1	REVISION 1
2	
3	Relaxation effects and re-entrant spin glass behaviour at low temperatures
4	in natural strunzite, ferristrunzite and ferrostrunzite
5	Antoine Van Alboom ^{1,2,*} and Eddy De Grave ²
6	¹ Physics, Faculty of Applied Engineering Sciences, University College Gent, Valentin
7	Vaerwyckweg 1, 9000 Gent, Belgium
8	² NUMAT, Department of Subatomic and Radiation Physics, University of Gent,
9	Proeftuinstraat 86, 9000 Gent, Belgium
10	* Present adress : Physics, Faculty of Applied Engineering Sciences, University College Gent,
11	Valentin Vaerwyckweg 1, 9000 Gent, Belgium. E-mail: toon.vanalboom@hogent.be
12	Keywords: strunzite, ferristunzite, ferrostrunzite, Mössbauer spectroscopy, hyperfine field
13	Abstract
14	The temperature variations of the ferric and ferrous hyperfine fields in natural samples of
15	strunzite, ferristrunzite and ferrostrunzite are examined by Mössbauer spectroscopy between
16	4.2 K and their respective magnetic transition temperatures. The spectra of all three strunzite
17	species are composed of two dominant sextets with equal contribution and with hyperfine
18	fields at 4.2 K of approximately 48.5 T and 48.0 T, respectively. They are assigned to Fe^{3+}
19	cations in the Fe(1) and Fe(2) sites in the crystallographic structure. A third magnetic
20	component, with field of 54.8 T \pm 0.1 T, is well resolved in the 4.2 K spectrum of
21	ferristrunzite and is attributed to Fe^{3+} in the Mn sites of the structure. This third ferric
22	contribution also appears in the spectra of strunzite and ferrostrunzite, however with smaller
23	relative spectral areas as compared to ferristrunzite, i.e., ~10% and ~3% of total spectral area,
24	respectively, where it is 30% for ferristrunzite. For ferrostrunzite, two additional magnetic
25	components are recognized. They are both attributed to Fe ²⁺ substituting in the Mn sites.

26 Their hyperfine fields at 4.2 K are 21.8 T \pm 0.5 T and 28.6 T \pm 0.5 T, respectively, and their 27 relative contributions 11% and 15%. With increasing temperature, enhanced asymmetric 28 broadening of the absorption lines is observed for all three samples. This phenomenon is due 29 to relaxation of the magnetic moments, thus causing gradually broader distribution ranges of 30 the respective hyperfine fields. The spectra have been fitted with superpositions of model-31 independent hyperfine-field distributions. The temperature dependencies of the maximum-32 probability hyperfine fields of the two ferric subspectra associated with the Fe(1) and Fe(2)33 sites are interpreted in terms of the molecular field theory taking into account the occurrence 34 of exchange magnetostriction. The variations with temperature of the magnetic hyperfine 35 fields acting at the Mn sites indicate that the magnetization of the Mn sublattice shows re-36 entrant spin glass behaviour. This feature is most prominent for the strunzite sample.

37 **1. Introduction**

38 Strunzite, ferristrunzite and ferrostrunzite are three isostructural minerals with triclinic 39 crystallographic structure (P1) (Frondel 1957, Fanfani et al. 1978, Peacor et al. 1983). The $Mn^{2+}Fe^{3+}_{2}(PO_{4})_{2}(OH)_{2}\cdot 6(H_{2}O),$ 40 ideal chemical formulas are $Fe^{3+}Fe^{3+}_{2}(PO_{4})_{2}(OH)_{2}\cdot[(H_{2}O)_{5}(OH)]$ 41 (Vochten and De Grave 1990) and 42 Fe²⁺Fe³⁺₂(PO₄)₂(OH)₂·6(H₂O) (Vochten et al. 1995), respectively. Ferristrunzite may be 43 regarded as the fully oxidized form of ferrostrunzite.

The strunzite structure (Figure 1) consists of infinite chains of octahedral ferric sites along the c axis which are linked to one another by sharing hydroxyl groups and by PO₄ tetrahedra. The latter also bind adjacent chains, thus forming slabs that are connected to each other by Mn octahedra between remaining PO₄ vertices. Within the chains two ferric sites Fe(1) and Fe(2) alternate, of which Fe(1) is somewhat more distorted than Fe(2) as indicated by a slight difference in average Fe-O bond length and average O-Fe-O bond angle (Vochten and De Grave 1990). The crystallographic unit cell contains two octahedra of each type. 51 In ferristrunzite the manganese is substituted by Fe^{3+} , in ferrostrunzite by Fe^{2+} . In the first 52 case the charge balance is re-established by substitution of an H₂O by an OH⁻ at a non-53 bridging vertex of the Mn octahedron. In the following, this position is referred to as the Mn 54 site.

55 Mössbauer spectra (MS) at room temperature (RT) and at 4.2 K for these three mineral 56 species have been reported earlier (Vochten and De Grave 1990, Vochten et al. 1995, Van 57 Tassel and De Grave 1992). In this paper the detailed temperature dependencies of the 58 Mössbauer parameters between 4.2 K and their respective magnetic transition temperatures 59 (T_N) are presented and discussed, with emphasis on the magnetic hyperfine fields, in order to 60 gain an insight into the relation between the crystallographic structure and the magnetic 61 properties of these minerals.

62 Preliminary numerical results were reported earlier (Van Alboom and De Grave 2005).

However, the data at that time could not be explained satisfactorily and no sound conclusionswere drawn.

65 **2. Experimental**

The samples used in this study were described earlier (Vochten and De Grave 1990,
Vochten et al. 1995, Van Tassel and De Grave 1992) and were characterised by these authors
using X-ray diffraction, chemical analysis and Mössbauer spectroscopy at room temperature
(RT) and 4.2 K.

In the present work, Mössbauer spectra for powder samples were collected at variable temperatures in the range where the samples exhibit magnetic ordering. Prior to these experiments, the respective magnetic order-disorder transition temperatures (T_N) were determined by the Mössbauer thermoscanning technique, in which the temperature (T) variation of the γ -ray transmission through the absorber is recorded with the source at zero velocity (Chambaere and De Grave 1984). For the acquisition of MS conventional time-mode spectrometers were used operating with a triangular reference signal. The sources were 57 Co in Rh matrix, but all center shift (δ) values quoted hereafter are relative to α -Fe at room temperature.

79 The obtained spectra were numerically analyzed assuming either discrete Lorentzianshaped magnetic components (fitting program referred to hereafter as SFP) or, where found to 80 81 be recommended, by model-independent distributions (Vandenberghe et al. 1994) of such 82 components (program DFP). The positions and intensities of the various absorption lines were 83 calculated by diagonalization of the full nuclear interaction hamiltonian (Hoy and Chandra 1967, Kündig 1967). Most generally, a magnetically split ⁵⁷Fe-MS contains a maximum of 84 85 eight absorption lines of which the respective positions are determined by the so-called Mössbauer parameters, particularly: the center shift δ , the hyperfine field B_{hf} and the polar 86 87 (Ω) and the azimuthal angle (Ψ) of the hyperfine field in the principal axis system (x, y, z) of 88 the electric field gradient (EFG) with components (V_{xx} , V_{yy} , V_{zz}). Finally, two additional 89 parameters, related to the quadrupole interaction, affect the line positions, i.e., the quadrupole 90 splitting ΔE_0 and the asymmetry parameter η . They are given by, respectively,

91
$$\Delta E_{Q} = \frac{1}{2}e^{2}qQ = \frac{1}{2}V_{zz}eQ, \qquad (1)$$

92 and

93
$$\eta = \frac{V_{xx} - V_{yy}}{V_{zz}},$$
 (2)

94 with V_{zz} the principal EFG component and Q the nuclear quadrupole moment.

However, in the case of Fe³⁺ the quadrupole interaction is generally weak in comparison with the magnetic dipole interaction. As a result, the peak positions are not independently determined by ΔE_Q and Ω , but instead by the quadrupole shift ($2\epsilon_Q$), which is given by

1/16

99
$$2\varepsilon_{q} = \frac{1}{2}\Delta E_{q} (3\cos^{2}\Omega - 1). \qquad (3)$$

Further in this case, the number of lines is reduced to six (sextet) with ideally a mutual line-intensity ratio of 3:2:1:1:2:3 for a random-powder absorber.

In addition to the above mentioned parameters the line width (Γ : full width at half maximum height) and the relative area (RA) for each spectral component appearing in a multiphase spectrum are determined by fitting the experimental MS.

105 Because of the diversity in types of Mössbauer spectra, four different fitting programs were 106 used in this study. SFP1 and SFP2 (single component fit programs) were aimed to analyse MS 107 consisting of a superposition of a number of well-defined discrete magnetically split 108 subspectra, which are composed of only one subcomponent. Distribution fit programs (DFP1 109 and DFP2) were applied if subspectra themselves were the superposition of a number of 110 magnetically split subcomponents related to a distribution of hyperfine fields. In these cases, 111 only model independent distributions of hyperfine fields (Vandenberghe et al. 1994) were 112 considered. In SFP1 and DFP1, the positions of the absorption lines (either discrete sextets or 113 distributions of sextets, respectively) are calculated from their analytical expressions as 114 function of the various Mössbauer parameters. This approach applies to Fe sites for which the 115 quadrupole interaction is weak compared to the magnetic hyperfine interaction. If the latter 116 condition is not met, the programs SFP2 and DFP2 are used which are both based on the 117 diagonalization of the full nuclear interaction hamiltonian.

- 118 **3. Mössbauer analysis**
- 119 3.1 Thermoscanning

The Mössbauer thermoscanning curves for the three strunzite varieties are reproduced in Figure 2. Expanding on previously reported data (Van Alboom and De Grave 2005), the magnetic order-disorder transition temperatures were accurately and consistently determined, yielding the values 50.5 ± 0.5 K, 44 ± 1 K and 45 ± 3 K for respectively strunzite, ferristrunzite and ferrostrunzite. Hence, the Fe-for-Mn substitution in the strunzite structure seems to weaken the strength of the overall magnetic exchange interaction. The T_N -value as determined from thermoscanning gives an overall value for a sample, and therefore does not correspond necessarily to the values of all the magnetic sublattices of the sample.

128 3.2 Ferristrunzite

At 4.2 K the experimental MS of ferristrunzite was satisfactorily described using SFP1 assuming a superposition of three ferric Lorentzian shaped sextets, S1, S2, and S3 (Vochten and De Grave 1990), attributed to the three distinct Fe sites available in the ferristrunzite structure. The respective center shift values are very similar ($\delta \approx 0.50$ mm/s) and are consistent with an octahedral co-ordination.

Two of these sextets (S1 and S2) have slightly different hyperfine fields ($B_{hf} = 48.4 \text{ T}$ 134 \pm 0.1 T and B_{hf} = 47.8 T \pm 0.1 T) while the third one (S3) has a remarkably higher hyperfine 135 field ($B_{hf} = 54.8 \text{ T} \pm 0.1 \text{ T}$) at 4.2 K. S1 and S2 are assigned to Fe³⁺ at the Fe(1 or 2) and Fe(2 136 or 1) sites in the structure. Although the quadrupole shifts for $Fe^{3+}(S1)$ and $Fe^{3+}(S2)$ are 137 138 different, further unambiguous assignment to the Fe(1) or Fe(2) sites isn't possible, because 139 the difference in $2\epsilon_0$ is related to the difference in orientation of the local hyperfine field with 140 respect to the local EFG in both sites, of which the orientation isn't known until now. S3 is attributed to Fe³⁺ at the Mn position. The center shifts corresponding to S1 and S2 were 141 142 constrained to have equal values. The subspectral area fractions were fixed on the ratio 143 0.35:0.35:0.30 which corresponds to a model for which the aluminum, which was detected by 144 EDX to be present in the sample by an amount of 0.10 per formula unit, is located on the Mn 145 sites in the structure. It was experienced, however, that a forced random distribution of the Al 146 among the three available lattice sites or among sites Fe(1) and Fe(2) did not significantly 147 affect the adjusted hyperfine parameter values. The line area ratios 3:x:y of the outer to 148 middle to inner lines for a particular subspectrum were assumed to be the same for all three 149 components and were found to be 3:2.43:1.14. The deviation from the ideal 3:2:1 ratio can be 150 due to thickness effects or texture effects, however here, also relaxation effects cannot be 151 excluded (see further). As it is experienced that the deviation is irrelevant for the subsequent 152 determination and interpretation of the temperature variations of the various hyperfine 153 parameters, the hyperfine field in particular, no further attention is paid to the line area ratios.

154 At 8 K and higher temperatures T, significant line broadening is noticed, as well as 155 additional absorption at low velocities which becomes more prominent with increasing T. It 156 was found that its contribution could be taken into account by introducing an extra doublet 157 component D1 in the foregoing analyzing procedure, hereby keeping the same constraints for 158 the sextets as those imposed on the fit of the 4.2 K spectrum. The adjusted values of the D1 159 center shift values are, within experimental error limits, equal to those of the sextet 160 components. Further, the MS acquired at 8 K and 14 K show strongly overlapping sextet 161 subspectra S1, S2, and S3 and consequently their Mössbauer parameters could not be 162 determined accurately using the SFP1 fitting approach. Nevertheless, a remarkably different 163 T-dependence of the hyperfine fields among these subspectra is observed. While $B_{hf,S1}$ and 164 $B_{\rm hf,S2}$ show nearly the same T-dependence, $B_{\rm hf,S3}$ decreases more rapidly with increasing T 165 than $B_{hf,S1}$ and $B_{hf,S2}$ and above 14 K the field $B_{hf,S3}$ becomes smaller than the corresponding S1 and S2 values. 166

From 17 K on, the asymmetric broadening of the absorption lines is considerable and increases drastically with increasing *T*. As a result, the analysis of these MS using a superposition of three discrete sextets and one doublet (SFP1 approach) failed. Therefore, the theoretical spectra were calculated by the superposition of three model independent distributions of hyperfine fields, respectively, related to the already mentioned Fe sites, using the distribution fitting program DFP1. In this case, a subspectrum is generally the superposition of a certain number of subcomponents (given by #div), of which the hyperfine 174 fields have different values as given by the distribution, whereas the center shifts, the 175 quadrupole shifts, the line widths and the corresponding line area ratios are constraint to equal 176 values for all subcomponents. Here the line area ratios of all these subcomponents were fixed 177 at 3:2:1. The central component D1 was fitted as a discrete, Lorentzian-shaped quadrupole 178 doublet. At 17 K and higher T, the center shifts of all subspectra were constrained to have 179 equal values. Regrettably, in the minimization routine used in the distribution program for 180 fitting the MS, it is not possible to impose fixed values for the RA of the subspectra. Such 181 possibility would indeed be very useful when strongly overlapping subspectra determine the 182 overall Mössbauer line shape. As a consequence and also because of the nearly total overlap 183 of S1 and S2, the RA results for S1 and S2 fluctuated strongly depending on the input 184 parameter values. Therefore, at each T, several trial fits were done with different distribution 185 ranges (lower and upper limits for the allowed hyperfine fields) and/or different fixed $2\epsilon_0$ 186 values for S1 and S2 in order to get nearly equal RA values for S1 and S2. It was found that 187 fixing or not fixing the elemental line width Γ , only slightly affected the adjusted RA values, 188 while the positions of the respective maximum probability hyperfine fields did not change 189 substantially.

190 Applying the calculation approaches as outlined above, the MS at temperatures 191 between 4.2 K and T_N could satisfactorily be reproduced. Figure 3 shows selected 192 experimental and calculated spectra as well as the probability distribution profiles $p(B_{hf})$ of 193 the three sextet components. It is noticed that the $p(B_{hf})$ profiles for S1 and S2 are very close 194 to one another, both in shape and in probability for a given field value. In spite of the strong 195 overlap of the sextet components at higher T, and hence their ill-defined parameter values, 196 reasonably consistent results for these parameters were obtained, especially for the hyperfine 197 fields B_{max} at maximum probability.

Table I gives the results of the relevant Mössbauer parameters for which the *RA* of the S1 and S2 components are in good approximation the same and for which the Mössbauer parameters of all the subspectra show a consistent and physical acceptable dependence of temperature. The quadrupole shifts of the various sextets are not sharply defined, but it is clear that their values for S1 and S2 have opposite signs. Figure 3 contains the experimental and calculated MS. At 17 K and higher *T*, the *RA* of S1 and S2 decrease substantially.

The quadrupole splitting of component D1 was determined for all *T* to be approximately 0.75mm/s, which is in good agreement with the value of the quadrupole splitting of the doublet absorption at maximum probability above T_N (Vochten and De Grave 1990). $2\varepsilon_{Q,S1}$ and $2\varepsilon_{Q,S2}$ had always a different sign, and vary between -0.63 and -0.42 mm/s for S1 and between 0.11 and 0.32 mm/s for S2. Most probably these fluctuations are a consequence of the overlap of S1 and S2, by which it was not possible to determine these values more precisely. A change in sign of $2\varepsilon_{Q,S3}$ below 8 K and above 14 K is determined.

Because the sign of $2\varepsilon_Q$ is related to the orientation of the local hyperfine field to the principal axis of the EFG, the difference in sign of $2\varepsilon_{Q,S1}$ and $2\varepsilon_{Q,S2}$ indicates a difference in that orientation at Fe(1) and Fe(2) sites. For the same reason the difference in sign of $2\varepsilon_{Q,S3}$ below 8 K and above 14 K should be interpreted as a change in the direction of the local hyperfine field with respect to the principal axis of the local EFG. However, taking into account the constraints on some of the parameters during the analysis, one best takes any reservation concerning this finding.

The effect of increasing RA_{DI} and decreasing RA of the other components with increasing Tabove 17 K, is interpreted in the context of relaxation of the magnetic moments, by which the magnetic moments remain increasingly less time in their equilibrium positions as T increases. When the relaxation time is sufficiently shorter than the nuclear Larmor precession time (of the order 10⁻⁷ s), then the Mössbauer absorption is of doublet type. In the opposite case, when the magnetic moments stay long enough in their equilibrium state, one observes a magnetically split spectrum. In the intermediate case, the absorption manifests itself as the superposition of a continuum of hyperfine fields between 0 T (doublet situation) and a maximum value.

From these considerations, the D1 component has to be associated with Fe³⁺ for which the 227 relaxation time is already short enough to get a more doublet-like absorption. That the 228 229 relaxation phenomenon is spread over a relatively wide temperature range, means that the 230 magnetic exchange interaction with neighbouring magnetic ions, which is responsible for the 231 orientation of the magnetic moments, is not equally strong at each site, and being weakest for 232 Fe³⁺ at Mn sites. This last assertion can be easily understood because the Mn site in strunzite 233 is a relatively isolated octahedrally co-ordinated site between the slabs of chains of octahedral ferric sites in the crystallographic structure. Therefore it is plausible that the Fe^{3+} at the Mn 234 235 position has weaker magnetic exchange interactions with neighbouring magnetic ions than $Fe^{3+}(1)$ and $Fe^{3+}(2)$ do. 236

237 The broadening of the absorption lines is interpreted as being the consequence of local 238 differences in the strength of the magnetic exchange interactions between neighbouring 239 magnetic ions caused by local differences in the co-ordination of the probe ions. Consequently 240 the T-dependence of the hyperfine field at a certain position can be different from the one at 241 another position of the same site, whereas the saturation values at lowest T are approximately 242 the same. In the former case, a broad B_{hf} distribution range arises and an asymmetric 243 broadening of the outer absorption lines of the spectral component is observed when T244 increases. In order to analyse spectra showing relaxation effects, a special fitting routine is 245 designed in the past (Blume and Tjon 1968), which is successfully applied to Mössbauer 246 spectra with single component subspectra. However because of the strong overlap of the

subspectra, which themselves show a distribution of hyperfine fields, this fitting routine is notapplicable for the minerals in this study.

249 3.3 Strunzite

250 Using SFP1 the MS of strunzite at 4.2 K was calculated by three sextets and an additional 251 doublet component D2 at small velocities with RA less than 3 %. This doublet was assigned to 252 some iron in the window of the equipment. The center shifts and line widths of S1 and S2 253 were constrained to equal values. Broadening of the outer lines of these components was 254 taken into account by a linear broadening parameter ($\Delta\Gamma$): the inner line to middle to outer 255 lines having line widths Γ , $\Gamma + \Delta \Gamma$ and $\Gamma + 2\Delta \Gamma$. Additionally a coupled 3:x:1 ratio for the line 256 area of the sextet absorption lines was used. In this way the MS was satisfactorilly reproduced 257 and the results for the Mössbauer parameters of the sextets at 4.2 K (see Table II) were very 258 similar to those at 4.2 K in ferristrunzite. In comparison with ferristrunzite and the 259 components S1 and S2, the component with the highest B_{hf} (S3) had now a substantially 260 smaller area fraction of 0.11 approximately, S1 and S2 having equal area fractions of 0.44 261 approximately. However the sign of $2\varepsilon_{Q,S3}$ at 4.2 K in strunzite was opposite to the 262 corresponding one in ferristrunzite.

The MS at 10 K was also satisfactorilly reproduced by an analogous superposition of discrete components for which the *RA* of the sextets were fixed at 0.44:0.44:0.11 and the Mössbauer parameters of D2 at the 4.2 K values (except the line width). In this case S3 overlapped totally with S1 and S2, and its parameters therefore are somewhat more inaccurate.

At increasing temperatures, $B_{hf,S3}$ decreased very rapidly and already at 15 K, its value was much smaller than $B_{hf,S1}$ and $B_{hf,S2}$. Meanwhile, using SFP1, the line width of S3 increased strongly with increasing *T*, most probably because of relaxation effects. Therefore, from 15 K and higher *T*, the MS were analysed using DFP1 with the distribution fitting model for S3, whereas S1 and S2 were reproduced by discrete sextet components with a coupled linear 272 broadening of the absorption lines. All sextet components were constrained to equal 3:x:1 line 273 area ratios and to equal center shifts. However DFP1 has the disadvantage that the relative 274 areas of the components could not be fixed. Up to 20 K the parameters of D2 were fixed to 275 the values found at 4.2 K. Above 20 K up to 47 K the lower limit of the distribution range for 276 $B_{hf,S3}$ was lowered to 0 and consequently overlapped the absorption corresponding to D2. A 277 separate D2 doublet therefore was not taken into consideration for these T. At 43 and 47 K the 278 MS showed also asymmetric broadening for the S1 and S2 absorption. Therefore these MS 279 were reproduced using DFP1 with distribution models for the hyperfine fields related to the 280 S1, S2 and S3 absorption. At 47 K additional doublet absorption D1 was resolved. As for 281 ferristrunzite, at each T, several fits were done with different distribution ranges and/or 282 different fixed $2\epsilon_Q$ values for S1 and S2 in order to get nearly equal RA for S1 and S2. Again 283 it was clear that the dominating sextet absorption was the superposition of two components 284 with very similar hyperfine field distributions and with a different sign for the related 285 quadrupole shifts, indicating that the hyperfine field orientation is different with respect to the 286 local principal axis of the EFG at the corresponding sites. At 43 K and at 47 K the subspectra 287 in the MS are clearly collapsing and at these T, the highest value of the hyperfine field 288 distribution of S3 was only 12.0 T and 10.0 T, respectively. The results for which the RA of 289 the S1 and S2 components are in good approximation the same and for which the Mössbauer 290 parameters of all the subspectra show a consistent and physical acceptable dependence of 291 temperature, are given in Table II. Figure 4 contains the experimental and calculated MS.

292 3.4 Ferrostrunzite

In the MS of ferrostrunzite below $T_N = 45 \pm 3$ K, because of the presence of Fe²⁺ in the sample, magnetically split ferrous subspectra were observed in addition to the Fe³⁺ subspectra. Because for Fe²⁺, the quadrupole interaction and the magnetic dipole interaction are of the same order, the MS of ferrostrunzite below T_N were analysed with the method based 297 on the diagonalization of the full nuclear interaction hamiltonian. At 5 K the MS could be 298 reproduced by three discrete ferric (sextet) and two ferrous magnetically split subspectra 299 using SFP2. A position dependent width broadening parameter ($\Delta\Gamma$) accounts for some 300 broadening of the outher lines caused by small local variations of B_{hf} . As for the other samples, two of these ferric subspectra (S1 and S2) are related to Fe^{3+} at the Fe(1 or 2) and 301 302 Fe(2 or 1) sites and the third ferric component (S3) is assigned to ferric ions at the Mn site. 303 Keeping the area fractions of S1 and S2 equal to each other during the fitting process gives 304 no worse goodness-of-fit values, thus makes an equal ferric occupation of both sites very 305 plausible. Because the ferrous quadrupole splitting is temperature dependent as consequence 306 of the varying Boltzmann occupation of the lower ⁵D orbital electron energy levels, its value 307 saturates at low temperatures; this occurs already at about 80 K. From this consideration, 308 quadrupole splittings of the two ferrous magnetically split subspectra (S4 and S5) were fixed 309 at the values just above the magnetic transition temperature, 3.04 and 2.52 mm/s respectively 310 (Van Tassel and De Grave 1992). The area fractions of both ferrous subspectra were fixed at 311 the ratio determined just above the magnetic transition temperature. This was also done for 312 the center shifts of both components.

313 At 22 K and 28 K an asymmetric broadening of the outer absorption peaks was visually 314 observed, indicating an inhomogeneous hyperfine field decrease at Fe(1) and Fe(2) sites. 315 Therefore, at these temperatures, the MS were analysed using ferric subspectra with 316 distributions of hyperfine fields. Also an increase of the absorption near zero velocity was 317 observed and S3 could not be resolved anymore. In order to reproduce the MS at these T, 318 therefore a magnetically split subspectrum (S6) was added, for which the hyperfine field 319 range extends from 0.0 to 10.0 T. The quadrupole shift of it was fixed at the value of 0.755 320 mm/s for the quadrupole splitting with highest probability in the paramagnetic spectrum just 321 above T_N (Van Tassel and De Grave 1992). Because of the strong overlap of the subspectra,

322 by which their resolution is strongly limited, the S6 absorption is composed of the collapsing 323 S3 on the one hand, and parts of the S1 and S2 absorption on the other hand. Center shifts and 324 widths of the ferric subspectra were constrained to equal values. Because of the strong 325 overlap with the other subspectra, the ferrous absorption was reproduced by single 326 magnetically split subspectra with large line widths constrained equal to each other. For the 327 calculations at these T, DFP2 was used. However relative areas could not be fixed, and 328 because of the strong overlap with the ferric subspectra, the ferrous ones were not well 329 resolvable at these T. Therefore, the parameters determined for them are less accurate. The 330 results of all these calculations for which the RA of the S1 and S2 components are in good 331 approximation the same and for which the Mössbauer parameters of all the subspectra show a 332 consistent and physical acceptable dependence of temperature, are given in Table III. Figure 5 333 contains the experimental and calculated MS. It is clear that the area fractions of S1 and S2 334 decrease above 22 K.

335 **4. Hyperfine field analysis**

4.1 Temperature dependence of the hyperfine field

As is generally known, the *T*-dependence of the sublattice magnetisation (σ) in a classical magnetic structure can be described by a Brillouin curve within the molecular field theory of magnetism (Morrish 1965):

340
$$\sigma = B_{5/2}(\chi) = \frac{6}{5} \operatorname{coth}\left(\frac{6}{5}\chi\right) - \frac{1}{5} \operatorname{coth}\left(\frac{1}{5}\chi\right) \quad \text{with} \quad \chi = \frac{15}{7} \frac{T_N}{T} \sigma$$

Because the hyperfine field B_{hf} is in very good approximation proportional to the sublattice magnetization, here related to an iron site, the formula can also be applied to describe the *T*dependence of B_{hf} :

344
$$\sigma = \frac{\mathbf{B}_{hf}(T)}{\mathbf{B}_{hf}(0K)} \iff \mathbf{B}_{hf}(T) = \mathbf{B}_{hf}(0K) \times \mathbf{B}_{\frac{5}{2}}(\chi)$$

These $B_{hf}(T)$ Brillouin curves are determined by the mean spin value of the probe-ions (2.5 for ferric ions), the saturation value of the magnetization at low temperatures (here: the hyperfine 347 field at lowest T) and the corresponding magnetic transition temperature (here: the T_N value as 348 determined from thermoscanning). Figure 6 shows the experimental T-dependences of B_{hf} 349 (most probable values) and the calculated Brillouin curves for the respective ferric sites and 350 samples. It is clear that the molecular field model gives mostly only a raw description of the 351 T-dependences of $B_{hf,S1}$ and $B_{hf,S2}$, and is totally deficient for $B_{hf,S3}(T)$. However for strunzite 352 and ferrostrunzite a better description of $B_{hf,S1}$ and $B_{hf,S2}$ as function of T was obtained by a 353 curve which takes into account the exchange magnetostriction associated with a non-second-354 order magnetic transition (Bean and Rodbell 1962). The argument of the Brillouin curve is 355 than given by:

356
$$\chi = B_{\frac{5}{2}}^{-1}(\sigma) = [-2a - 4b\eta_{nso}\sigma^2]\frac{T_N}{T}\sigma$$
$$a = -\frac{15}{14} \qquad b = -\frac{2331}{9604}$$

Such a curve is calculated using an extra parameter, here noted by η_{nso} in order to distinguish 357 358 it from the asymmetry parameter. η_{nso} is a measure for the departure from an ideal second-359 order magnetic transition and is determined by the fractional volume change at the transition, 360 the compressibility, and the slope of the variation of the transition temperature with the lattice 361 volume (Bean and Rodbell 1962). In the case of an ideal second-order magnetic transition η_{nso} 362 = 0 and the curve is then the classical Brillouin curve; $\eta_{nso} > 1$ corresponds to a first-order 363 transition. η_{nso} can be determined by fitting this kind of curve to the experimental data using 364 the same values for the other parameters as for the foregoing calculation of the corresponding 365 classical Brillouin curve. Because the T-dependences of $B_{hf,S1}$ and $B_{hf,S2}$ are mutually very 366 similar in strunzite, one finds $\eta_{nso} = 0.43 \pm 0.01$ for both S1 and S2 in the case of strunzite, and 367 $\eta_{nso,SI} = 0.37 \pm 0.02$ and $\eta_{nso,S2} = 0.34 \pm 0.02$ in the case of ferrostrunzite, indicating that the 368 magnetic transition at the corresponding T_N is not a pure second-order. For ferristrunzite the 369 best agreement with the experimental data was obtained for $\eta_{nso} = 0\pm 0.02$, yielding the 370 classical Brillouin curve. Because the fractional volume changes at the magnetic transition 371 temperatures for strunzite and ferrostrunzite are not known, more quantitative data about 372 compressibility and slope of the variation of the transition temperature with the lattice volume 373 cannot be derived from the obtained values for η_{nso} . In ferrostrunzite (Figure 3) it can be seen 374 that $T_N(S1,S2)$ is different from $T_N(S3,S6)$, demonstrating that different sublattices can have 375 different magnetic transition temperatures.

376 *4.2 Re-entrant spin glass behaviour*

377 The $B_{hf,S3}$ T-dependence is remarkable. In strunzite (Figure 6) $B_{hf,S3}$ (most probable values) 378 decreases strongly when T increases from 0 to about 20 K, after which $B_{hf,S3}$ decreases less 379 quickly when T further increases up to T_N . This kind of T-dependence is typical for re-entrant 380 spin glass behaviour (De Bakker et al. 1997). This phenomenon occurs in systems in which 381 some spins, here the ferric ones at the Mn sites, are frustrated in the sense that they are liable 382 to conflicting magnetic exchange interactions which never can be satisfied simultaneously for 383 all interactions (Stein 1989). The kink in the curve determines the so-called spin glass freezing 384 temperature (T_{sg}). In strunzite, T_{sg} is estimated at 21 ± 1 K. Above T_{sg} , the frustrated spins are 385 in a local paramagnetic-like state, and $B_{hf,S3}(T)$ then reflects the normal course of the spontaneous magnetisation (of non-frustrated Fe³⁺ spins at the Mn site), which almost reaches 386 387 its saturation value at T_{sg} . Below T_{sg} , the frustrated spins are frozen in and their contribution is 388 superimposed on the mean spin value of the non-frustrated spins. Because the hyperfine field 389 of the non-frustrated spins was approximately saturated at T_{sg} , $B_{hf,S3}(T)$ then mainly reflects 390 the T-dependence of the frustrated spins (Huck and Hesse 1989). A less pronounced 391 analogous tendency is determined for $B_{hf,S3}$ in ferristrunzite and ferrostrunzite.

392 More quantitative discussion about this phenomenon can only be done when more data 393 concerning the *T*-dependence of the ferric hyperfine fields are determined and the magnetic 394 structure is better clarified. Complete strunzite-ferristrunzite and strunzite-ferrostrunzite

- 395 solution series are interesting mineral series for further research on this subject.
- 396 **5. Conclusion**
- 397 Strunzite, ferristrunzite and ferrostrunzite exhibit Mössbauer spectra at temperatures below 398 the magnetic ordering temperature that are rather complicated to analyze. The B_{hf} of the iron 399 located at the Mn sites decreases more strongly with temperature than these of the Fe(1) and 400 Fe(2) sites. When temperature increases sufficiently, the absorption lines of the dominant 401 sextets become asymmetrical and broaden as a consequence of relaxation effects. In strunzite 402 this becomes visible above 40 K, in ferristrunzite and ferrostrunzite already above about 17 K and 23 K respectively. The *T*-dependence of the hyperfine fields of the $\text{Fe}^{3+}(1)$ and $\text{Fe}^{3+}(2)$ 403 404 sites in strunzite and ferrostrunzite is succesfully described by a model which takes exchange 405 magnetostriction effects into account and shows that the magnetic transition is not of pure 406 second order. However in ferristrunzite, a classical Brillouin dependence is more likely. The T-dependence of the hyperfine field of the Fe^{3+} at the Mn sites in strunzite is rather 407 408 extraordinary and shows re-entrant spin glass behaviour.
- 409 Acknowledgements
- 410 The authors thank the Fund for Scientific Research Flanders for the continuous financial
- 411 support.

412

413 **References**

- 414 Bean, C.P. and Rodbell, D.S. (1962) Magnetic disorder as a first-order phase transformation.
- 415 Physical Review, 126, 104-115.
- 416 Blume, M., Tjon, T.A. (1968) Mössbauer spectra in a fluctuating environment. Physical
- 417 Review, 165, 446-461.
- 418 Chambaere, D. and De Grave, E. (1984) On the Néel temperature of β FeOOH : structural
- 419 dependence and its applications. Journal of Magnetism and Magnetic Materials, 42, 263-268.
- 420 De Bakker, P.M.A., Vandenberghe, R.E., and De Grave, E. (1997) Magnetic phase diagram
- 421 for the spinel ferrite series MgFe_{2-x}Cr_xO₄ and Mg_{1-x}Zn_xFe_{1,5}Cr_{0,5}O₄ as determined by
- 422 Mössbauer Spectroscopy. Journal Physique IV 7 C1, 267.
- 423 Frondel, C. (1957) Strunzit, ein neues Eisen-Mangan-Phosphat. Neues Jahrbuch fur
- 424 Mineralogie-Monatshefte, 222-226.
- 425 Fanfani, L., Tomassini, M., Zanazzi, P., and Zanzari, A.R. (1978) The crystal structure of
- 426 strunzite, a contribution of the crystal chemistry of basic ferric manganous hydrated
- 427 phosphates. Tschermaks Mineralogische und Petrographische Mitteilungen, 25, 77-87.
- 428 Hoy, G.R. and Chandra, S. (1967) Effective Field Parameters in Iron Mössbauer
- 429 Spectroscopy. Journal of Chemical Physics, 47, 961-965.
- 430 Huck, B. and Hesse, J. (1989), Mössbauer effect spectroscopy on the re-entrant spin glass
- 431 FeNiMn. Journal of Magnetism and Magnetic Materials, 78, 247-254.
- 432 Kündig, W. (1967) Evaluation of Mössbauer spectra for ⁵⁷Fe. Nuclear Instruments and
- 433 Methods in Physics Research, 48, 219-228.
- 434 Morrish, A.H. (1965) The Physical Principles of Magnetism, 680 p. Wiley, New-York.
- 435 Peacor, D.R., Dunn, P.J., and Simmons, W.B. (1983) Ferrostrunzite, the ferrous analogue of
- 436 strunzite from Mullica Hill, New Jersey. Neues Jahrbuch fur Mineralogie-Monatshefte, 524-
- 437 528.

- 438 Stein, D.L. (1989) Spin glasses. Scientific American, 261 (July), 36-42.
- 439 Van Alboom, A. and De Grave, E. (2005) Mössbauer effect in natural strunzite, ferristrunzite
- 440 and ferrostrunzite. Hyperfine Interactions 166, 679-685.
- 441 Vandenberghe, R.E., De Grave, E., and de Bakker, P.M.A. (1994) On the methodology of
- 442 Mössbauer spectra. Hyperfine Interactions 83, 29-49.
- 443 Van Tassel, R. and De Grave, E. (1992) Ferrostrunzite from Arnsberg, Sauerland, Germany.
- 444 Neues Jahrbuch fur Mineralogie-Monatshefte, 207-212.
- 445 Vochten, R. and De Grave, E. (1990) Mössbauer and infrared spectroscopic characterization
- 446 of ferristrunzite from Blaton, Belgium. Neues Jahrbuch fur Mineralogie-Monatshefte, 176-
- 447 190.
- 448 Vochten, R., De Grave, E., Van Springel, K., and Van Haverbeke, L. (1995) Mineralogical
- and Mössbauer spectroscopic study of some strunzite varieties of the Silbergrube, Waidhaus,
- 450 Oberpfalz, Germany. Neues Jahrbuch fur Mineralogie-Monatshefte, 11-25.
- 451

452 **Figure captions**

- 453 Figure 1. Part of the crystallographic structure of strunzite viewed along the c axis (adapted
- 454 from Fanfani 1978).
- 455 Figure 2. Mössbauer thermoscanning curves of the three strunzite varieties.
- 456 Figure 3a. Experimental and calculated MS for ferristrunzite at 4.2 K. S1: blue; S2: purple;
- 457 S3: magenta; calculated MS: red. Parameters are in Table I.
- 458 Figure 3b. Experimental and calculated MS for ferristrunzite (left) and corresponding
- 459 probability histograms for the hyperfine fields (right) at selected temperatures. S1: blue; S2:
- 460 purple; S3: magenta; D1: orange; calculated MS: red. Parameters are in Table I.
- 461 Figure 4. Experimental and calculated MS for strunzite at selected temperatures. S1: blue; S2:
- 462 purple; S3: magenta; D1: orange; D2: pink; calculated MS: red. Parameters are in Table II.
- 463 Figure 5a. Experimental and calculated MS for ferrostrunzite at 5 K and 10 K. S1: blue; S2:
- 464 purple; S3: magenta; S4: orange; S5: green; calculated MS: red. Parameters are in Table III.
- 465 Figure 5b. Experimental and calculated MS for ferrostrunzite (left) and corresponding
- 466 probability histograms for the hyperfine fields (right) at 22 K and at 28 K. S1: blue; S2:
- 467 purple; S4: orange; S5: green; S6: pink; calculated MS: red. Values of the relevant Mössbauer
- 468 parameters are listed in Table III.
- 469 Figure 6. *T*-dependence of the hyperfine fields (most probable values) in ferristrunzite,
- 470 strunzite and ferrostrunzite. Dotted lines are calculated Brillouin curves. Full lines are
- 471 calculated accounting for exchange magnetostriction (see text) (Bean and Rodbell 1962). S1:
- 472 blue; S2: purple; S3: magenta; S6: pink.
- 473

474 TABLES

Table I. Mössbauer parameters for ferristrunzite below T_N at selected temperatures. See text for the meaning of the symbols. Except at *T*=4.2 K (errors in the last digit), estimated errors are: 0.5 K on *T*, 0.01 mm/s on δ , 0.05 mm/s on $2\varepsilon_Q$, 0.03 mm/s on ΔE_Q , 0.02 mm/s on Γ , 0.5 T on B_{max} (B_{hf} with maximum probability), 0.5% on RA. At 4.2 K the given widths refer to the outer lines of the sextets. SFP1 (single component fit program), DFP1 (distribution fit program) are both based on analytical expressions for the calculation of the absorption line positions. B_{hfmax} , #div: underlimit, upperlimit and number of divisions of the distribution range of B_{hf} for use in DFP1.

482

Т (К)	Progr	Comp	B _{hfmin} (T)	B _{hfmax} (T)	#div	B _{max} (T)	δ (mm/s)	$2\epsilon_Q$ (mm/s)	$\begin{array}{c} \Delta E_Q \\ (\text{mm/s}) \end{array}$	Γ (mm/s)	RA (%)
		S1				48.4(1)	0.508(5)	-0.63(2)		0.48(1)	35
4.2	SFP1	S2				47.8(1)	0.508(5)	0.11(2)		0.42(1)	35
		S 3				54.8(1)	0.498(5)	-0.22(2)		0.35(1)	30
		S1	35.0	45.5	23	41.7	0.51	-0.48		0.40	24
17	DED1	S2	35.0	45.5	23	41.8	0.51	0.18		0.40	24
1/	DFPI	S 3	10.0	35.0	26	31.2	0.51	0.14		0.40	38
		D1					0.51		0.71	0.66	14
21 D		S1	30.0	45.0	31	40.1	0.51	-0.46		0.37	22.5
21	DED1	S2	30.0	45.0	31	40.4	0.51	0.16		0.37	22.5
21	DFFI	S 3	10.0	35.0	26	24.4	0.51	0.18		0.37	37
		D1					0.51		0.74	0.58	18
		S1	20.0	40.0	21	33.8	0.51	-0.46		0.35	16
20	DED1	S2	20.0	40.0	21	34.7	0.51	0.32		0.35	16
29	DFFI	S3	3.0	31.0	29	14.3	0.51	0.12		0.35	32
		D1					0.51		0.75	0.53	36
		S1	20.0	40.0	21	30.9	0.52	-0.42		0.31	14.5
22	DED1	S2	20.0	40.0	21	31.6	0.52	0.32		0.31	14.5
33	DFFI	S 3	3.0	31.0	29	12.3	0.52	0.17		0.31	29
		D1					0.52		0.76	0.39	42

483 484 1/16

This is a preprint, the final version is subject to change, of the American Mineralogist (MSA) Cite as Authors (Year) Title. American Mineralogist, in press. (DOI will not work until issue is live.) DOI: http://dx.doi.org/10.2138/am.2014.4367

485	Table II. Mössbauer parameters for strunzite below T_N . See text for the meaning of the symbols. Typical errors
486	are: 0.5 K on T, 0.005 mm/s on δ , 0.005 mm/s on ΔE_Q and $2\varepsilon_Q$, 0.005 mm/s on Γ , 0.5 T on B_{max} (B_{hf} with
487	maximum probability), 0.1 on x, 0.5% on RA. SFP1 (single component fit program), DFP1 (distribution fit
488	program) are both based on analytical expressions for the calculation of the absorption line positions. B_{hfmin}
489	B_{hfmax} , #div: underlimit, upperlimit and number of divisions of the distribution range of B_{hf} for use in DFP1. For
490	the discrete sextets, given line widths are for the outer lines. x is the line area ratio of the second (and fifth) line
491	relative to the third (and fourth) line in a sextet. The line area ratio of the first (and sixth) line to the third (and
492	the fourth) line of a sextet was always fixed at 3.

493

Т (К)	Progr	Comp	$egin{array}{c} B_{h fmin} \ ({ m T}) \end{array}$	B _{hfmax} (T)	#div	B _{max} (T)	δ (mm/s)	$2\epsilon_Q$ (mm/s)	ΔE_Q (mm/s)	Γ (mm/s)	X	RA (%)
4.2	SFP1	S1				48.6	0.518	-0.425		0.436	2.2	43.3
		S2				48.3	0.518	0.078		0.436	2.2	43.3
		S3				54.2	0.501	0.177		0.490	2.2	10.8
		D2					0.316		0.330	0.802		2.6
		S1				47.2	0.519	-0.354		0.393	2.1	43.5
10	SED1	S2				47.0	0.519	0.126		0.393	2.1	43.5
	SFPI	S3				44.1	0.519	0.068		0.687	2.1	10.9
		D2					0.316		0.330	0.540		2.1
		S1				45.8	0.515	-0.240		0.381	2.0	42.8
15	DED1	S2				45.8	0.515	0.185		0.381	2.0	42.8
15	DFP1	S3	30.0	45.0	31	37.5	0.515	0.413		0.236	2.0	12.0
		D2					0.316		0.330	0.510		2.4
		S1				44.6	0.507	-0.115		0.386	1.7	40.9
20	DED1	S2				44.6	0.507	0.190		0.386	1.7	40.9
20	DFF1	S3	5.0	35.0	61	22.0	0.507	0.371		0.449	1.7	16.6
		D2					0.316		0.257	0.500		1.6
	DFP1	S1				43.3	0.510	-0.112		0.384	1.7	41.2
25		S2				43.1	0.510	0.197		0.384	1.7	41.2
		S3	0.0	25.0	51	16.7	0.510	0.449		0.250	1.7	17.6
		S1				41.6	0.510	-0.107		0.376	1.7	40.9
30	DFP1	S2				41.5	0.510	0.190		0.376	1.7	40.9
		S3	0.0	25.0	51	13.5	0.510	0.446		0.239	1.7	18.2
	DFP1	S1				39.1	0.506	-0.100		0.381	1.7	40.0
35		S2				38.9	0.506	0.206		0.381	1.7	40.0
		S3	0.0	25.0	51	11.4	0.506	0.376		0.256	1.7	20.0
	DFP1	S1				35.3	0.508	-0.110		0.450	1.7	40.8
40		S2				35.2	0.508	0.190		0.450	1.7	40.8
		S3	0.0	20.0	41	9.2	0.508	0.643		0.260	1.7	18.4
		S1	25.0	35.0	21	31.8	0.508	-0.117		0.231	1.8	39.8
43	DFP1	S2	25.0	35.0	21	31.5	0.508	0.223		0.231	1.8	39.8
		S3	0.0	12.0	25	6.9	0.508	0.550		0.231	1.8	20.4
		S1	15.0	28.0	14	24.5	0.518	-0.100		0.286	1.8	35.1
47	DED1	S2	15.0	28.0	14	24.2	0.518	0.229		0.286	1.8	35.1
4/	DFPI	S3	0.0	10.0	21	5.1	0.518	0.452		0.286	1.8	19.5
		D1					0.518		0.749	0.396		10.3

494 495

- 496 Table III. Mössbauer parameters for strunzite below T_N . See text for the meaning of the symbols. Typical errors
- 497 are: 0.5 K on T, 0.01 mm/s on δ , 0.01 mm/s on ΔE_Q and $2\varepsilon_Q$, 0.10 on η , 0.01 mm/s on Γ , 5° on Ω , 0.5 T on B_{max}
- 498 (*B_{hf}* with maximum probability), 0.5% on RA. SFP2 (single component fit program) and DFP2 (distribution fit
- 499 program) are both based on the diagonalization of the full nuclear interaction hamiltonian. B_{hfmax} , #div:
- 500 underlimit, upperlimit and number of divisions of the distribution range of B_{hf} for use in DFP2.
- 501

Т (К)	Progr	Comp	B _{hfmin} (T)	B _{hfmax} (T)	#div	<i>B_{max}</i> (T)	δ (mm/s)	$2\epsilon_Q$ (mm/s)	ΔE_Q (mm/s)	η	Ω (°)	Г (mm/s)	<i>RA</i> (%)
5	SFP2	S1				48.2	0.51	-0.30				0.54	35.8
		S2				47.9	0.51	0.10				0.54	35.8
		S 3				55.3	0.51	0.16				0.30	2.7
		S4				21.8	1.37		3.04	0.30	80	1.02	10.9
		S 5				28.6	1.37		2.52	0.30	80	1.02	14.8
		S1				47.4	0.51	-0.34				0.56	35.5
10		S2				47.2	0.51	0.04				0.56	35.5
	SFP2	S 3				46.7	0.51	0.30				0.25	2.7
		S4				16.8	1.37		3.04	0.08	68	0.90	11.1
		S5				22.7	1.37		2.52	0.08	80	0.90	15.2
	DFP2	S1	35.0	45.5	21	44.4	0.52	-0.17				0.39	33.3
		S2	35.0	45.5	21	44.0	0.52	0.06				0.39	33.3
22		S4				8.7	1.37		3.04	0.09	76	0.59	12.1
		S5				9.0	1.37		2.52	0.09	85	0.59	14.2
		S 6	0.0	10.0	20	7.1	0.52	0.76				0.39	7.1
		S1	30.0	43.0	26	41.4	0.51	-0.19				0.33	31.8
		S2	30.0	43.0	26	41.0	0.51	0.16				0.33	31.8
28	DFP2	S4				6.5	1.37		3.04	0.01	79	0.44	10.2
		S5				5.9	1.37		2.52	0.01	80	0.44	16.5
		S 6	0.0	10.0	20	3.1	0.51	0.76				0.33	9.7

502 503

504

REVISION 1 FIGURES



Figure 1.



This is a preprint, the final version is subject to change, of the American Mineralogist (MSA) Cite as Authors (Year) Title. American Mineralogist, in press. (DOI will not work until issue is live.) DOI: http://dx.doi.org/10.2138/am.2014.4367

1/16



Always consult and cite the final, published document. See http://www.minsocam.org or GeoscienceWorld

This is a preprint, the final version is subject to change, of the American Mineralogist (MSA) Cite as Authors (Year) Title. American Mineralogist, in press. (DOI will not work until issue is live.) DOI: http://dx.doi.org/10.2138/am.2014.4367



1/16



1/16



Figure 6.