gamma_j)$	$s_j$	$\sigma(s_j)$	$\varphi_j$	$\sigma(\varphi_j)$	$\theta_j$
1	1053.3492	0.2690	9.8041	0.5202	161807.4260	8449.4644	95.0619	0.5386	90
2	1038.6875	0.4434	11.9789	1.1319	102002.5840	8297.6460	81.8839	0.7175	90
3	987.1730	1.1344	22.0697	2.3375	43276.2855	8424.1418	-58.3177	1.7571	90
4	961.9418	0.5321	21.8740	1.4044	328246.2610	13426.1056	107.2574	0.4212	90
5	908.9778	1.2771	34.1717	3.7573	112024.9180	10847.7536	88.2555	1.2813	90
6	851.9349	0.7393	52.0897	1.8660	276075.0410	8603.7134	40.6706	1.1639	90
7	799.5717	2.9359	82.3099	8.5987	133499.1160	11910.3499	140.7057	2.6738	90
8	694.3785	2.9357	139.0056	9.4790	184063.9650	11455.4558	35.9865	1.6679	90
9	625.9676	0.6184	20.9614	1.1518	85562.6394	3066.4869	111.9229	0.8129	90
10	526.8297	1.7909	48.7469	3.7816	97271.7196	11572.2554	45.1962	1.4457	90
11	509.9354	0.7947	14.1122	2.1296	16820.8129	2318.7730	-38.9575	4.1582	90
12	488.7702	1.0874	25.2207	3.2101	102617.7860	14753.3418	51.9415	1.9863	90
13	462.0420	0.8680	37.2480	2.3266	302939.5670	16231.8191	21.2456	0.8955	90

14	379.4407	0.6114	32.5722	1.2839	117538.4980	4685.8259	112.1053	0.9140	90	
15	326.4878	0.9753	26.4854	2.6635	108243.6900	14930.1059	-66.8801	1.1360	90	
16	297.4023	1.4361	27.9642	5.2131	98661.8029	17147.5147	-59.4064	4.2180	90	
17	293.0303	0.7230	16.9140	1.9692	26130.5743	3123.2116	20.9335	12.1318	90	
18	236.8182	2.6280	14.3421	3.0202	49054.5289	8115.4718	10.1390	6.1075	90	
19	1065.2649	61.4940	50.5334	1.8536	136867.0000	139.1562	0	--	0	
20	954.2127	50.9689	35.3480	3.5371	65646.0001	54.6979	0	--	0	
21	913.3331	114.3238	31.7870	6.9236	42070.0001	24.6549	0	--	0	
22	863.7441	367.8407	126.8820	24.7606	195784.2720	54551.0765	0	--	0	
23	737.9597	2678.3443	140.9497	56.2231	92809.1745	52723.1287	0	--	0	
24	636.0029	552.8693	93.0399	9.7740	89888.9079	17968.0509	0	--	0	
25	504.2384	645.0038	31.7429	1.4460	163435.8470	13745.7938	0	--	0	
26	472.0973	792.9559	28.4750	1.9606	276342.9090	15861.7259	0	--	0	
27	413.8197	324.9801	26.0624	3.2501	70136.7282	7278.4287	0	--	0	
28	249.2397	277.7731	9.7763	1.2649	18139.7810	2824.9225	0	--	0	
$\epsilon_{xx}=2.2120(0.0092)$		$\epsilon_{xy}=0.0084(0.0109)$			$\epsilon_{yy}=3.2076(0.0107)$			$\epsilon_{zz}=2.3310(0.0111)$		

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785 Table 5.

j	$v_i$	$\sigma(v_i)$	$\gamma_i$	$\sigma(\gamma_i)$	$s_i$	$\sigma(s_i)$	$\varphi_i$	$\sigma(\varphi_i)$	$\theta_i$
1	1085.5086	1.0619	53.6032	2.0797	71057.9148	10.0624	77.0368	1.6560	90
2	1059.2960	0.5518	45.6650	1.1514	159377.0460	3792.4402	-48.5292	0.5769	90
3	993.1131	1.1689	26.9090	3.5555	9922.7969	2381.2746	38.0265	7.7690	90
4	967.4469	0.8350	45.8181	2.5602	95361.6473	4969.2601	84.6362	1.1185	90
5	904.4503	1.5764	39.7557	4.0140	17736.2350	2895.3918	9.7179	4.8795	90
6	854.4819	0.3215	34.8501	0.7845	359324.3050	3685.3273	8.2315	0.2746	90

7	627.1273	0.4077	18.5918	0.9474	22454.7292	809.0312	70.7968	1.0661	90	
8	554.0051	0.8223	20.0375	2.2529	11125.4731	935.6338	80.3744	1.5719	90	
9	498.4405	0.5100	36.7979	1.1952	47045.1989	1737.4115	31.5953	0.6441	90	
10	444.5136	0.2360	25.4438	0.5064	259681.5030	2531.5769	7.0115	0.1641	90	
11	365.7561	0.4065	40.0052	1.1436	115793.2740	2579.3708	-17.1310	0.3263	90	
12	316.5364	0.2868	17.5481	0.7074	79478.0308	2327.4479	146.8989	0.4168	90	
13	295.8607	0.2829	11.2035	0.8214	38037.8982	2341.3446	-10.7030	0.9746	90	
14	269.8787	0.2434	11.4372	0.5673	24979.2863	1003.2629	31.7790	0.9470	90	
15	1058.5000	0.1466	13.5591	0.1493	298081.4700	1586.6960	0	--	0	
16	954.2000	0.1756	15.9576	0.3435	155456.8690	2858.5771	0	--	0	
17	905.3000	0.3818	37.5829	1.0258	283403.9260	5357.2963	0	--	0	
18	661.1000	1.2062	61.1123	3.4700	68518.4267	3042.9618	0	--	0	
19	509.4000	0.2300	25.7632	0.3148	236134.3190	3062.6966	0	--	0	
20	471.8000	0.2540	14.0770	0.6940	81073.6183	2729.1871	0	--	0	
21	398.1000	0.3743	23.0772	1.0692	64178.4958	2420.5236	0	--	0	
22	363.7127	0.6361	14.3740	2.4526	17497.1025	3048.7496	0	--	0	
23	330.9739	1.4859	43.8902	5.1669	49615.4221	5331.1122	0	--	0	
24	277.0999	0.3965	9.3079	1.2061	11291.1774	1162.9514	0	--	0	
$\epsilon_{xx}=2.323433(0.005738)$		$\epsilon_{xy}=0.011083(0.004933)$			$\epsilon_{yy}=1.811383(0.004054)$			$\epsilon_{zz}=3.476333(0.0063)$		

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787 Table 6.

j	$v_j$	$\sigma(v_j)$	$\gamma_j$	$\sigma(\gamma_j)$	$s_j$	$\sigma(s_j)$	$\varphi_j$	$\sigma(\varphi_j)$	$\theta_j$
1	1128.3686	0.3431	31.2995	0.7423	54556.3167	2123.2030	144.4339	1.2192	90
2	1108.8571	0.5588	31.1108	0.5308	212226.6890	3795.8550	24.3628	0.4137	90
3	1027.3708	0.3793	33.1247	0.6044	363554.0780	18.0570	96.9297	1.5021	90
4	1037.7416	0.5378	30.7390	1.3144	141912.7300	17.9877	4.6421	2.7087	90
5	1009.9518	0.4677	22.2564	1.0482	172752.3130	5177.9700	175.9980	1.8392	90

6	991.7254	0.5793	24.2304	2.0259	150428.1450	11517.6300	103.3195	1.8847	90	
7	765.4734	3.4041	42.0367	10.9087	15482.5298	3241.5640	5.9023	6.1221	90	
8	718.5118	1.2856	27.2765	3.9460	20263.4125	2306.1400	-51.0158	3.5496	90	
9	638.1094	0.4900	17.9200	1.1941	28666.8966	1392.3170	145.0234	1.2033	90	
10	578.8337	1.6291	24.5589	1.0170	42223.7579	9406.2510	43.3327	1.7890	90	
11	570.6447	0.5394	11.6388	0.7779	73786.7114	488.4382	42.3799	1.2196	90	
12	563.4729	0.7157	9.2861	1.9959	35232.0422	9229.9400	39.1535	2.2532	90	
13	415.7642	0.5353	34.7887	0.7423	114579.3000	1911.6290	-41.7658	0.4592	90	
14	1135.3801	0.8506	39.8303	2.1129	14854.1612	1579.1328	0	--	0	
15	1102.8937	0.5060	42.5629	2.0172	32799.5356	2104.3277	0	--	0	
16	1020.6898	0.5366	42.7108	0.7767	259002.7780	13008.0113	0	--	0	
17	992.4225	0.2744	32.1287	0.8942	387229.1360	12639.2520	0	--	0	
28	774.4806	1.3876	22.3337	4.0335	15639.1124	4074.5787	0	--	0	
19	752.3726	1.4388	23.5478	7.5651	15823.7692	6397.6618	0	--	0	
20	725.7595	1.3542	25.5395	3.9808	18424.9031	3558.0395	0	--	0	
21	610.5486	0.5855	17.5118	1.7004	11599.0969	836.4948	0	--	0	
22	543.0700	0.2560	16.9498	0.6631	31705.6149	1032.8273	0	--	0	
$\epsilon_{xx}=2.0658(0.0111)$		$\epsilon_{xy}=0.0043(0.0085)$			$\epsilon_{yy}=2.0126(0.0107)$			$\epsilon_{zz}=2.4461(0.0033)$		

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