## Revision 1

# Structural complexity of lead silicates: crystal structure of $\mathbf{P b}_{21}\left[\mathrm{Si}_{7} \mathbf{O}_{22}\right]_{2}\left[\mathrm{Si}_{4} \mathbf{O}_{13}\right]$ and its comparison to hyttsjöite 

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#### Abstract

The crystal structure of $\mathrm{Pb}_{21}\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]_{2}\left[\mathrm{Si}_{4} \mathrm{O}_{13}\right]$ has been solved on crystals grown by crystallization from melt. The compound is hexagonal, $P 6_{3} / m, a=9.9244(5), c=34.2357(16) ~ \AA \AA, \mathrm{~V}=$ $795.28(6) \AA^{3}, R_{1}=0.042$ for 3361 unique observed reflections. The structure contains five symmetrically independent Si sites tetrahedrally coordinated by O atoms. The $\mathrm{SilO}_{4}, \mathrm{Si}_{3} \mathrm{O}_{4}$ and $\mathrm{Si}_{4} \mathrm{O}_{4}$ tetrahedra share corners to form branched heptameric $\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]^{16-}$ units, whereas the $\mathrm{Si}_{2} \mathrm{O}_{4}$ and $\mathrm{Si5O}_{4}$ tetrahedra form the tetrameric $\left[\mathrm{Si}_{4} \mathrm{O}_{13}\right]^{10-}$ anions. The structure contains six symmetrically independent Pb sites with the $\mathrm{PbO}_{\mathrm{n}}$ coordination polyhedra distorted due to the stereochemical activity of the lone electron pairs. The structure can be described as a stacking of layers of the two types, A and B. The Atype layer contains $\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]^{16-}$ units, $\mathrm{Pb} 1, \mathrm{~Pb} 2, \mathrm{~Pb} 3$, and Pb 4 sites, whereas the B -type layer contains $\left[\mathrm{Si}_{4} \mathrm{O}_{13}\right]^{10-}$ anions, together with $\mathrm{Pb} 5, \mathrm{~Pb} 6$, and Pb 6 A sites. Stacking of the layers can be described as a sequence ...AA'BAA'B...., where A and $\mathrm{A}^{\prime}$ denote A layers with opposite orientations of the tripodshaped silicate heptamers. The crystal structure of $\mathrm{Pb}_{21}\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]_{2}\left[\mathrm{Si}_{4} \mathrm{O}_{13}\right]$ has many similarities to that of


hyttsjoiite, which contains the same layers consisting of tripod-shaped $\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]^{16-}$ anions. In both title compound and hyttsjöite, the anions are stacked together in such a way that ellipsoidal cavities with dimensions of ca. $10 \times 6 \times 6 \AA^{3}$ are created. The cavities are occupied by the $\mathrm{ClPb}_{6}$ octahedra in hyttsjöite and by 'empty' $\mathrm{Pb}_{6}$ octahedra in $\mathrm{Pb}_{21}\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]_{2}\left[\mathrm{Si}_{4} \mathrm{O}_{13}\right]$. Analysis of structural and chemical complexity in the $\mathrm{PbO}-\mathrm{SiO}_{2}$ system indicates that the most chemically complex phases (in terms of complexity of relations between chemical components) appear to be the most complex from the structural point of view as well. The title phase is the most structurally and chemically complex phase in the system. Structural organization of crystalline phases in the $\mathrm{PbO}-\mathrm{SiO}_{2}$ system can be described is controlled by the $\mathrm{Pb}: \mathrm{Si}$ ratio. For the phases with $\mathrm{Pb}: \mathrm{Si}<2$, their structures contain $\mathrm{Pb}^{2+}$ ions and silicate anions. For the phases with $\mathrm{Pb}: \mathrm{Si} \geq 2$, the structures contain 'additional' O atoms, i.e. atoms that are not bonded to Si. These atoms form $\mathrm{OPb}_{4}$ tetrahedra, which are the next strongest structural subunits in the structure after silicate anions. The structures of the phases with $\mathrm{Pb}: \mathrm{Si}<2$ can therefore be described as based upon silicate anions and polynuclear cationic units consisting of edge- and corner-sharing $\mathrm{OPb}_{4}$ tetrahedra.

Keywords: lead silicate; new mineral; crystal structure; hyttsjöite; lone electron pair stereoactivity; structural complexity.

## Introduction

Within recent years, a number of novel Pb -containing silicate mineral species have been reported in the literature (Chukanov et al. 2008; Yakubovich et al. 2008; Kampf et al. 2009; Belokoneva and Dimitrova 2011; Kolitsch et al. 2012; Turner et al. 2012; Siidra et al. 2013; Kampf et al. 2013; Yang et al. 2013; Pinch et al. 2013). These minerals possess unusual and unique structures due to the adaptation of topology and geometry of silicate (or aluminosilicate) anions to the arrangements of the $\mathrm{Pb}^{2+}$ cations possessing stereochemically active $6 s^{2}$ lone electron pairs. For
instance, the structures of maricopaite, $\mathrm{Ca}_{2} \mathrm{~Pb}_{7}\left(\mathrm{Si}_{36} \mathrm{Al}_{12}\right) \mathrm{O}_{99} \cdot n\left(\mathrm{H}_{2} \mathrm{O}, \mathrm{OH}\right)$ (Rouse and Peacor 1994) and rongibbsite, $\left[\mathrm{Pb}_{2} \mathrm{OH}\right]\left[\left(\mathrm{Si}_{4} \mathrm{Al}\right) \mathrm{O}_{11}\right]$ (Yang et al. 2013), contain interrupted tetrahedral frameworks with interruptions induced by the interaction of silicate species with polynuclear $\mathrm{Pb}-\mathrm{OH}$ clusters present as complex extraframework cations. The structures of britvinite,
$\left[\mathrm{Pb}_{7}(\mathrm{OH})_{3} \mathrm{~F}\left(\mathrm{BO}_{3}\right)_{2}\left(\mathrm{CO}_{3}\right)\right]\left[\mathrm{Mg}_{4.5}(\mathrm{OH})_{3}\left(\mathrm{Si}_{5} \mathrm{O}_{14}\right)\right]$ (Yakubovich et al. 2008) and molybdophyllite, $\left(\mathrm{Pb}_{4} \mathrm{O}\right)_{2} \mathrm{Mg}_{9}\left[\mathrm{Si}_{10} \mathrm{O}_{28}\right](\mathrm{OH})_{8}\left(\mathrm{CO}_{3}\right)_{3} \cdot \mathrm{H}_{2} \mathrm{O}$ (Kolitsch et al. 2012) possess complex [ $\left.\mathrm{Si}_{5} \mathrm{O}_{14}\right]$ tetrahedral sheets that can be viewed as interrupted mica-like silicate sheets. Stereochemical activity of lone electron pairs on $\mathrm{Pb}^{2+}$ cations seems to have a profound influence upon the structural features of silicate anions. The structure emerges as a result of a synergy between electronic and bonding requirements of the $\mathrm{Pb}^{2+}$ cations and flexibility of silicate tetrahedral units. The best way to understand this synergy is to investigate structures of 'pure' Pb silicates in the $\mathrm{PbO}-\mathrm{SiO}_{2}$ system, containing no additional cations and anions. However, only few data on these phases are available in the current literature. The only 'pure' Pb silicate mineral, alamosite, $\mathrm{PbSiO}_{3}$ (Boucher and Peacor 1968; Krivovichev and Burns 2004), contains unusually distorted single silicate chain with 12 tetrahedra within its identity period (Liebau 1985). Other phases known in the $\mathrm{PbO}-\mathrm{SiO}_{2}$ system are synthetic $\mathrm{Pb}_{2} \mathrm{SiO}_{4}\left(=\mathrm{Pb}_{2} \mathrm{O}\left[\mathrm{SiO}_{3}\right]\right)($ Kato 1980 ; Dent Glasser et al. 1981), $\mathrm{Pb}_{3} \mathrm{Si}_{2} \mathrm{O}_{7}$ (Petter et al. 1971), and $\mathrm{Pb}_{11} \mathrm{Si}_{3} \mathrm{O}_{17}\left(=\mathrm{Pb}_{11} \mathrm{O}_{6}\left[\mathrm{SiO}_{4}\right]\left[\mathrm{Si}_{2} \mathrm{O}_{7}\right]\right)($ Kato 1982). Herein we report on the synthesis and structural investigation of another synthetic Pb silicate, $\mathrm{Pb}_{7} \mathrm{Si}_{6} \mathrm{O}_{19}\left(=\mathrm{Pb}_{21}\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]_{2}\left[\mathrm{Si}_{4} \mathrm{O}_{13}\right]\right)$, and discuss its relations to hyttsjöite, $\mathrm{Pb}_{18} \mathrm{Ba}_{2} \mathrm{Ca}_{5} \mathrm{Mn}^{2+}{ }_{2} \mathrm{Fe}^{3+}{ }_{2} \mathrm{Si}_{30} \mathrm{O}_{90} \mathrm{Cl}^{-} 6 \mathrm{H}_{2} \mathrm{O}$ (Grew et al. 1996), a rare mineral from the Långban mines, Sweden.

## Experimental

Crystals of the title compound have been prepared by crystallization from melt. The mixture of $\mathrm{PbO}(0.595 \mathrm{~g}$, Aldrich, $99.95 \%)$ and $\mathrm{SiO}_{2}(0.040 \mathrm{~g}$, Aldrich, $99.98 \%)$ was placed into a platinum crucible on the substrate of $\mathrm{PbCl}_{2}(2.000 \mathrm{~g}$, Aldrich, $98 \%)$ used as a flux. The loaded crucible was heated to $920^{\circ} \mathrm{C}$ at the rate of $10^{\circ} \mathrm{C} / \mathrm{min}$ and kept at this temperature for 30 min . Then it was cooled to
$700^{\circ} \mathrm{C}$ at the cooling rate of $3^{\circ} \mathrm{C} / \mathrm{min}$ and kept for 30 min , followed by cooling to room temperature at the rate of $2^{\circ} \mathrm{C} / \mathrm{min}$. The product consisted of perfect hexagonal crystals of $\mathrm{Pb}_{21}\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]_{2}\left[\mathrm{Si}_{4} \mathrm{O}_{13}\right]$ (Figure 1) grown on the bottom of crucible.

Qualitative analysis performed using the Hitachi TM 3000 electron microscope indicated the absence of other elements with atomic number greater than $11(\mathrm{Na})$, except $\mathrm{Pb}^{2+}$ and $\mathrm{Si}^{4+}$.

One of the crystals obtained was crashed and suitable grain was mounted on a thin glass fiber for the X-ray diffraction analysis. More than a hemisphere of X-ray diffraction data with frame widths of $0.3^{\circ}$ in $\omega$, and 10 s spent counting for each frame were collected at room temperature using a Bruker three-circle Smart APEX II X-ray diffractometer operated at 50 kV and 40 mA with MoK $\alpha$ radiation. The data were integrated and corrected for absorption using an empirical ellipsoidal model by means of the Bruker programs $A P E X$ and $X P R E P$. The observed systematic absences were consistent with the space group $P-3$. The structure was solved by direct methods and refined to $R_{1}=0.061$ on the basis of $F^{2}$ for all unique data. The obtained structure model was transformed to the space group $P 6_{3} / m$ using the ADDSYM algorithm incorporated in the PLATON program package (Le Page, 1988; Speck, 2003). Structure refinement in this group resulted in the crystallographic agreement index $R_{1}=0.042$ (Table 1). The SHELX program package was used for all structure calculations (Sheldrick, 2008). Positional disorder was observed for one of the Pb sites, which appears to be splitted into three sites, $\mathrm{Pb} 6, \mathrm{~Pb} 6 \mathrm{~A}$ and Pb 6 A ' (note that the Pb 6 A and Pb 6 A ' sites are symmetrically equivalent), with a total occupancy factor of 1 . This kind of disorder for the Pb sites is typical for Pb silicates and aluminosilicates (see Yang et al. (2013) and references therein). The final model included anisotropic displacement parameters for all atoms. The final atomic coordinates and anisotropic displacement parameters are given in Table 2 and selected interatomic distances in Table 3.

## Silicate anions

The structure of the title compound contains five symmetrically independent Si sites tetrahedrally coordinated by O atoms. The $\mathrm{SilO}_{4}, \mathrm{Si3O}_{4}$ and $\mathrm{Si4O}_{4}$ tetrahedra share corners to form branched heptameric $\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]^{16-}$ unit shown in Figure 2a. The unit has a shape of a tripod with the
 very similar geometry have been observed previously in hyttsjöite (Grew et al. 1996). The $\mathrm{Si}_{2} \mathrm{O}_{4}$ and $\mathrm{Si5O}_{4}$ tetrahedra form the tetrameric $\left[\mathrm{Si}_{4} \mathrm{O}_{13}\right]^{10-}$ anion shown in Figure 2 b . Note that the $\mathrm{Si5}$ site is halfoccupied, which corresponds to the orientation of the $\mathrm{Si5O}_{4}$ tetrahedron either up or down relative to the plane of the unit. The $\left[\mathrm{Si}_{4} \mathrm{O}_{13}\right]^{10-}$ anions of the type shown in Figure 2 b have not been observed in minerals, but were described in the structure of synthetic $\mathrm{NaBa}_{3} \mathrm{Nd}_{3}\left[\mathrm{Si}_{2} \mathrm{O}_{7}\right]\left[\mathrm{Si}_{4} \mathrm{O}_{13}\right]$ (Malinovskii et al. 1983; Pushcharovsky, 1986).

## Pb coordination

The structure contains six symmetrically independent Pb sites (Figure 3). The Pb 6 and Pb 6 A sites have the site occupation factors (SOFs) of $1 / 3$. The coordinations of the Pb atoms demonstrate very different degree of distortion. In general, there are three $\mathrm{Pb}-\mathrm{O}$ bonds shorter than $2.6 \AA$ that form $\mathrm{PbO}_{3}$ trigonal pyramid with Pb at its apex. The $\mathrm{PbO}_{3}$ configuration is complemented by five or six $\mathrm{Pb}-$ O bonds in the range of 2.6-3.5 $\AA$. The coordination environments of the Pb 6 and Pb 6 A sites are rather irregular, obviously, due to their low occupancies. The relative distortion of the $\mathrm{PbO}_{\mathrm{n}}$ coordination polyhedra is due to the stereochemical activity of the lone electron pairs, which explains large variations in the $\mathrm{Pb}-\mathrm{O}$ bond lengths within the same polyhedron.

## Bond-valence analysis

Bond-valence calculations were performed using bond-valence parameters taken from Krivovichev and Brown (2001) for the $\mathrm{Pb}-\mathrm{O}$ bonds and from Brown and Altermatt (1985) for the Si-O bonds. The results are presented in Table 4. As can be seen, there is a general agreement between the expected and calculated oxidation states for all atomic sites, except those that are either low-occupied
( $\mathrm{Pb} 6, \mathrm{~Pb} 6 \mathrm{~A}, \mathrm{Si5}$, and O 12 ) or bonded to the low-occupied sites (e.g., O11). Since bond-valence parameters are derived from the data obtained for fully ordered structures, there is still no accepted procedure in dealing with the bond-valence sums for the low-occupied sites, which usually essentialy deviate from the expected values.

## Structure description

The structure of $\mathrm{Pb}_{21}\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]_{2}\left[\mathrm{Si}_{4} \mathrm{O}_{13}\right]$ is shown in Figure 4 a . It can be described as a stacking of layers of the two types, A and B . The A-type layer (Figure $4 b$ ) contains $\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]^{16-}$ units, $\mathrm{Pb} 1, \mathrm{~Pb} 2, \mathrm{~Pb} 3$, and Pb 4 sites, whereas the B -type layer (Figure 4c) contains $\left[\mathrm{Si}_{4} \mathrm{O}_{13}\right]^{10-}$ anions, together with $\mathrm{Pb} 5, \mathrm{~Pb} 6$, and Pb 6 A sites. The disorder observed for the Pb 6 and Pb 6 A sites correlates with disordered orientation of the $\mathrm{Si5O}_{4}$ tetrahedron (see above). The tripod-shaped $\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]^{16-}$ units have the same orientation within one A layer.

Stacking of the layers can be described as...AA'BAA'B...., where A and A' denote A layers with opposite orientations of the tripod-shaped silicate heptamers.

## Discussion

## Relations to hyttsjöite

The crystal structure of $\mathrm{Pb}_{21}\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]_{2}\left[\mathrm{Si}_{4} \mathrm{O}_{13}\right]$ has many similarities to that of hyttsjöite, $\mathrm{Pb}_{18} \mathrm{Ba}_{2} \mathrm{Ca}_{5} \mathrm{Mn}^{2+}{ }_{2} \mathrm{Fe}^{3+}{ }_{2} \mathrm{Si}_{30} \mathrm{O}_{90} \mathrm{Cl}^{6} 6 \mathrm{H}_{2} \mathrm{O}($ Grew et al. 1996 $)=$ $\mathrm{Pb}_{18} \mathrm{Ba}_{2} \mathrm{Ca}_{5} \mathrm{Mn}^{2+}{ }_{2} \mathrm{Fe}^{3+}{ }_{2}\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]_{2}\left[\mathrm{Si}_{8} \mathrm{O}_{23}\right]_{2} \mathrm{Cl}^{6} 6 \mathrm{H}_{2} \mathrm{O}$. As described by Grew et al. (1996), the latter consists of two types of plumbosilicate layers, L1 and L2. The L1 layers contain continuous $\left[\mathrm{Si}_{8} \mathrm{O}_{23}\right]^{14-}$ silicate sheets and $\mathrm{Pb}^{2+}$ cations. The L2 layers are virtually identical to the A sheets in the structure of $\mathrm{Pb}_{21}\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]_{2}\left[\mathrm{Si}_{4} \mathrm{O}_{13}\right]$. The layers are based upon the same type of tripod-shaped silicate heptamers, $\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]^{16-}$. The stacking sequence of the layers in hyttsjöite can be described as ...L1 L1' L2 L2' L1 L1' L2 L2'..., where ' is used to identify opposite orientation of the sheets.

The most interesting common aspects of the structures of $\mathrm{Pb}_{21}\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]_{2}\left[\mathrm{Si}_{4} \mathrm{O}_{13}\right]$ and hyttsjöite are the arrangements of tripod-shaped silicate heptamers in the adjacent plumbosilicate layers. They are stacked together in such a way that ellipsoidal cavities with dimensions of ca. $10 \times 6 \times 6 \AA^{3}$ are created. The cavity in hyttsjöite (Figure 5c) is occupied by the $\mathrm{ClPb}_{6}$ octahedron (Figure 5d), i.e. the octahedron formed by six Pb atoms and centered by Cl . In contrast, in $\mathrm{Pb}_{21}\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]_{2}\left[\mathrm{Si}_{4} \mathrm{O}_{13}\right]$, there is no Cl in the center of the cavity (Figure 5a), yielding an 'empty' $\square \mathrm{Pb}_{6}$ octahedron (Figure 5b). Figure 5 shows that the size of the $\mathrm{Pb}_{6}$ octahedra are approximately the same in the two structures, except in hyttsjöite it is larger, which can be explained by the large size of the $\mathrm{Cl}^{-}$anion. The structural similarity of the double $\left\{\mathrm{L} 2 \mathrm{~L} 2^{\prime}\right\}$ and $\left\{\mathrm{AA}^{\prime}\right\}$ layers in hyttsjöite and the title compound, respectively, is striking and may be interpreted as follows. The octahedral $\mathrm{Pb}_{6}$ cavity in $\mathrm{Pb}_{21}\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]_{2}\left[\mathrm{Si}_{4} \mathrm{O}_{13}\right]$ is occupied by the stereoactive lone electron pairs of the $\mathrm{Pb}^{2+}$ cations, which tend to associate into specific regions of the structure called lone-pair micelles (Makovicky 1997; Krivovichev et al. 2004; Siidra et al. 2009, 2010) or lone-pair self-containments (Johnston and Harrison 2002). The probable explanation for this effect is the electrophilicity of lone electron pairs that can be understood in terms of the hard-soft acid-base (HSAB) concept (Pearson 1988). The lone electron pairs on the cations behave as soft ligands that tend to associate together in a 'hard' environment. Therefore, the octahedral lone-pair micelles in $\mathrm{Pb}_{21}\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]_{2}\left[\mathrm{Si}_{4} \mathrm{O}_{13}\right]$ with partially delocalized electron density are replaced by highly polarizable and soft Cl- anions in hyttsjöite. Very similar phenomenon has been observed by Frit et al. (1983) for two compounds, $\mathrm{Tl}^{3+}{ }_{6} \mathrm{O}_{6}\left(\mathrm{TeO}_{6}\right)$ and $\mathrm{Tl}_{6}^{+}\left(\mathrm{TeO}_{6}\right)$, which are isotypic, except for one symmetrically independent O site, which is present in the former and absent in the latter. In the structure of $\mathrm{Tl}^{3+}{ }_{6} \mathrm{O}_{6}\left(\mathrm{TeO}_{6}\right)$, this O atom occupies $\left[\mathrm{Tl}_{4}\right]$ tetrahedron formed by four $\mathrm{Tl}^{3+}$ cations, whereas, in $\mathrm{Tl}^{+}{ }_{6}\left(\mathrm{TeO}_{6}\right)$, the corresponding tetrahedron is empty.

According to Grew et al. (1996), hyttsjöite forms at temperatures near $500-600{ }^{\circ} \mathrm{C}$ and pressures 2-4 kbar. Similar conditions may also be suitable for the formation of $\mathrm{Pb}_{21}\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]_{2}\left[\mathrm{Si}_{4} \mathrm{O}_{13}\right]$. Although no traces of Cl were observed by electron microprobe or single-crystal structure analysis, it seems possible that the presence of chlorine in the synthesis mixture may be necessary for the crystallization
of the title phase. One may speculate that tripod-shaped silicate heptamers with $\mathrm{ClPb}_{6}$ octahedra at the center serve as prenucleation building blocks with subsequent removal of Cl .

## Chemical and structural complexity of crystalline phases in the $\mathrm{PbO}_{-\mathrm{SiO}_{2}}$ system

At the time of writing, there are four known structurally characterized crystalline phases in the $\mathrm{PbO}-\mathrm{SiO}_{2}$ system, and the title compound is the fifth. Crystallographic data for these compounds are given in Table 5, which also provides quantitative characteristics of their structural and chemical complexity calculated as follows.

The structural complexity is evaluated as an amount of structural information per atom $\left(I_{G}\right)$ and per unit cell (u.c., $I_{G, \text { total }}$ ) according to the formulas (Krivovichev, 2012, 2013a, b):

$$
\begin{gather*}
I_{G}=-p_{i} \log _{2} p_{i} \quad \text { (bits/atom) }  \tag{1}\\
I_{G, \text { total }}=-v I_{G}=-v p_{i} \log _{2} p_{i} \quad \text { (bits/u.c.) }
\end{gather*}
$$

where $k$ is the number of different crystallographic orbits and $p_{i}$ is the random choice probability for an atom from the $i$ th crystallographic orbit, that is:

$$
\begin{equation*}
p_{i}=m_{i} / v \tag{3}
\end{equation*}
$$

where $m_{i}$ is a multiplicity of a crystallographic orbit relative to the reduced unit cell, and $v$ is the number of atoms in the reduced unit cell.

The chemical complexity is estimated by considering chemical formula as a message, where symbols correspond to different chemical elements. For instance, $\mathrm{Pb}_{2} \mathrm{SiO}_{4}$ should be considered as PbPbSiOOOO , i.e. as a message of seven symbols that can be separated into three equivalence classes, $\{\mathrm{Pb}\},\{\mathrm{Si}\},\{\mathrm{O}\}$, containing two, one, and four symbols, respectively. The amount of chemical information per formula then can be calculated as

$$
\begin{equation*}
{ }^{\text {chem }} I=-n p_{i}^{\prime} \log _{2} p_{i}^{\prime} \quad \text { (bits/formula) } \tag{4}
\end{equation*}
$$

where $n$ is the number of atoms in the chemical formula, $k$ is the number of different elements (= the number of equivalence classes), and is the random choice probability for an atom from the $i$ th class:

$$
\begin{equation*}
p_{i}^{\prime}=q_{i} / n \tag{5}
\end{equation*}
$$

where $q_{i}$ is the number of atoms in the $i$ th equivalence class.
For $\mathrm{Pb}_{2} \mathrm{SiO}_{4}, q_{1}\{\mathrm{~Pb}\}=2, q_{2}\{\mathrm{Si}\}=1, q_{3}\{\mathrm{O}\}=4, n=7$, which results in: $p_{1}^{\prime}=2 / 7, p_{2}^{\prime}=1 / 7, p_{3}^{\prime}=$ $4 / 7$. Therefore, ${ }^{\text {chem }} I=-2 \log _{2}(2 / 7)-\log _{2}(1 / 7)-4 \log _{2}(4 / 7)=9.651$ bits/f.u. (f.u. $=$ formula unit).

As can be seen from the data given in Table 5, there is a general trend of increasing structural information with the increasing chemical complexity. The most chemically complex phases appear to be the most complex from the structural point of view as well. The title phase appears to be the most structurally and chemically complex phase in the $\mathrm{PbO}-\mathrm{SiO}_{2}$ system known so far.

In general, structural organization of crystalline phases in the system can be described as follows. For the phases with $\mathrm{Pb}: \mathrm{Si}<2$, their structures contain $\mathrm{Pb}^{2+}$ ions and silicate anions. The topological diversity of the silicate anions is controlled via the chemical complexity, i.e. complexity of the chemical formula. For the phases with $\mathrm{Pb}: \mathrm{Si} \geq 2$, the structures contain 'additional' O atoms, i.e. atoms that are not bonded to Si . These atoms form strong bonds to the Pb atoms, which results in the formation of cationic $\mathrm{OPb}_{4}$ tetrahedra, which are the next strongest structural subunits in the structure after silicate anions. Therefore the structure is considered as based upon silicate anions and polynuclear cationic units consisting of edge- and corner-sharing $\mathrm{OPb}_{4}$ tetrahedra (Krivovichev and Filatov 1999; Siidra et al. 2008; Krivovichev et al. 2013). For instance, the structure of $\mathrm{Pb}_{2} \mathrm{O}\left[\mathrm{SiO}_{3}\right]$ (Dent Glasser et al. 1981) contains $\left[\mathrm{Si}_{4} \mathrm{O}_{12}\right]^{8-}$ rings of $\mathrm{SiO}_{4}$ tetrahedra and $\left[\mathrm{OPb}_{2}\right]^{2+}$ chains of $\mathrm{OPb}_{4}$ tetrahedra. More details on structural systematics of anion-centered tetrahedra and their occurence in minerals and inorganic compounds can be found in the recent review (Krivovichev et al. 2013).

## Implications

In this study, using the example of the title compound, we demonstrated how structural complexity of lead silicates emerges as a result of interplay between stereochemical behaviour of lone electron pairs on $\mathrm{Pb}^{2+}$ cations and flexibility of tetrahedral silicate anions. In particular, comparison of the structure of $\mathrm{Pb}_{21}\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]_{2}\left[\mathrm{Si}_{4} \mathrm{O}_{13}\right]$ with that of hyttsjöite shows that structural complexity of the latter is indeed induced by the electronic properties of Pb atoms rather than by general complex chemical composition. This kind of behaviour is observed in other natural Pb silicates as well, where lone electron pair stereoactivity induces either high periodicity of silicate chains (as in alamosite) or formation of interrupted tetrahedral frameworks (as in maricopaite and rongibbsite).

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## References

Belokoneva, E.L. and Dimitrova, O.V. (2011) Crystal structure $\left(\mathrm{Pb}_{4.8} \mathrm{Na}_{1.2}\right)\left[\mathrm{Si}_{8}\left(\mathrm{Si}_{1.2} \mathrm{~B}_{0.8}\right) \mathrm{O}_{25}\right]$ with a new double tetrahedral layer and its comparison with hyttsjoeite, barisilite, benitoite, and langasite. Crystallography Reports, 56, 116-122.

Boucher, M.L. and Peacor, D.R. (1968) The crystal structure of alamosite, $\mathrm{PbSiO}_{3}$. Zeitschrift für Kristallographie, 126, 98-111.

Brown, I.D. and Altermatt, D. (1985) Bond-valence parameters from a systematic analysis of the inorganic crystal structure database. Acta Crystallographica, B41, 244-247.

Chukanov, N.V., Yakubovich, O.V., Pekov, I.V., Belakovsky, D.I., and Massa, W. (2008) Britvinite,
$\mathrm{Pb}_{15} \mathrm{Mg}_{9}\left(\mathrm{Si}_{10} \mathrm{O}_{28}\right)\left(\mathrm{BO}_{3}\right)_{4}\left(\mathrm{CO}_{3}\right)_{2}(\mathrm{OH})_{12} \mathrm{O}_{2}$, a new mineral species from Långban, Sweden. Geology of Ore Deposits, 50, 713-719.

Dent Glasser, L.S., Howie, R.A., and Smart, R.M. (1981) The structure of lead 'orthosilicate', $(\mathrm{PbO})_{2} \mathrm{SiO}_{2}$. Acta Crystallographica, B37, 303-306.

Frit, B., Roult, G., and Galy, J. (1983) Cristallochimie de $\mathrm{Tl}(\mathrm{III})_{6} \mathrm{Te}(\mathrm{VI}) \mathrm{O}_{12}$ et $\mathrm{Tl}(\mathrm{I})_{6} \mathrm{Te}(\mathrm{VI}) \mathrm{O}_{6} E_{6}$ : un exemple original de l'activite stereochimique de la paire electronique $6 s^{2}(E)$ du thallium(I). Journal of Solid State Chemistry, 48, 246-255.

Grew, E.S., Peacor, D.R., Rouse, R.C., Yates, M.G., Su, S.-C., and Marquez, N. (1996) Hyttsjoeite, a new, complex layered plumbosilicate with unique tetrahedral sheets from Langban, Sweden. American Mineralogist, 81, 743-753.

Johnston, M.G. and Harrison, W.T.A. (2002) Lone-pair self-containment in tellurite tubes: hydrothermal syntheses and structures of $\mathrm{BaTeO}_{7}$ and $\mathrm{BaTe}_{4} \mathrm{O}_{9}$. Journal of the American Chemical Society, 124, 4576-4577.

Kampf, A.R., Rossman, G.R., and Housley, R.M. (2009) Plumbophyllite, a new species from the Blue Bell claims near Baker, San Bernardino County, California. American Mineralogist, 94, 11981204.

Kampf, A.R., Pluth, J.J., Chen, Y.-S., Roberts, A.C., and Housley, R.M. (2013) Bobmeyerite, a new mineral from Tiger, Arizona, USA, structurally related to cerchiaraite and ashburtonite. Mineralogical Magazine, 77, 81-91.

Kato, K. (1980) Die OD-Struktur von Bleisilicat $\mathrm{Pb}_{2} \mathrm{SiO}_{4}$ und Bleisilicat-Germanat-Mischkristall $\mathrm{Pb}_{2}(\mathrm{SiGe}) \mathrm{O}_{4}$. Acta Crystallographica, B36, 2539-2545.

Kato, K. (1982) Die Kristallstruktur des Bleisilicats $\mathrm{Pb}_{11} \mathrm{Si}_{3} \mathrm{O}_{17}$. Acta Crystallographica, B38, 57-62. Kolitsch, U., Merlino, S., and Holtstam, D. (2012) Molybdophyllite: crystal chemistry, crystal structure, OD character and modular relationships with britvinite. Mineralogical Magazine, 76, 493-516.

Krivovichev, S.V (2012) Topological complexity of crystal structures: quantitative approach. Acta
Crystallographica, A68, 393-398.
Krivovichev, S.V. (2013a) Structural complexity of minerals: information storage and processing in the mineral world. Mineralogical Magazine, 77, 275-326.

Krivovichev, S.V. (2013b) Which inorganic structures are the most complex? Angewandte Chemie International Edition, accepted. DOI: 10.1002/anie.201304374.

Krivovichev, S.V. and Brown, I.D. (2001) Are the compressive effects of encapsulation an artefact of the bond valence parameters? Zeitschrift für Kristallographie, 216, 245-247.

Krivovichev, S.V. and Burns, P.C. (2004) Crystal structure of synthetic alamosite $\mathrm{Pb}\left(\mathrm{SiO}_{3}\right)$. Zapiski Vserossijskogo Mineralogicheskogo Obshchestva, 133(5), 70-76 (in Russian).

Krivovichev, S.V. and Filatov, S.K. (1999) Structural principles for minerals and inorganic compounds containing anion-centered tetrahedra. American Mineralogist, 84, 1099-1106.

Krivovichev, S.V., Armbruster, T., and Depmeier, W. (2004) One-dimensional lone electron pair micelles in the crystal structure of $\mathrm{Pb}_{5}\left(\mathrm{SiO}_{4}\right)\left(\mathrm{VO}_{4}\right)_{2}$. Materials Research Bulletin, 39, 1717-1722.

Krivovichev, S.V., Mentré, O., Siidra, O.I., Colmont, M., and Filatov, S.K. (2013) Anion-centered tetrahedra in inorganic compounds. Chemical Reviews, 113, 6459-6535.

Le Page, Y. (1988) MISSYM1.1 - a flexible new release. Journal of Applied Crystallography, 21, 983984.

Liebau, F. (1985) Structural Chemistry of Silicates. Structure, Bonding and Classification. Springer Verlag, Berlin.

Makovicky, E. (1997) Modular and crystal chemistry of sulfosalts and other complex sulfides. In:
Merlino, S. (ed). Modular Aspects of Minerals. EMU Notes in Mineralogy, Vol. 1. Eotvos Univesity Press, Budapest, pp. 237-271.

Malinovskii, Yu.A., Baturin, S.V., and Bondareva, O.S. (1983) A new island silicate radical [ $\mathrm{Si}_{4} \mathrm{O}_{13}$ ] in the structure of $\mathrm{NaBa}_{3} \mathrm{Nd}_{3}\left[\mathrm{Si}_{2} \mathrm{O}_{7}\right]\left[\mathrm{Si}_{4} \mathrm{O}_{13}\right]$. Soviet Physics Doklady 28, 809-812.

Pearson, R.G. (1988) Absolute electronegativity and hardness: application to inorganic chemistry.

Inorganic Chemistry, 27, 734-740.
Petter, W., Harnik, A.B., and Keppler, U. (1971) Die Kristallstruktur von Blei-Barysilit, $\mathrm{Pb}_{3} \mathrm{Si}_{2} \mathrm{O}_{7}$.
Zeitschrift für Kristallographie, 133, 445-458.
Pinch, W.W., Downs, R.T., Evans, S.H., Megaw, L., and Bloch, E.M. (2013) Yangite, IMA 2012-052. CNMNC Newsletter No. 15. Mineralogical Magazine, 77, 1-12.

Pushcharovsky, D.Yu. (1986) Structural Mineralogy of Silicates and Their Synthetic Analogs. Moscow Nedra (in Russian).

Rouse, R.C. and Peacor, D.R. (1994) Maricopaite, an unusual lead calcium zeolite with an interrupted mordenite-like framework and intrachannel $\mathrm{Pb}_{4}$ tetrahedral clusters. American Mineralogist, 79, 175-184.

Sheldrick, G.M. (2008) A short history of SHELX. Acta Crystallographica, A64, 112-122.
Siidra, O.I., Krivovichev, S.V., and Filatov, S.K. (2008) Minerals and synthetic Pb (II) compounds with oxocentered tetrahedra: review and classification. Zeitschrift für Kristallographie, 223, 114-125. Siidra, O.I., Britvin, S.N., and Krivovichev, S.V. (2009) Hydroxocentered $\left[(\mathrm{OH}) \mathrm{Tl}_{3}\right]^{2+}$ triangle as a building unit in thallium compounds: Synthesis and crystal structure of $\mathrm{Tl}_{4}(\mathrm{OH})_{2} \mathrm{CO}_{3}$. Zeitschrift für Kristallographie, 224, 563-567.

Siidra, O.I., Britvin, S.N., Krivovichev, S.V., and Depmeier, W. (2010) Polytypism of layered alkaline hydroxides: Crystal structure of TlOH . Zeitschrift für Anorganische und Allgemeine Chemie, 636, 595-599.

Siidra, O.I., Krivovichev, S.V., Turner, R.W., Rumsey, M.S., and Spratt, J. (2013) Crystal chemistry of layered Pb oxychloride minerals with PbO -related structures: Part I. Crystal structure of hereroite, $\left[\mathrm{Pb}_{32} \mathrm{O}_{20}(\mathrm{O}, \square)\right]\left(\mathrm{AsO}_{4}\right)_{2}\left[(\mathrm{Si}, \mathrm{As}, \mathrm{V}, \mathrm{Mo}) \mathrm{O}_{4}\right]_{2} \mathrm{Cl}_{10}$. American Mineralogist, 98, 248-255. Speck, A.L. (2003) Single-crystal structure validation with the program PLATON. Journal of Applied Crystallography, 36, 7-13.

Turner, R., Siidra, O.I., Rumsey, M.S., Krivovichev, S.V., Stanley, C.J., and Spratt, J. (2012) Hereroite and vladkrivovichevite: two novel lead oxychlorides from the Kombat mine, Namibia. Mineralogical Magazine, 76, 883-890.

Yakubovich, O.V., Massa, W., and Chukanov, N.V. (2008) Crystal structure of britvinite $\left[\mathrm{Pb}_{7}(\mathrm{OH})_{3} \mathrm{~F}\left(\mathrm{BO}_{3}\right)_{2}\left(\mathrm{CO}_{3}\right)\right]\left[\mathrm{Mg}_{4.5}(\mathrm{OH})_{3}\left(\mathrm{Si}_{5} \mathrm{O}_{14}\right)\right]$ : a new layered silicate with an original type of silicon-oxygen networks. Crystallography Reports, 53, 206-215.

Yang, H., Downs, R.T., Evans, S.H., Jenkins, R.A., and Bloch, E.M. (2013) Rongibbsite, $\mathrm{Pb}_{2}\left(\mathrm{Si}_{4} \mathrm{Al}^{2}\right) \mathrm{O}_{11}(\mathrm{OH})$, a new zeolitic aluminosilicate mineral with an interrupted framework from Maricopa County, Arizona, U.S.A. American Mineralogist, 98, 236-241.

Figures


Figure 1. Yellow hexagonal crystals of $\mathrm{Pb}_{21}\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]_{2}\left[\mathrm{Si}_{4} \mathrm{O}_{13}\right]$ under optical microscope.


Figure 2. Silicate anions in the crystal structure of $\mathrm{Pb}_{21}\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]_{2}\left[\mathrm{Si}_{4} \mathrm{O}_{13}\right]$.



Figure 3. Coordination of Pb atoms in the crystal structure of $\mathrm{Pb}_{21}\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]_{2}\left[\mathrm{Si}_{4} \mathrm{O}_{13}\right]$.


Figure 4. The crystal structure of $\mathrm{Pb}_{21}\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]_{2}\left[\mathrm{Si}_{4} \mathrm{O}_{13}\right]$ projected along [110] (a) and projections of the A and B layers onto the (001) plane (b).


Figure 5. Ellipsoidal cavities formed by $\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]$ silicate anions in the crystal structures of
$\mathrm{Pb}_{21}\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]_{2}\left[\mathrm{Si}_{4} \mathrm{O}_{13}\right]$ (a) and hyttsjöite (c) are occupied by the $\square \mathrm{Pb}_{6}$ (b) and $\mathrm{ClPb}_{6}$ (d) octahedral clusters, respectively.

Table 1. Crystallographic data and refinement parameters for $\mathrm{Pb}_{21}\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]_{2}\left[\mathrm{Si}_{4} \mathrm{O}_{13}\right]$

| Crystal size $\left(\mathrm{mm}^{3}\right)$ | $0.14 \times 0.15 \times 0.09$ |
| :--- | :--- |
| Space group | $P 6_{3} / m$ |
| $a(\AA)$ | $9.9244(5)$ |
| $c(\AA)$ | $34.2357(16)$ |
| $V\left(\AA^{3}\right)$ | $795.28(6)$ |
| $\mu\left(\mathrm{mm}^{-1}\right)$ | 60.771 |
| $\mathrm{D}_{\text {calc }}\left(\mathrm{g} / \mathrm{cm}^{3}\right)$ | 6.560 |
| Radiation wavelength $(\AA)$ ) | $0.71073(\mathrm{MoKa})$ |
| $\theta$-range (deg.) | $1.19-36.36$ |
| Total Ref. | 28731 |
| Unique Ref. | 4625 |
| Unique \|Fol $\geq 4 \mathrm{~s}_{F}$ | 3361 |
| $R_{\text {int }}$ | 0.08 |
| $R_{1}$ | 0.042 |
| $R_{1}($ all data $)$ | 0.072 |
| GoF | 1.090 |
| $\rho_{\text {max,min }}\left(\mathrm{e} \cdot \AA^{-3}\right)$ | $+4.283 /-3.306$ |

Table 2. Atomic coordinates and displacement parameters $\left(\AA^{2}\right)$ for $\mathrm{Pb}_{21}\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]_{2}\left[\mathrm{Si}_{4} \mathrm{O}_{13}\right]$

| Atom | $x$ | $y$ | $z$ | $U_{\text {eq }}$ | $U_{11}$ | $U_{22}$ | $U_{33}$ | $U_{23}$ | $U_{13}$ | $U_{12}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Pb1 | 2/3 | 1/3 | 0.594983(16) | 0.01184(10) | $0.01273(14)$ | 0.01273(14) | $0.0101(2)$ | 0 | 0 | 0.00637(7) |
| Pb2 | 2/3 | 1/3 | 0.490651(16) | 0.01352(11) | 0.01453(15) | 0.01453(15) | 0.0115(2) | 0 | 0 | 0.00726(7) |
| Pb3 | 0.75175(4) | $0.01367(4)$ | 0.545490(10) | $0.01375(7)$ | 0.01206(13) | $0.01489(14)$ | $0.01423(13)$ | 0.00059(12) | -0.0002(1) | 0.00668(12) |
| Pb4 | 0.08035(4) | 0.64380(4) | $0.653630(10)$ | $0.01664(7)$ | $0.01375(14)$ | 0.01605(15) | $0.01811(15)$ | 0.00081 (12) | $0.00214(12)$ | 0.00594(12) |
| Pb5 | $2 / 3$ | $1 / 3$ | 0.698105(18) | 0.02069(12) | $0.02583(18)$ | 0.02583(18) | 0.0104(2) | 0 | 0 | 0.01292(9) |
| Pb6* | 0.8208(5) | $0.7603(5)$ | $3 / 4$ | 0.0122(6) | $0.0061(7)$ | 0.0120(9) | 0.0181(15) | 0 | 0 | 0.0042(7) |
| Pb6A* | 0.8013(4) | 0.7563(4) | $0.74109(15)$ | 0.0481(10) | 0.0445(16) | 0.0481(13) | 0.066(3) | 0.0258(15) | 0.0321(16) | 0.0335(12) |
| Si 1 | 0 | 0 | $0.66788(15)$ | 0.0206(9) | 0.0195(13) | 0.0195(13) | 0.023(2) | 0 | 0 | 0.0098(6) |
| Si2 | 0.9873(4) | 0.5707(4) | $3 / 4$ | 0.0118(6) | 0.0093(14) | 0.0103(14) | 0.0145(15) | 0 | 0 | 0.0040(12) |
| Si3 | 0.6029(3) | 0.6077 (3) | 0.55296(7) | 0.0101(4) | 0.0115(10) | 0.0108(10) | 0.0090(9) | 0.0005(8) | -0.0019(8) | 0.0062(8) |
| Si4 | 0.9360(3) | 0.2492(3) | 0.63335(7) | 0.0134(4) | 0.0131(10) | 0.0148(11) | 0.0104(10) | 0.0009(9) | 0.0000(9) | 0.0056(9) |
| Si5** | 1/3 | 2/3 | 0.7398(2) | 0.0085(14) | $0.0071(17)$ | 0.0071 (17) | 0.011(3) | 0 | 0 | 0.0036(8) |
| 01 | 0.8553(10) | 0.3855(10) | 3/4 | 0.0125(16) | 0.015(4) | 0.013(4) | 0.010(4) | 0 | 0 | 0.008(3) |
| 02 | 0.6955(7) | 0.5135(7) | $0.54715(18)$ | $0.0137(11)$ | 0.017(3) | 0.012(3) | 0.013(3) | -0.002(2) | 0.001(2) | 0.007(2) |
| O3 | 0.6446(8) | 0.7344(8) | 0.5187(2) | 0.0191(13) | 0.031(4) | 0.017(3) | 0.015(3) | 0.005(3) | 0.003(3) | 0.016(3) |
| 04 | 0.7599(7) | 0.1252(8) | $0.6215(2)$ | $0.0213(14)$ | 0.009(3) | 0.026(3) | 0.024(4) | 0.008(3) | 0.005(3) | 0.005(3) |
| 05 | 0.0389(7) | 0.3315(9) | $0.59424(19)$ | 0.0224(15) | 0.012(3) | 0.033(4) | 0.012(3) | 0.011(3) | 0.001(3) | 0.003(3) |
| 06 | 0 | 0 | $0.7141(4)$ | 0.022(2) | 0.021(3) | 0.021(3) | 0.024(6) | 0 | 0 | 0.0105(17) |
| 07 | 0.9638(9) | 0.6542(8) | 0.7121(2) | 0.0245(15) | 0.037(4) | 0.028(4) | 0.017(3) | 0.009(3) | 0.012(3) | 0.023(3) |
| 08 | 0.1412(9) | 0.9762(9) | 0.6532(3) | 0.0301(18) | 0.026(4) | 0.027(4) | 0.042(5) | 0.006(4) | 0.016(4) | 0.016(3) |
| 09 | 0.4174(8) | 0.4943(8) | 0.5569(3) | 0.0284(18) | 0.011(3) | 0.014(3) | 0.058(6) | -0.004(3) | -0.002(3) | 0.005(3) |
| 010 | 0.9442(9) | $0.3821(9)$ | $0.6617(3)$ | 0.037(2) | 0.032(4) | 0.021(4) | 0.042(5) | -0.013(4) | 0.016(4) | 0.002(3) |
| 011 | 0.1581(11) | 0.5831(12) | $3 / 4$ | 0.048(4) | 0.005(4) | 0.015(5) | 0.119(14) | 0 | 0 | 0.001(4) |
| O12** | 1/3 | 2/3 | 0.6943(8) | 0.027(5) |  |  |  |  |  |  |

$369 *$ SOF $=0.333$. **SOF $=0.5$.

Table 3. Selected interatomic distances in the structure of $\mathrm{Pb}_{21}\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]_{2}\left[\mathrm{Si}_{4} \mathrm{O}_{13}\right]$

| $\mathrm{Pb} 1-\mathrm{O} 2(\times 3)$ | $2.334(6)$ | $\mathrm{Pb} 6 \mathrm{~A}-\mathrm{Pb} 6 \mathrm{~A}$ | $0.610(10)$ |
| :--- | :--- | :--- | :--- |
| $\mathrm{Pb} 1-\mathrm{O} 4(\times 3)$ | $2.804(7)$ | $\mathrm{Pb} 6 \mathrm{~A}-\mathrm{O} 6$ | $2.413(6)$ |
| $\mathrm{Pb} 1-\mathrm{O} 10(\times 3)$ | $3.420(10)$ | $\mathrm{Pb} 6 \mathrm{~A}-\mathrm{O} 7$ | $2.498(8)$ |
|  |  | $\mathrm{Pb} 6 \mathrm{~A}-\mathrm{O} 1$ | $2.518(9)$ |
| $\mathrm{Pb} 2-\mathrm{O} 2(\times 3)$ | $2.551(6)$ | $\mathrm{Pb} 6 \mathrm{~A}-\mathrm{O} 6$ | $2.705(8)$ |
| $\mathrm{Pb} 2-\mathrm{O}(\times 3)$ | $2.775(8)$ | $\mathrm{Pb} 6 \mathrm{~A}-\mathrm{O} 7$ | $2.797(8)$ |
| $\mathrm{Pb} 2-\mathrm{O} 3(\times 3)$ | $2.832(7)$ | $\mathrm{Pb} 6 \mathrm{~A}-\mathrm{O} 8$ | $3.142(10)$ |
|  |  | $\mathrm{Pb} 6 \mathrm{~A}-\mathrm{O} 10$ | $3.304(11)$ |
| $\mathrm{Pb} 3-\mathrm{O} 9$ | $2.175(7)$ | $\mathrm{Pb} 6 \mathrm{~A}-\mathrm{O} 11$ | $3.320(11)$ |
| $\mathrm{Pb} 3-\mathrm{O} 3$ | $2.359(7)$ |  |  |
| $\mathrm{Pb} 3-\mathrm{O} 3$ | $2.589(7)$ | $\mathrm{Si1-O6}$ | $1.584(14)$ |
| $\mathrm{Pb} 3-\mathrm{O} 2$ | $2.621(6)$ | $\mathrm{Si1-O8( } \mathrm{\times 3)}$ | $1.613(7)$ |
| $\mathrm{Pb} 3-\mathrm{O} 4$ | $2.814(8)$ |  |  |
| $\mathrm{Pb} 3-\mathrm{O} 5$ | $3.193(7)$ | $\mathrm{Si2-O7( } \mathrm{\times 2)}$ | $1.617(7)$ |
| $\mathrm{Pb} 3-\mathrm{O} 5$ | $3.445(7)$ | $\mathrm{Si} 2-\mathrm{O} 11$ | $1.637(11)$ |
| $\mathrm{Pb} 3-\mathrm{O} 9$ | $3.496(7)$ | $\mathrm{Si} 2-\mathrm{O} 1$ | $1.639(9)$ |


| Pb4-O10 | $2.267(8)$ | $\mathrm{Si} 3-\mathrm{O} 9$ | $1.613(7)$ |
| :--- | :--- | :--- | :--- |
| $\mathrm{Pb} 4-\mathrm{O} 4$ | $2.279(6)$ | $\mathrm{Si} 3-\mathrm{O} 3$ | $1.614(7)$ |
| $\mathrm{Pb} 4-\mathrm{O} 7$ | $2.341(7)$ | $\mathrm{Si} 3-\mathrm{O} 2$ | $1.618(6)$ |
| $\mathrm{Pb} 4-\mathrm{O} 12$ | $2.780(14)$ | $\mathrm{Si} 3-\mathrm{O} 5$ | $1.660(7)$ |
| $\mathrm{Pb} 4-\mathrm{O} 8$ | $3.043(7)$ |  |  |
| $\mathrm{Pb} 4-\mathrm{O9}$ | $3.365(9)$ | $\mathrm{Si4-O10}$ | $1.606(8)$ |
| $\mathrm{Pb} 4-\mathrm{O} 5$ | $3.443(7)$ | $\mathrm{Si4-O4}$ | $1.606(7)$ |
| $\mathrm{Pb} 4-\mathrm{O} 11$ | $3.508(4)$ | $\mathrm{Si4-O8}$ | $1.627(8)$ |
|  |  | $\mathrm{Si4-O5}$ | $1.634(7)$ |
| $\mathrm{Pb5-O1( } \mathrm{\times 3)}$ | $2.442(6)$ |  |  |
| $\mathrm{Pb} 5-\mathrm{O} 10(\times 3)$ | $2.836(8)$ | $\mathrm{Si5-Si5}$ | $0.697(15)$ |
| $\mathrm{Pb} 5-\mathrm{O}(\times 3)$ | $3.111(8)$ | $\mathrm{Si5-O11( } \mathrm{\times 3)}$ | $1.547(10)$ |
|  |  | $\mathrm{Si5-O12}$ | $1.56(3)$ |
| $\mathrm{Pb} 6-\mathrm{Pb} 6 \mathrm{~A}$ | $0.353(4)$ | $\mathrm{Si5-O12}$ | $2.25(3)$ |
| $\mathrm{Pb} 6-\mathrm{O}(\times 2)$ | $2.469(7)$ |  |  |
| $\mathrm{Pb} 6-\mathrm{O}(\times 2)$ | $2.509(8)$ |  |  |
| $\mathrm{Pb} 6-\mathrm{O} 1$ | $2.569(9)$ |  |  |
| $\mathrm{Pb} 6-\mathrm{O}(\times 2)$ | $3.439(9)$ |  |  |
| $\mathrm{Pb} 6-\mathrm{O} 11$ | $3.476(11)$ |  |  |

Table 4. Bond-valence analysis (v.u.) for the crystal structure of $\mathrm{Pb}_{21}\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]_{2}\left[\mathrm{Si}_{4} \mathrm{O}_{13}\right]$

| Atom | O1 | O 2 | O3 | O4 | O5 | 06 | 07 | O8 | 09 | 010 | 011 | O12* | Sum |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Pb1 |  | $0.47{ }^{3 x \rightarrow}$ |  | $0.18{ }^{3 x \rightarrow}$ |  |  |  |  |  | $0.05{ }^{3 x \rightarrow}$ |  |  | 2.10 |
| Pb2 |  | $0.30{ }^{3 x \rightarrow}$ | $0.17{ }^{3 x \rightarrow}$ |  |  |  |  |  | $0.19^{3 x \rightarrow}$ |  |  |  | 1.98 |
| Pb3 |  | 0.26 | $0.45+0.28$ | 0.18 | $0.08+0.05$ |  |  |  | $0.65+0.04$ |  |  |  | 1.99 |
| Pb4 |  |  |  | 0.53 | 0.05 |  | 0.46 | 0.11 | 0.06 | 0.54 | $0.04{ }^{2 \times \downarrow}$ | $0.19^{3 \times \downarrow}$ | 1.98 |
| Pb5 | $0.38{ }^{3 x \rightarrow 2 \times \downarrow}$ |  |  |  |  |  | $0.10^{3 x \rightarrow}$ |  |  | $0.17{ }^{3 x \rightarrow}$ |  |  | 1.95 |
| Pb6** | 0.29 |  |  |  |  | $0.36{ }^{2 \times \rightarrow 3 \times \downarrow}$ | $0.33{ }^{2 x \rightarrow}$ | $0.05^{2 x \rightarrow}$ |  |  | 0.05 |  | 1.77 |
| Pb6A** | 0.32 |  |  |  |  | $0.40+0.22^{3 \times \downarrow}$ | $0.34+0.18$ | 0.09 |  | 0.07 | 0.06 |  | 1.68 |
| Si1 |  |  |  |  |  | 1.11 |  | $1.03{ }^{3 x \rightarrow}$ |  |  |  |  | 4.20 |
| Si2 | 0.96 |  |  |  |  |  | $1.02^{2 \times \rightarrow}$ |  |  |  | 0.97 |  | 3.97 |
| Si3 |  | 1.02 | 1.03 |  | 0.91 |  |  |  | 1.03 |  |  |  | 3.99 |
| Si4 |  |  |  | 1.05 | 0.97 |  |  | 0.99 |  | 1.05 |  |  | 4.06 |
| Si5* |  |  |  |  |  |  |  |  |  |  | $1.23{ }^{3 x \rightarrow}$ | 1.19 | 4.88 |
| Sum | 2.03 | 2.05 | 1.93 | 1.94 | 2.06 | 2.13 | 2.17 | 2.23 | 1.97 | 1.88 | 2.39 | 1.76 |  |

Table 5. Crystallographic data and complexity parameters for Pb silicates

| $\mathrm{Pb}: \mathrm{Si}$ | Chemical formula | Sp.gr. | $a[\AA] / \alpha[\mathrm{deg}]$ | $b[\AA] / \beta$ [deg] | $c[\AA] / \gamma[\mathrm{deg}]$ | [atoms] | $I_{G}$ [bits/atom] | $I_{G, \text { total }}$ [bits/u.c.] | $I_{\text {chem }}$ <br> [bits/f.u.] | Ref. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.000 | $\mathrm{Pb}\left[\mathrm{SiO}_{3}\right]$ | P2/n | 11.209 / 90 | 7.0410 / 113.0 | 12.220 / 90 | 60 | 3.974 | 238.413 | 6.855 | 1 |
| 1.167 | $\mathrm{Pb}_{21}\left[\mathrm{Si}_{7} \mathrm{O}_{22}\right]_{2}\left[\mathrm{Si}_{4} \mathrm{O}_{13}\right]$ | $\mathrm{Pb}_{3} / \mathrm{m}$ | 9.924 / 90 | 9.924 / 90 | 34.236 / 120 | 194 | 4.361 | 846.109 | 44.128 | 2 |
| 1.500 | $\mathrm{Pb}_{3}\left[\mathrm{Si}_{2} \mathrm{O}_{7}\right]$ | $R-3 c$ | 10.126 / 90 | 10.1264 / 90 | 38.678 / 120 | 72 | 2.828 | 203.627 | 16.613 | 3 |
| 2.000 | $\mathrm{Pb}_{2} \mathrm{O}\left[\mathrm{SiO}_{3}\right]$ | A121 | 19.43 / 90 | 7.64 / 99.33 | 12.24 / 90 | 56 | 4.879 | 273.212 | 9.651 | 4 |
| 3.667 | $\left(\mathrm{Pb}_{2} \mathrm{O}\right)_{2}\left(\mathrm{~Pb}_{7} \mathrm{O}_{3}\right) \mathrm{O}\left[\mathrm{SiO}_{4}\right]\left[\mathrm{Si}_{2} \mathrm{O}_{7}\right]$ | $P-1$ | 22.502 / 92.5 | 12.982 / 99.2 | 7.313 / 100.3 | 124 | 5.954 | 738.320 | 41.285 | 5 |

377 References: (1) Krivovichev and Burns 2004; (2) this work; (3) Petter et al. 1971; (4) Dent Glasser et al. 1981; (5) Kato 1982. 378





b


